Finite dimensional spin glass in magnetic field

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Edwards-Anderson model

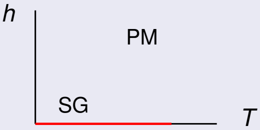
$$\mathcal{H} = -\sum_{\langle ij \rangle} \sigma_i J_{ij} \sigma_j - h \sum_i \sigma_i$$

- Bimodal couplings $J_{ij} = \pm 1$, Ising spins, $\sigma_i = \pm 1$, external magnetic field, $h > 0$

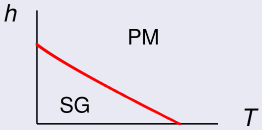
Several conflicting theoretical pictures

- There exist several theoretical pictures for a spin glass in the presence of an external magnetic field, $h \neq 0$.

- Droplet:
- Any infinitesimal field destroys the transition.



- RSB (Replica Symmetry Breaking):
- The transition remains, and the so-called de Almeida-Thouless line exists.



Introduction

Edwards-Anderson model

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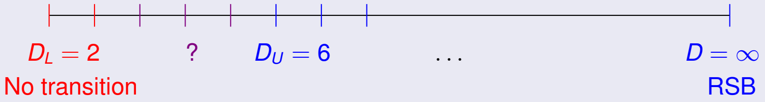
Several conflicting theoretical pictures

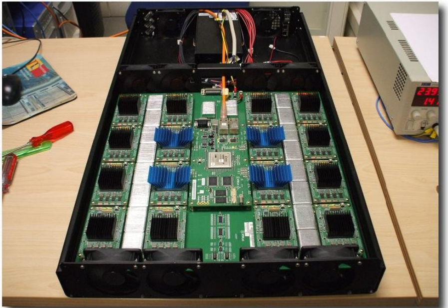
- Which of them best describes the behaviour?



- Does the phase transition survive below the upper critical dimension, $D_U = 6$?

- Numerical work is needed to make these theories quantitative and determine which one best describes the SG phase.

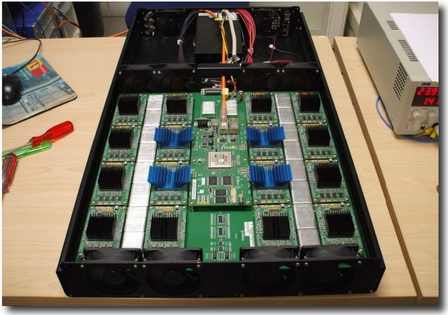
Our simulations



Our implementation

- $L < 16$ systems on conventional PC's with multi spin coding technique.
- $L = 16$ system on Janus.
- Simulation times
  - Janus: 72 days, 52 processes.
- Offline analysis of the stored configurations

Our simulations



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Our parameters

- SG transition below $T_c\,(h = 0) = 2.03(3)$ (E. Marinari & F. Zuliani, 1999).
- $L < 16$ systems $\rightarrow$ 25600 samples ($T_{\rm min} = 0.833$).
- $L = 16$ system $\rightarrow$ 4000 samples ($T_{\rm min} = 1.304$).
- Magnetic field values: $h = 0.15, 0.3$
- Sample-dependent thermalization protocol

Correlation functions: definition

- We consider the two point correlation functions, defined as

$$C^{(1)}(\boldsymbol{r}) = \overline{\sum_{\boldsymbol{x}} \left(\langle \sigma_{\boldsymbol{x}} \sigma_{\boldsymbol{r}} \rangle - \langle \sigma_{\boldsymbol{x}} \rangle \langle \sigma_{\boldsymbol{x}+r} \rangle \right)^2}$$

$$C^{(2)}(\boldsymbol{r}) = \overline{\sum_{\boldsymbol{x}} \left(\langle \sigma_{\boldsymbol{x}} \sigma_{\boldsymbol{r}} \rangle^2 - \langle \sigma_{\boldsymbol{x}} \rangle^2 \langle \sigma_{\boldsymbol{x}+r} \rangle^2 \right)}$$

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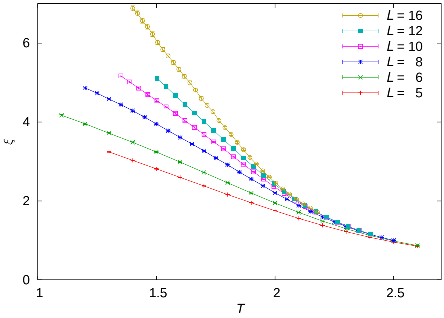
- In the limits $T > T_c$, $L \to \infty$ and $k \ll 0$, the Fourier transform behaves as

$$\hat{C}(k)^{-1} \sim \xi^{-2} + 4 \sum_{\mu} \sin^2(k_\mu/2) + \dots$$

leading to the definition of the correlation length and the spin glass susceptibility

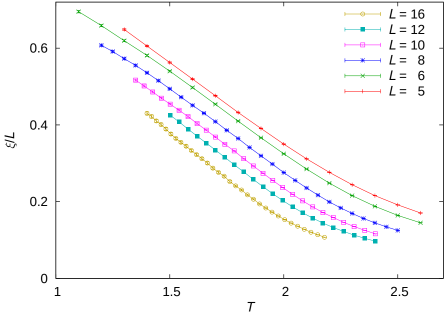
$$\xi_L = \frac{1}{2\sin(\pi/L)} \sqrt{\frac{\hat{C}(0)}{\hat{C}(2\pi/L)} - 1}, \qquad \chi_{SG} = \hat{C}(0)$$

Correlation length (I)



- Correlation length for our different system sizes and $h = 0.15$
- Growing behaviour at low temperatures

Correlation length (I)



- The correlation length should scale as

$$\xi/L = f_\xi\left(L^{1/\nu}t\right)$$

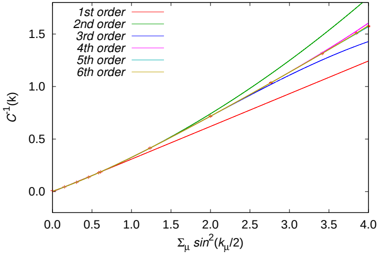
where $t = (T - T_c)/T_c$.

- ξ/L from different systems must cross at T_c (up to scaling corrections).

- Similar results lead other researchers to conclude there is no phase transition
- But that is a hasty conclusion

Anomalous behaviour of the lowest momentum propagator

- ξ/L depends on $\hat{C}(0)$
- $\hat{C}^{-1}(k)$ does not extrapolate smoothly to $\hat{C}^{-1}(0)$



- Lagrange interpolating polynomial in $\not{k}^2 = \sum_\mu \sin^2(k_\mu/2)$

$$C^{-1}\left(\not{k}\right) = a_0^{(n)} + a_2^{(n)}\left(\not{k}^2\right) + \cdots + a_{2n}^{(n-1)}\left(\not{k}^2\right)^{n-1}$$

- extrapolation to $\not{k} = 0$

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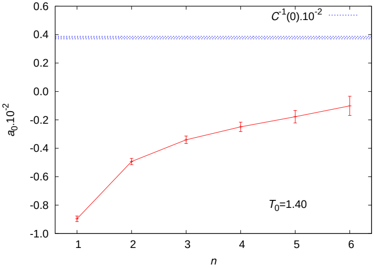
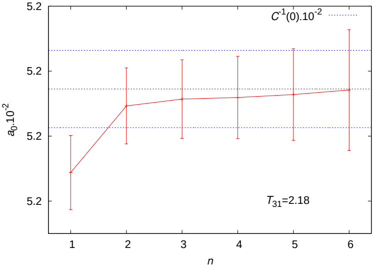
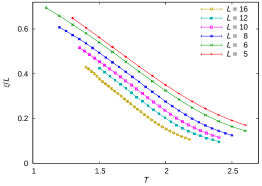
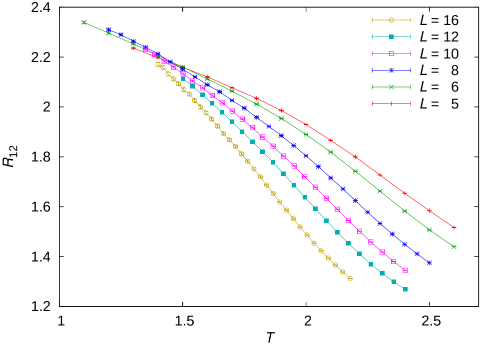






$$R_{12} = \frac{\hat{C}\,(2\pi/L, 0, 0, 0)}{\hat{C}\,(2\pi/L, 2\pi/L, 0, 0)}$$

Universal scaling invariant

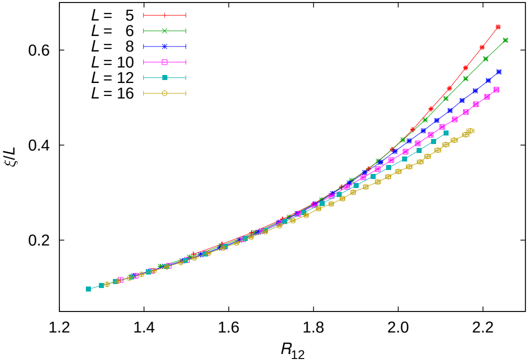


- Scaling behaviour
  $R_{12} = f_R\left(L^{1/\nu} t\right)$
- Curves from different sizes cross at T_c^*
- Thermalization is extremely difficult as h increases
- For low h results may be influenced by the zero field transition

- We focus on fixed values of R_{12} instead of its crossing points

The anomalous behaviour of $\hat{C}(0)$ as scaling corrections

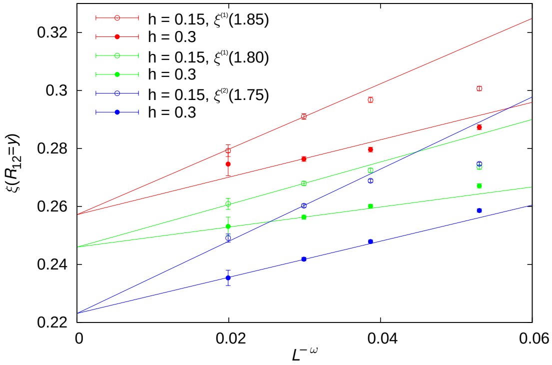
- We wonder why R_{12} shows crossings while ξ/L does not $\rightarrow$ corrections to scaling?
- Qualitative determination of how important they are $\rightarrow$ ξ/L vs. R_{12}



- Curves merge at low R_{12} (high T)
- They differ at high R_{12} (low T)
- Quantitative determination of scaling corrections is possible

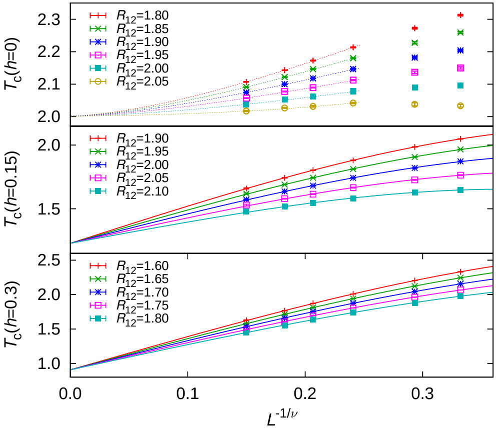
- We plot ξ/L vs. $L^{-\omega}$ at high fixed $R_{12} = y$ (low temperatures)
- Data from $h = 0.3, 0.15$, $C^{(1)}$, and $C^{(2)}$ (two largest lattices) $\rightarrow$ fit to

$$A(y, C) + B(y, C, h)\, L^{-\omega}$$



- Joint fit for $h = 0.15, 0.3$, all $R_{12} = y$ and both correlation functions
- $\omega = 1.43(37)$
- $\chi^2/\mathrm{d.o.f.} = 9.2/11$

$$T_c = T_c^*(h) + A(h, R_{12})\, L^{1/\nu}\left(1 + B(h, R_{12})\, L^{-\omega}\right)$$



- Joint fit of data from $h = 0.15, 0.3$
- Linear behaviour as $L \to \infty$ for $h > 0$
- Non-linear behaviour for $h = 0$

$$\nu = 1.46(7)[6]$$

$$\chi^2/\text{d.o.f} = 40.2/37$$

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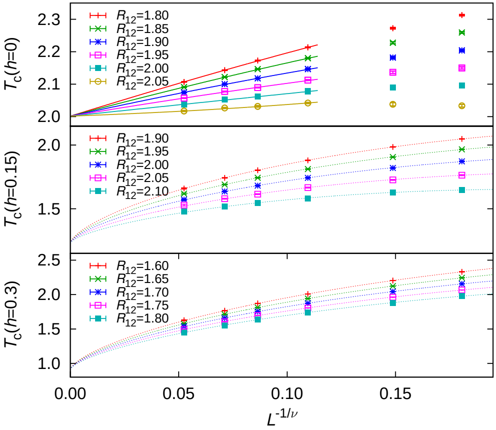

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$$\nu = 0.96(8)$$

$$\chi^2/\text{d.o.f} = 9.8/10$$

Conclusions

- Strong finite size effects on ξ/L do not allow us to see the phase transition
- R_{12} does show a critical behaviour (crossing points)
- The determination of ω allows us to include corrections to scaling
- We have been able to determine the critical exponent ν and the critical temperatures $T_c\,(h)$
- ν clearly differs from $\nu\,(h=0) \rightarrow$ they belong to different universality classes

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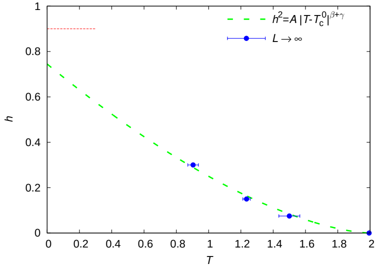

- We have been able to draw a sketch of the dAT line
- Infinite volume extrapolation for $h = 0.075, 0.15, 0.3$
- Fisher & Sompolinsky fit (1985)
- β and γ taken from literature