

Ising spin-glass transition in magnetic field out of mean-field: Numerical simulations and experiments

J. J. Ruiz-Lorenzo

with L. Leuzzi, G. Parisi and F. Ricci-Tersenghi (Rome).

Dep. Física, Universidad de Extremadura & BIFI
<http://www.unex.es/eweb/fisteor/juan>

Leipzig, November 26th, 2010

Phys. Rev. Lett. 103, 267201 (2009) and arXiv:1006.3450v1.

Outline of the Talk

- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
- The one dimensional diluted spin glass with long range interactions.

Outline of the Talk

- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
- The one dimensional diluted spin glass with long range interactions.
 - Observables and Numerical Simulations.

Outline of the Talk

- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
- The one dimensional diluted spin glass with long range interactions.
 - Observables and Numerical Simulations.
 - Numerical Analysis of the Spin Glass correlation functions.

Outline of the Talk

- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
- The one dimensional diluted spin glass with long range interactions.
 - Observables and Numerical Simulations.
 - Numerical Analysis of the Spin Glass correlation functions.
 - Evidences for a spin glass phase in magnetic field out of the MF region.

Outline of the Talk

- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
- The one dimensional diluted spin glass with long range interactions.
 - Observables and Numerical Simulations.
 - Numerical Analysis of the Spin Glass correlation functions.
 - Evidences for a spin glass phase in magnetic field out of the MF region.
 - Experiments.

Outline of the Talk

- What are spin glasses?
- Different Theories and Models (droplet, TNT and RSB).
- Spin Glasses with Long Range Interactions.
- The one dimensional diluted spin glass with long range interactions.
 - Observables and Numerical Simulations.
 - Numerical Analysis of the Spin Glass correlation functions.
 - Evidences for a spin glass phase in magnetic field out of the MF region.
 - Experiments.
- Conclusions.

What are Spin glasses

- Materials with disorder and frustration.
- Quenched disorder.
- Canonical Spin Glass: Metallic host (Cu) with magnetic impurities (Mn). RKKY interaction between magnetic moments:

$$J(r) \sim \frac{\cos(2k_F r)}{r^3}.$$

Some Definitions

- The typical Spin Glass Hamiltonian:

$$\mathcal{H} = - \sum_{i,j} J_{ij} \sigma_i \sigma_j$$

- The order parameter is:

$$q_{\text{EA}} = \overline{\langle \sigma_i \rangle^2}$$

Using two real replicas:

$$\mathcal{H} = - \sum_{i,j} J_{ij} (\sigma_i \sigma_j + \tau_i \tau_j)$$

Let $q_i = \sigma_i \tau_i$ be the normal overlap, then: $q_{\text{EA}} = \overline{\langle \sigma_i \tau_i \rangle}$.

We also define the link overlap: $q_{i,\mu}^{\text{link}} = q_i q_{i+\mu}$.

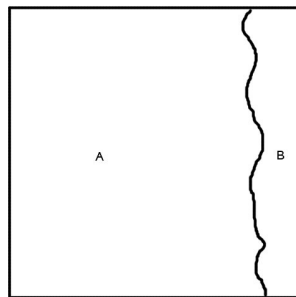
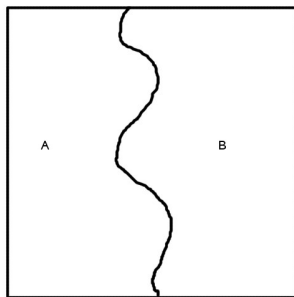
The Droplet Model

- Based on the Migdal-Kadanoff implementation (approximate) of the Renormalization Group (exact in $D = 1$).
- *Disguised Ferromagnet*: Only two pure states with order parameter $\pm q_{\text{EA}}$ (related by spin flip).
- Compact Excitations of fractal dimension d_f . The energy of a excitation of linear size L grows as L^θ .
- Any amount of magnetic field destroys the spin glass phase (even for Heisenberg spin glasses).
- Trivial probability distributions of the overlaps (both the normal overlap and the link one).

Different Theories and Models

The Trivial Non Trivial (TNT) Model

- *Disguised Ferromagnet with Anti Periodic Boundary conditions.*
- Trivial probability distributions for the link overlap (the interface has no effect) but Non Trivial probability distribution for the normal one (induced by the interface).



Replica Symmetry Breaking (RSB) Theory

- Exact in $D = \infty$.
- Infinite number of phases (pure states) not related by any kind of symmetry.
- These (pure) states are organized in a ultrametric fashion.
- The spin glass phase is stable under (small) magnetic field.
- The excitations of the ground state are space filling.

Long Range Interactions

- Hamiltonian (Action) for the long range model ($J(r) \simeq r^{-\rho/2}$):

$$S_n = H_n \propto \int d^d k \left(k^{\rho-d} + \tau \right) \text{Tr} Q^2 + \int d^D x \left[g_3 \text{Tr}(Q^3) + \lambda \sum Q_{ab}^4 \right]$$

- $\dim_{\rho}(g_3) = d - \frac{3}{4}\rho$. In MF: $\eta = d + 2 - \rho$ (holds in IRD!) and $1/\nu = \rho - d$.
- Hence, the Mean Field and Infrared region are ($d = 1$):

ρ	$D(\rho)$	transition type
≤ 1	∞	Bethe lattice like
$(1, 4/3]$	$[6, \infty)$	2 nd order, MF
$(4/3, 2]$	$[2.5, 6)$	2 nd order, non-MF
2	2.5	Kosterlitz-Thouless or $T = 0$ -like
> 2	< 2.5	none

- It is possible to show (equivalence D -SR and $1d$ -LR):

$$\frac{2 - \eta(D)}{D} = \rho - 1 ; \rho = 1.8 \rightarrow D = 3$$

Numerical Simulations

- The spins live on a finite connectivity network ($z = 6$) with periodic boundary conditions: $J_{ij} = 0, \pm 1$ with $P(J_{ij} \neq 0) \propto r_{ij}^{-\rho}$. With this choice one has $\overline{J_{ij}^2} \propto r_{ij}^{-\rho}$.
- We have implemented the Parallel Tempering Method.
- We have used multispin coding (64 bits) on a C++ code.
- We have simulated a Gaussian magnetic field and only two replicas.
- We have run on PC's Clusters.

Some Observables

- The spin glass correlation function:

$$C(x) = \sum_{i=1}^L \overline{(\langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle)^2}$$

Some Observables

- The spin glass correlation function:

$$C(x) = \sum_{i=1}^L \overline{(\langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle)^2}$$

- The associated spin glass correlation length:

$$\xi \equiv \frac{1}{2 \sin(\pi/L)} \left[\frac{\tilde{C}(0)}{\tilde{C}(2\pi/L)} - 1 \right]^{\frac{1}{\rho-1}}$$

Some Observables

- The spin glass correlation function:

$$C(x) = \sum_{i=1}^L \overline{(\langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle)^2}$$

- The associated spin glass correlation length:

$$\xi \equiv \frac{1}{2 \sin(\pi/L)} \left[\frac{\tilde{C}(0)}{\tilde{C}(2\pi/L)} - 1 \right]^{\frac{1}{\rho-1}}$$

- FSSA in the MF regime ($1 < \rho \leq 4/3$):

$$\frac{\chi_{\text{sg}}}{L^{1/3}} = \tilde{\chi} \left(L^{1/3} (T - T_c) \right), \quad \frac{\xi}{L^{\nu/3}} = \tilde{\xi} \left(L^{1/3} (T - T_c) \right)$$

with $\nu = 1/(\rho - 1)$,

Some Observables

- The spin glass correlation function:

$$C(x) = \sum_{i=1}^L \overline{(\langle \sigma_i \sigma_{i+x} \rangle - \langle \sigma_i \rangle \langle \sigma_{i+x} \rangle)^2}$$

- The associated spin glass correlation length:

$$\xi \equiv \frac{1}{2 \sin(\pi/L)} \left[\frac{\tilde{C}(0)}{\tilde{C}(2\pi/L)} - 1 \right]^{\frac{1}{\rho-1}}$$

- FSSA in the MF regime ($1 < \rho \leq 4/3$):

$$\frac{\chi_{\text{sg}}}{L^{1/3}} = \tilde{\chi} \left(L^{1/3} (T - T_c) \right), \quad \frac{\xi}{L^{\nu/3}} = \tilde{\xi} \left(L^{1/3} (T - T_c) \right)$$

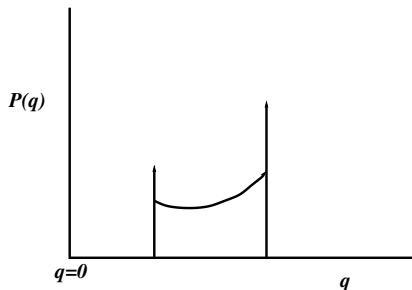
with $\nu = 1/(\rho - 1)$,

- FSSA in the IRD regime ($\rho > 4/3$):

$$\frac{\chi_{\text{sg}}}{L^{2-\eta}} = \tilde{\chi} \left(L^{1/\nu} (T - T_c) \right), \quad \frac{\xi}{L} = \tilde{\xi} \left(L^{1/\nu} (T - T_c) \right).$$

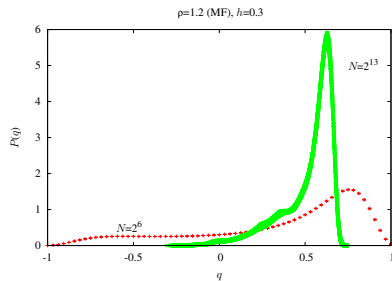
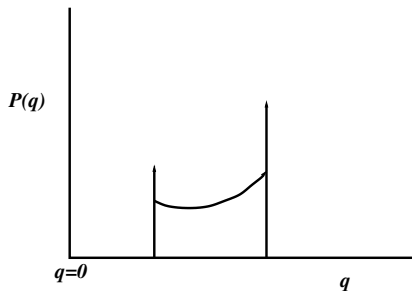
The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.



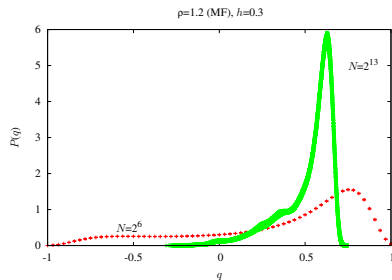
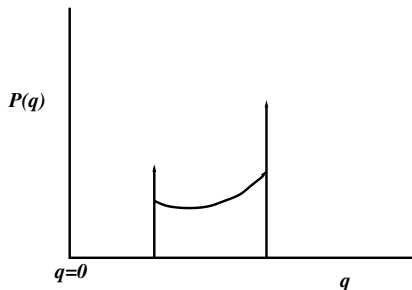
The negative overlap problem

- $P(q)$ in a magnetic field: SK results and numerical ones.



The negative overlap problem

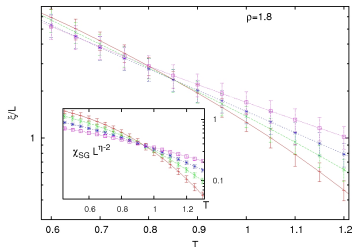
- $P(q)$ in a magnetic field: SK results and numerical ones.



- The negative overlap region induces large corrections in $\tilde{C}(0)$!!

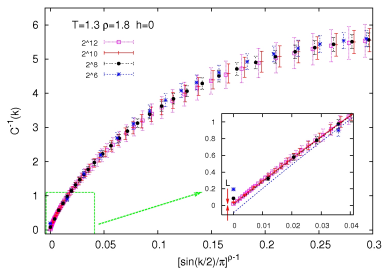
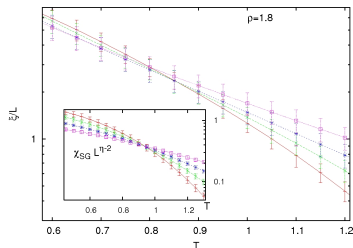
Numerical Analysis of the Correlation function ($h = 0$)

- $h = 0$ and $\rho = 1.8$.



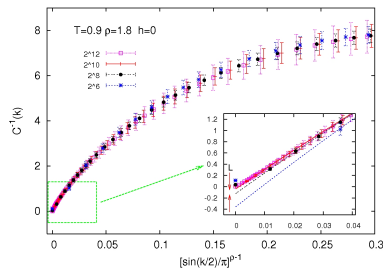
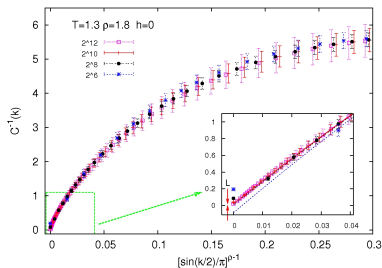
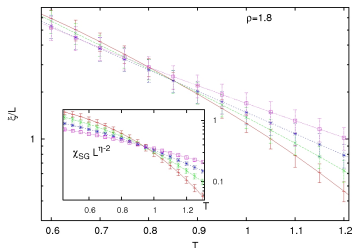
Numerical Analysis of the Correlation function ($h = 0$)

- $h = 0$ and $\rho = 1.8$.



Numerical Analysis of the Correlation function ($h = 0$)

- $h = 0$ and $\rho = 1.8$.



Numerical Analysis of the Correlation function

- We will avoid the $k = 0$ value by fitting ($k > 0$):

$$\left(\frac{1}{\tilde{C}_4(k)} \right)^{\text{fit}} = A(L, T) + B(L, T)[\sin(k/2)/\pi]^{\rho-1}$$

Numerical Analysis of the Correlation function

- We will avoid the $k = 0$ value by fitting ($k > 0$):

$$\left(\frac{1}{\tilde{C}_4(k)} \right)^{\text{fit}} = A(L, T) + B(L, T) [\sin(k/2)/\pi]^{\rho-1}$$

- We can analyze the L and T dependence of

$$A(L, T) \equiv \lim_{k \rightarrow 0} \frac{1}{\tilde{C}_4(k)}$$

Numerical Analysis of the Correlation function

- We will avoid the $k = 0$ value by fitting ($k > 0$):

$$\left(\frac{1}{\tilde{C}_4(k)} \right)^{\text{fit}} = A(L, T) + B(L, T)[\sin(k/2)/\pi]^{\rho-1}$$

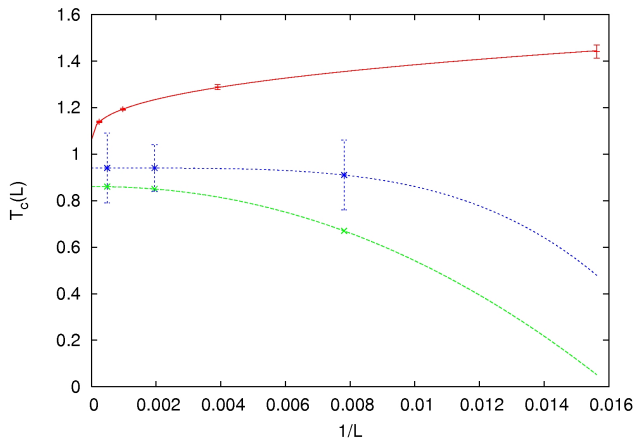
- We can analyze the L and T dependence of

$$A(L, T) \equiv \lim_{k \rightarrow 0} \frac{1}{\tilde{C}_4(k)}$$

- We fix the L -dependent critical temperature by means:

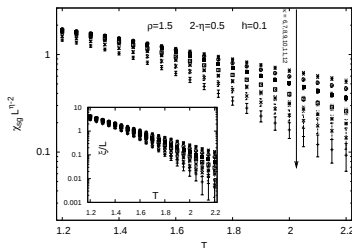
$$A(L, T_c(L)) = 0$$

Numerical Analysis of the Correlation function



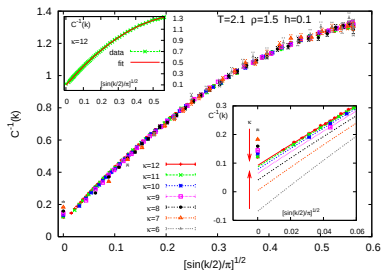
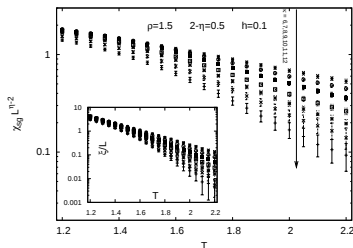
Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$.



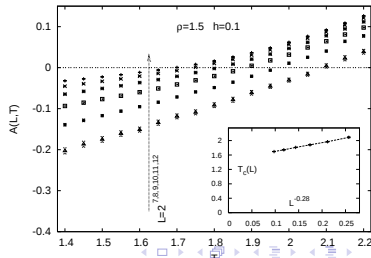
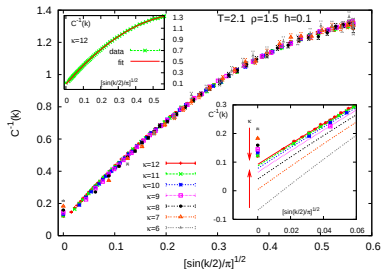
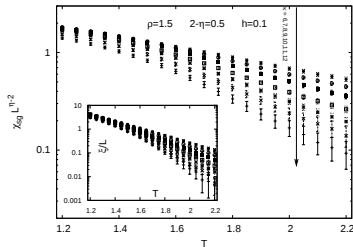
Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$.



Numerical Analysis of the Correlation function ($h \neq 0$)

- $h = 0.1$ and $\rho = 1.5$.

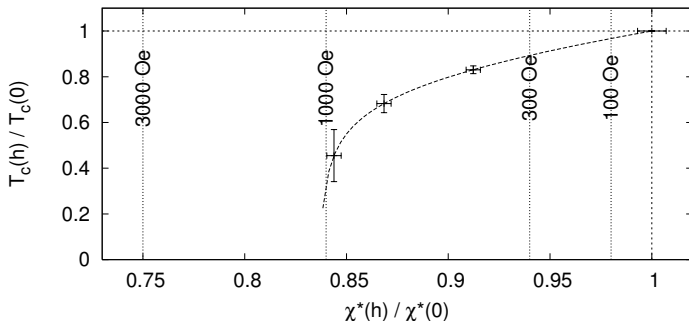


Characterization of the de Almeida-Thouless line

	ρ	" D "	h	T_c from $\tilde{C}(0)$	T_c from $A(L, T)$
MF	1.2	10	0.0	2.24(1)	2.34(3)
	1.2	10	0.1	2.02(2)	1.9(2)
	1.2	10	0.2	1.67(3)	1.4(2)
	1.2	10	0.3	1.46(3)	1.5(4)
	1.25	8	0.0	2.191(5)	2.23(2)
IRD	1.4	5	0.0	1.954(3)	1.970(2)
	1.4	5	0.1	~ 1.5	1.67(7)
	1.4	5	0.2	~ 1.1	1.2(2)
	1.5	4	0.0	1.758(4)	1.770(5)
	1.5	4	0.1	—	1.46(3)
	1.5	4	0.15	—	1.20(7)

Experiments

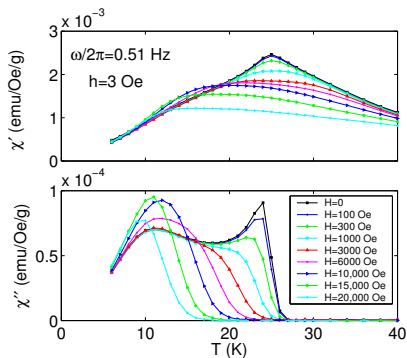
- Relative decrease of $T_c(h)/T_c(0)$ with increase field for $\rho = 1.5$ and $h = 0, 0.1, 0.15$ and 0.2 versus the relative decrease of χ^* (ZFC susceptibility). Experimental data from $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$ (Jönsson et al.).



- Hence, the critical field should be $H_c < 1000$ Oe.

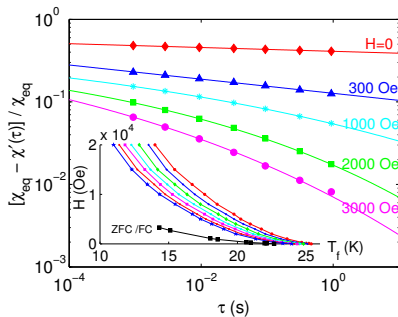
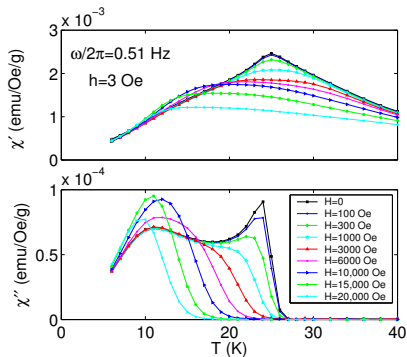
More on Experiments

- Experimental data from $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$ (Jönsson et al.).



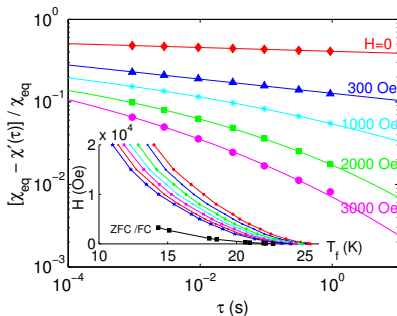
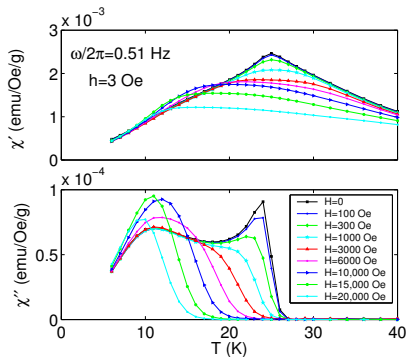
More on Experiments

- Experimental data from $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$ (Jönsson et al.).



More on Experiments

- Experimental data from $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$ (Jönsson et al.).



- $q(t) \simeq 1/t^x$ clear signature of a Spin Glass Phase (Ogileski).

Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.

Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.
- We have found strong evidences for a stable spin glass phase in presence of a magnetic field.

Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.
- We have found strong evidences for a stable spin glass phase in presence of a magnetic field.
- Experimental studies in Heisenberg spin glass find Phase Transition (Campbell et al.).

Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.
- We have found strong evidences for a stable spin glass phase in presence of a magnetic field.
- Experimental studies in Heisenberg spin glass find Phase Transition (Campbell et al.).
- Experimental studies in Ising spin glass find NO Phase Transition (Jönsson et al.) for fields $H > 1000\text{Oe}$.

Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.
- We have found strong evidences for a stable spin glass phase in presence of a magnetic field.
- Experimental studies in Heisenberg spin glass find Phase Transition (Campbell et al.).
- Experimental studies in Ising spin glass find NO Phase Transition (Jönsson et al.) for fields $H > 1000\text{Oe}$.
- We suggest to reanalyze the experimental data for $H < 1000\text{ Oe}$ on $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$.

Conclusions

- We have introduced a new analysis method to bypass the bias which induces the large plateau (at negative overlap) in $\tilde{C}(0)$.
- We have found strong evidences for a stable spin glass phase in presence of a magnetic field.
- Experimental studies in Heisenberg spin glass find Phase Transition (Campbell et al.).
- Experimental studies in Ising spin glass find NO Phase Transition (Jönsson et al.) for fields $H > 1000\text{Oe}$.
- We suggest to reanalyze the experimental data for $H < 1000\text{ Oe}$ on $\text{Fe}_{0.5}\text{Mn}_{0.5}\text{TiO}_3$.
- Recent experiments find spin glass order in a magnetic field for small external fields ($H \simeq 500\text{ Oe}$) in RKKY Spin Glasses.

Some (additional) References:

- H. Katzgraber and A. P. Young, PRB 68, 224408 (2003); 72, 184416.
- H. Katzgraber and A. Hartmann, PRL 102, 037207 (2009).
- L. Leuzzi, G. Parisi, F. Ricci-Tersenghi and J. J. Ruiz-Lorenzo, PRL
- H. Katzgraber, D. Larson and A. P. Young, PRL 102, 177205 (2009).
- D. Petit, L. Fruchter, I.A. Campbell. PRL 88, 207206 (2002).
- P. E. Jönsson, H. Takayama, H. Aruga Katori and A. Ito, PRB 71, 180412(R) (2005).
- Y. Tabata, K. Matsuda, S. Kanada, T. Yamazaki, T. Waki, H. Nakamura, K. Sato and K. Kindo, arXiv:1009.6115v2.