Domain walls and Schramm Loewner evolution in the Random Field Ising Model

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Outline

Motivation

- conformal field theory (CFT) allows for a complete classification of (pure) critical systems in two dimensions
- disordered systems, however, are not translationally (and thus conformally) invariant $\rightarrow$ no CFT results
- Schramm Loewner evolution (SLE) describes critical curves such as domain boundaries, with implications that might go beyond CFT
- first observations of consistency with SLE in random systems:
  - 2D $\pm J$ Ising spin glass (Amoruso et al., 2006; Bernard et al., 2007)
  - 3-state random-bond Potts model (Jacobsen et al., 2009)
  - disordered SOS model (Schwarz et al., 2009)

Outline

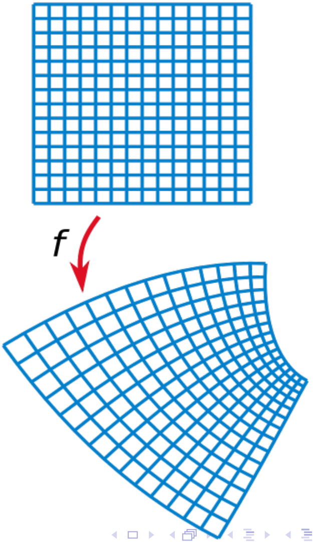
Conformal invariance

Conformal map

a function $f : U \to V$ which preserves angles between curves

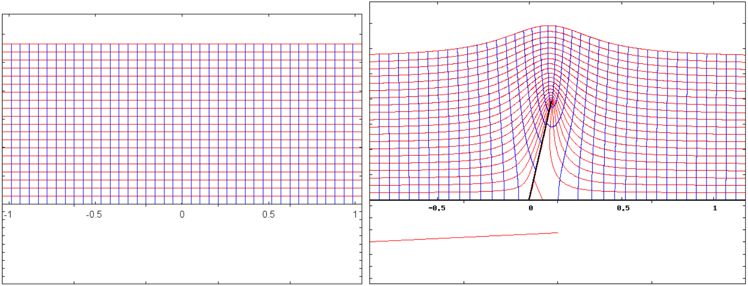
Riemann mapping theorem

in 2 dimensions there exists a conformal map between any two simply connected domains



Loewner evolution

- Relation between a 1D function and a curve in 2D via conformal mapping
- Curve is "extruded" into the upper half plane from the origin



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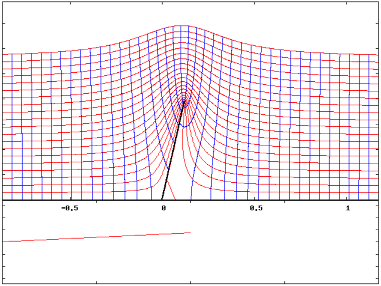

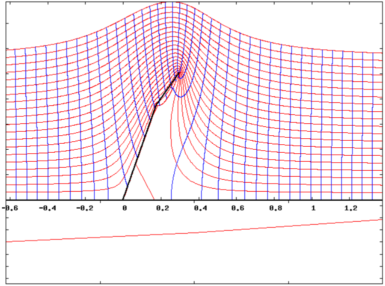

animation

Schramm (stochastic) Loewner evolution

Loewner evolution

ξ_t: one dimensional function $\longleftrightarrow$ γ: Curve in the plane

Schramm (stochastic) Loewner evolution

Loewner evolution

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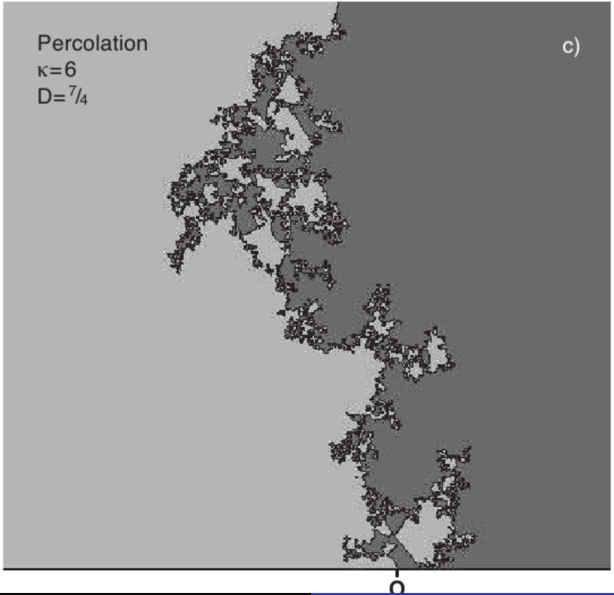
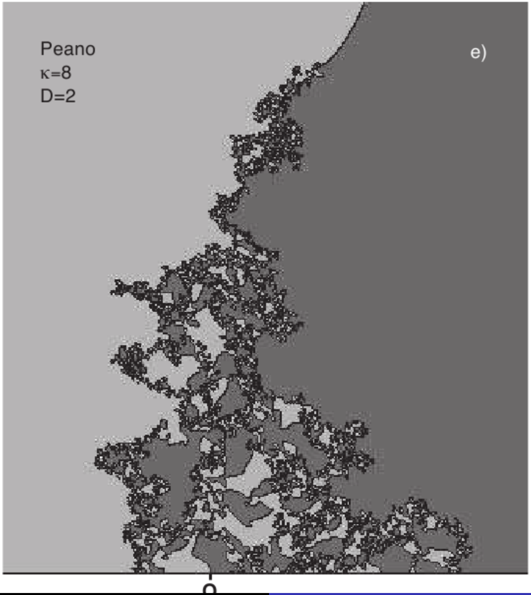
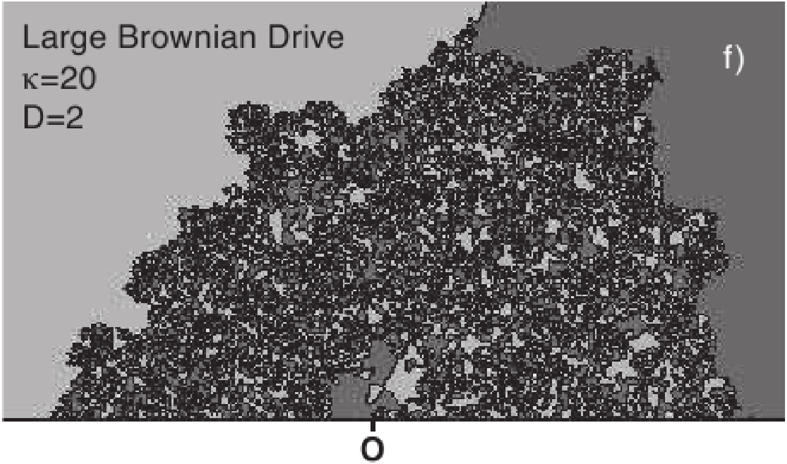
Schramm (stochastic) Loewner evolution

ξ_t is a Brownian motion with diffusion constant κ	$\longleftrightarrow$	The generated family of curves γ is conformally invariant and satisfies the domain Markov property


LERW
κ=2
D=5/4
a)


Percolation
κ=6
D=7/4
c)
0


Peano
κ=8
D=2
e)
0


Large Brownian Drive
κ=20
D=2
f)
0

Outline

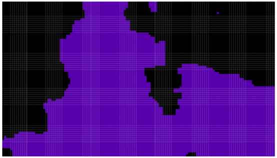
Random field Ising model

$$H = -J\sum_{ij} s_i s_j - \sum_i h_i s_i$$

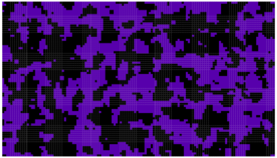
RFIM

- h_i: Gaussian quenched disorder
- Δ: disorder strength
- H: mean (external) field
- spin clusters grow with decreasing Δ and increasing H
- no thermodynamic transition in 2D

$\Delta = 1$

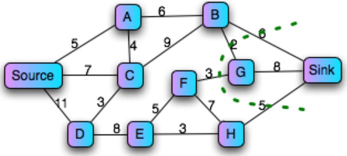


$\Delta = 2$



Ground-state computations: use graph theory

Maximum flow problem: find the minimum cut (bottlenecks)



Each link has maximum capacity c_{ij}

Statistical mechanics solution: minimize an "energy" function

$H = \sum_{ij} s_i(1 - s_j)c_{ij} \qquad s_i \in \{0, 1\}$

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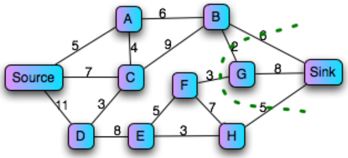

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$H = -\sum_{ij} c_{ij} s_i s_j + \sum_i \left(\sum_j c_{ij}\right) s_i$

exactly analogous to the RFIM Hamiltonian

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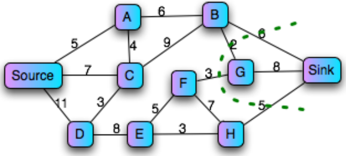

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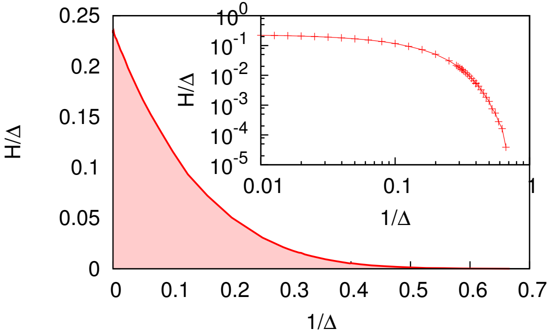
Graph theory algorithm

- polynomial in time
- no ambiguity about solution

Zero temperature phase diagram



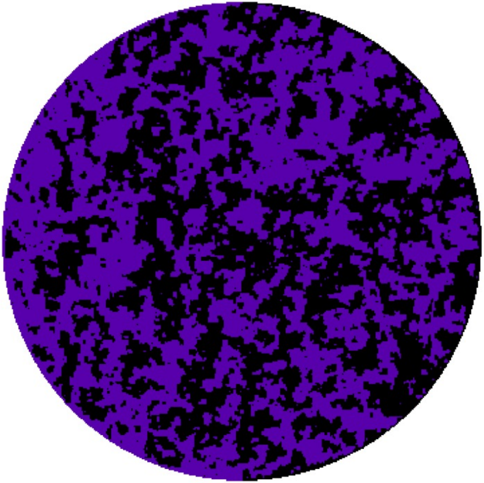
Geometric critical behavior

- $H < H_c(\Delta)$: spin clusters are finite
- $H > H_c(\Delta)$: spin clusters diverge

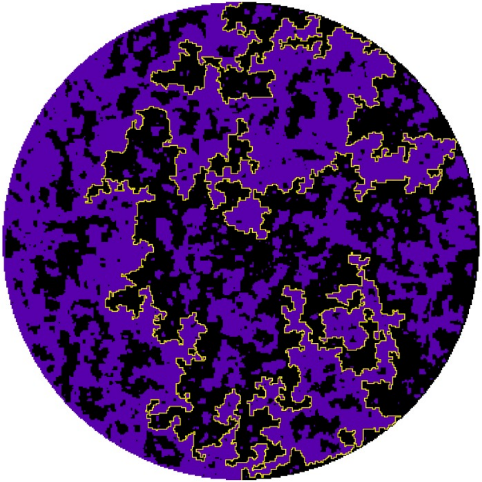
(Seppälä and Alava, PRE 2001)

(Környei and Iglói, PRE 2007)

Ground state spin configuration



Spin cluster boundary



Outline

SLE numerical evidence

SLE preconditions

1. conformal invariance
2. domain Markov property

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   - transformation properties between different domains

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SLE predictions

1. fractal dimension
2. left passage probability
3. is the driving function Brownian motion?

fractal dimension

SLE$_\kappa$ prediction: $d_f = 1 + \kappa/8$

fractal dimension

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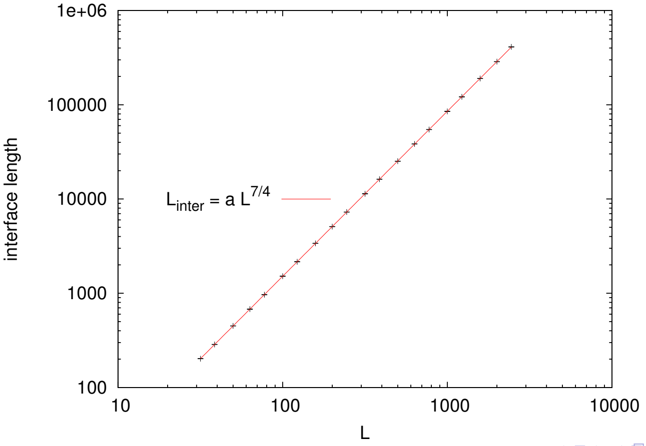


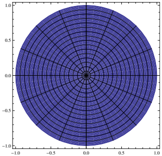
satisfied with $\kappa = 6$

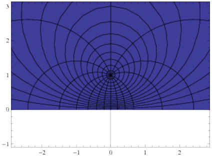
Schramm's left passage probability

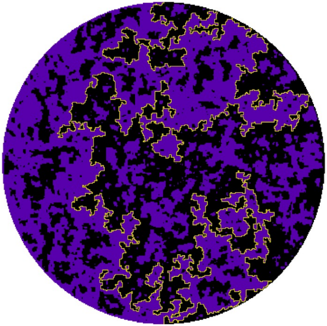
The probability a curve passes to the left of point (x, y)

- The probability is known exactly for SLE curves in the upper half plane
- Strategy: do simulations on a finite geometry then use conformal map to test against the exact formula
- Bonus: acts as a check of conformal invariance

circle geometry

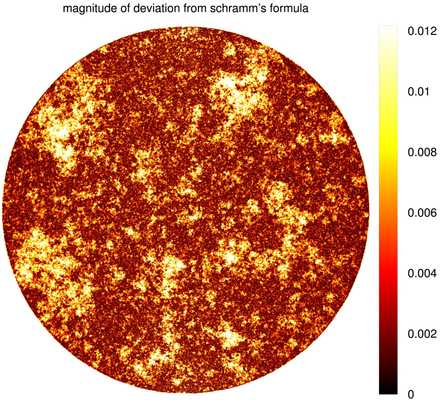




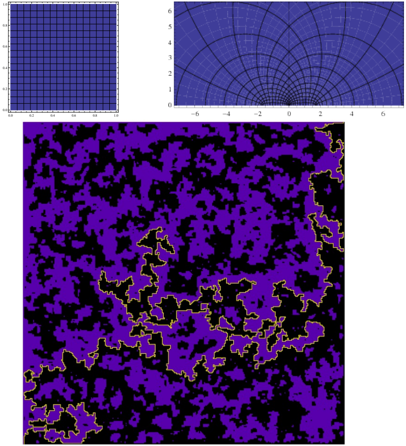


Deviation from exact formula

$\kappa = 6$



square geometry



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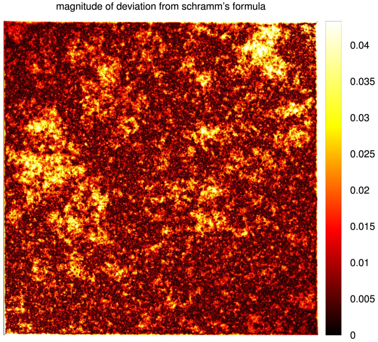
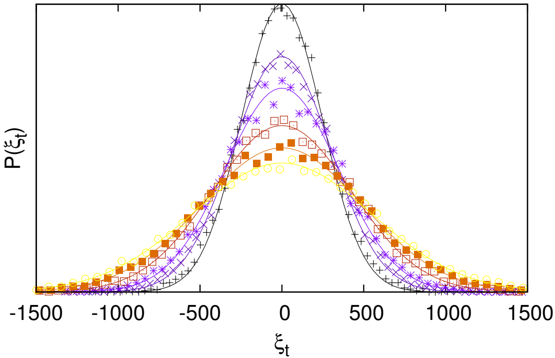


Is the generating function Brownian motion?

- points give the distribution of the driving function after "time" t
- solid lines are Gaussian curves with variance $\kappa * t$ with $\kappa = 6$



Conclusions

Domain walls in the RFIM satisfy Schramm Loewner evolution with $\kappa = 6$

evidence

- ✓ fractal dimension
- ✓ left passage probability
- ✓ is the driving function Brownian motion?

implications

1. conformal invariance
2. other disordered systems?

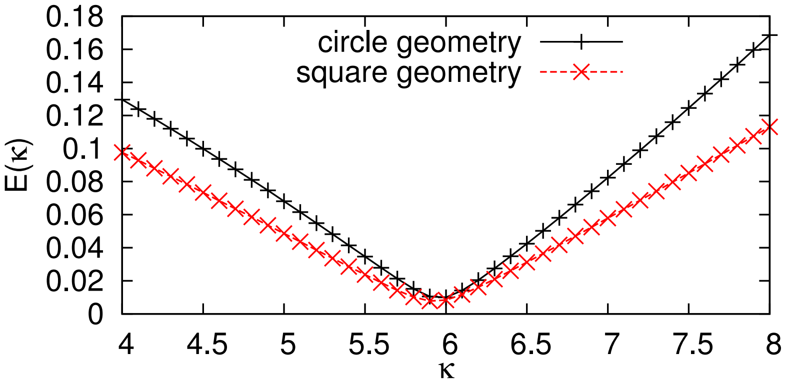
Definition: domain Markov property

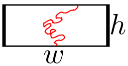
Domain Markov property

Given a domain D, boundary points a and c, and b a point on the interior of the domain, then the probability of curve γ_{bc} *given* curve γ_{ab} is the same as the probability of γ_{bc} on a domain excluding the curve γ_{ab}

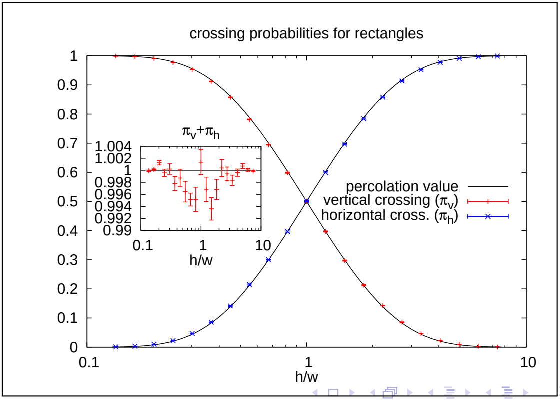
$$P(\gamma_{bc} \in D|\gamma_{ab}) = P(\gamma_{bc} \in (D \setminus \gamma_{bc}))$$

deviation from exact left passage as a function of κ



Prob $\Bigg[$ 

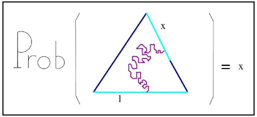
$\Bigg] = f\left(\frac{h}{w}\right)$



inset

symmetry relation

$\pi_v + \pi_h = 1$



inset

deviation from "exact"