Formulation of Time
from Aristotle to Monte Carlo Simulations and to Noncommutative Geometry

Harald Markum
Technische Universitaet Wien

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Abstract

Aristotle defines time as a quantity which can only be derived from motion. In Classical Mechanics time enters as a parameter into the Newtonian equations of motion. In Quantum Mechanics and in Quantum Field Theory time also is a parameter in the Schroedinger or Dirac equation. In Special Theory of Relativity a time dilatation appears, but time stays as a (relative) parameter. The same is the case in General Theory of Relativity, where for some metrics one finds trajectories into the past. The fundamental equations of Physics are invariant with respect to inversion of time. The Second Law of Thermodynamics violates this symmetry and leads to an arrow of time. In noncommutative Geometry time becomes an operator. In

Monte Carlo simulations, the relation between computer time and real time is an interesting question. Using atomic clocks, time and frequency are the most precisely measurable observables. An overview of time in different fields of physics is given.

Aristotle

Aristotle defines time as a quantity which can only be derived from motion. We measure the duration of a day by watching the reappearance of the Sun.

Classical Newtonian Mechanics

In second Newtons Law the time t enters as a **Parameter**, both in the infinitesimal derivative in the equation of motion

$$m\frac{\mathrm{d}^2\,\mathbf{s}}{\mathrm{d}t^2} = \mathbf{F}$$

and in the trajectory $s(t)$ of a body.

Quantum Mechanics and Quantum Field Theory

Both in the Schroedinger equation and in the Dirac equation the time t

is a **Parameter**:

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\, \psi(\mathbf{x}, t) = -\frac{\hbar^2}{2m} \left(\sum_{k=1}^{3} \frac{\partial^2}{\partial (x^k)^2} + V(\mathbf{x}, t) \right) \psi(\mathbf{x}, t)$$

$$\left(\mathrm{i}\hbar \sum_{\mu=0}^{3} \gamma^\mu \frac{\partial}{\partial x^\mu} - m \right) \psi(x) = 0\,.$$

Uncertainty relation between time t and energy E

$$\Delta E \Delta t \gtrsim h$$

Special Theory of Relativity

The Lorentz transformations relate the time and space coordinates of observers and events moving relatively to ach other. In Lorentz transformations the speed of light c does not change. On the other hand the constance of the speed of light was a prerequisite for Einsteins derivation of the Lorentz transformation.

$$t' = \frac{t - \frac{v}{c^2}\, x}{\sqrt{1 - \frac{v^2}{c^2}}}\;, \quad x' = \frac{x - v\, t}{\sqrt{1 - \frac{v^2}{c^2}}}\;, \quad y' = y\;, \quad z' = z$$

The time t stays the same **Parameter** as in classisal mechanics and in quantum mechanics.

General Theory of Relativity

Similar to Special Theory of Relativity what concerns time as a parameter.

Thermodynamics

In the Second Law of Thermodynamics

$$dS = \frac{\delta Q}{T} + \frac{\delta W_{\mathrm{diss}}}{T}$$

δW_{diss} represents the dissipative work (friction, collisions, ...) within the system. It is always positive in contrast to δQ which can also be negative.

For an adiabatic system with $\delta Q = 0$ we can conclude: In a closed adiabatic system the entropy generally increases. Only for reversible processes it stays constant.

This leads us to the (thermodynamic) Arrow of Time: The future is that direction of time where the entropy increases. It is interesting that the Arrow of Time does not exist im thermodynamic equilibrium: In an equilibrium state there is no thermodynamically defined Past or Future.

Non-commutative Geometry

In non-commutative geometry the coordinate operators $\hat{x}_\mu$ satisfy the commutation relation

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}.$$

This is similar to the commutation relation of space and momentum but now time becomes an operator in addition to ordinary space. As a consequence space and time may become discrete.

Monte Carlo Simulations

Setting the speed of light and the Planck constant to natural units

$$c = 1, \ \hbar = 1 \, ,$$

length and time can be expressed in energy

Energy: 1eV $= 1.60218 \cdot 10^{-19}$ J

Length: $\frac{1}{1\text{eV}} = 1.97327 \cdot 10^{-7}$ m

Time: $\frac{1}{1\text{eV}} = 6.58212 \cdot 10^{-16}$ s .

The uncertainty relation between time t and energy E becomes

$$\Delta E \Delta t \gtrsim 1 \, .$$

For a quantum field theory with a characteristic energy scale of 1 eV (atomic physics, chemistry, biophysics) this means a vacuum fluctuation takes $6.58212 \cdot 10^{-16}$ s.

Expectation values of physical observables $\mathcal{A}$ with some measure $P(x)$in a space Ω

$$\langle \mathcal{A} \rangle = \sum_{x \in \Omega} P(x)\, \mathcal{A}(x)$$

can be estimated with Monte-Carlo simulations and importance sampling over a finite number N of iterations

$$\langle \mathcal{A} \rangle \approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{A}(x_i)\ .$$

Depending on computer power, the update of one element (spin, electromagnetic or gluonic field quantum) may take $1p$s on graphic cards. So MC simulations are 4 - 10 orders of magnitude away from real time. For many systems like spin glasses it would be interesting to simulate the jump from one energy minimum to another and to compare the energy scale with experiment. With 10 000 graphic cards the update time (one sweep, MC time) in MC simulations could be related to the physical time in a box containing some matter/vacuum. Using them with IBERCIVIS (GPU-IBERCIVIS) real physical time Monte Carlo could be possible.

Atomic Clocks

Since 1967, the International System of Units (SI) has defined the Second as the duration of 9192631770 cycles of radiation corresponding to the transition between two energy levels of the caesium-133 atom. This definition makes the caesium oscillator the primary standard for time and frequency measurements, called the caesium standard. Other physical quantities, e.g., the volt and the metre, rely on the definition of the second in their own definitions.

A different branch beside these highly precise clocks follows the construction of cheap, small, light und energy saving clocks, e.g., for satellites and satellite-navigation systems like GPS, GLONASS or Galileo.

Conclusion

Aristotle Classical Newtonian Mechanics

Quantum Mechanics and Quantum Field Theory

Special Theory of Relativity

General Theory of Relativity

Thermodynamics

Non-commutative Geometry

Monte Carlo Simulations

Atomic Clocks