The thermodynamic Casimir effect in the neighbourhood of the λ-transition: A Monte Carlo study

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Overview

- An improved lattice model
- Finite size scaling of the thermodynamic Casimir force
- Numerical results
- Comparison with other MC studies, field theory and experiment

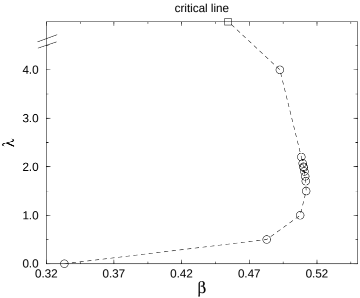
2-component ϕ^4 model:

$$H = -\beta \sum_{x,\mu} \vec{\phi}_x \vec{\phi}_{x+\hat{\mu}} + \sum_x \left[\vec{\phi}_x^2 + \lambda(\vec{\phi}_x^2 - 1)^2 \right]$$

where the field variable $\vec{\phi}_x$ is a vector with 2 real components.
x is a site on a simple cubic lattice and $\hat{\mu}$ a unit vector in μ direction.
$\lambda = 0$: Gaussian model; $\lambda \to \infty$: XY model.

The partition function is given by

$$Z = \left[\prod_x \prod_{i=1}^{2} \int d\phi_x^{(i)} \right] \exp(-H)$$



The correlation length behaves as

$$\xi = \xi_{0,\pm}(\lambda) \, |t|^{-\nu} \; (1 + c(\lambda) t^{\theta} + ...)$$

where $t = \beta_c - \beta$ is the reduced temperature. $\theta = \nu\omega \approx 0.5$ is the exponent of leading corrections.

The improved model: $c(\lambda^*) = 0$ Numerically: $\lambda^* = 2.15(5)$

Here we study $\lambda = 2.1$:

$$\beta_c = 0.5091503(6) \qquad \xi_{0,+} = 0.26362(8) \qquad \left|\frac{c(2.1)}{c(XY)}\right| \lesssim \frac{1}{30}$$

Film geometry:

System is finite in one direction and infinite in the other two

In our simulations: $L_0 \ll L_1 = L_2$

The range of fluctuations is characterized by the correlation length ξ.
For $L_0 \lessapprox \xi$ fluctuations are restricted by the geometry of the film

$\Longrightarrow$ a force $F_{Casimir}$ per area acts on the walls of the film.

Boundary conditions:

- periodic boundary conditions: theoretically relatively simple; no experimental realization
- free boundary conditions: Dirichlet boundary conditions with vanishing order parameter; relevant for films of ^{4}He in the neighbourhood of the λ-transition.

  Corrections $\propto L_0^{-1}$; Can be cast into the form $L_{0,eff} = L_0 + L_s$. For our model $L_s = 1.02(7)$. (Obtained from the numerical study of other quantities)

The thermal (or critical) Casimir force is given by

$$F_{Casimir} = -\frac{\partial f_{ex}}{\partial L_0}$$

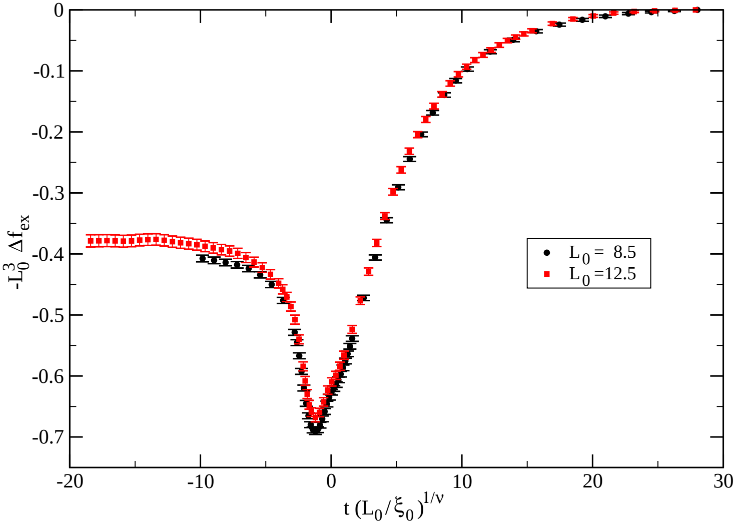
where L_0 is the thickness of the film.
The excess free energy per area:

$$f_{ex} = f(L_0) - L_0 f_{bulk}$$

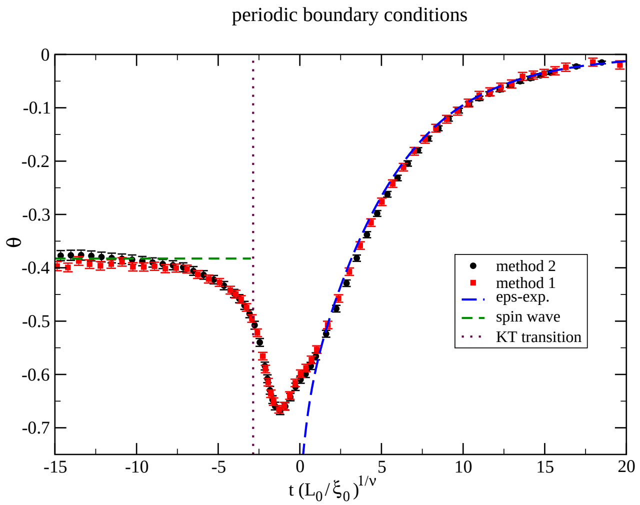
Finite size scaling predicts:

$$F_{Casimir} \simeq k_B T L_0^{-3} \theta(t[L_0/\xi_0]^{1/\nu})$$

where the function $\theta(x)$ is universal periodic boundary conditions



ϵ-expansion: Krech, Dietrich (1992), Grüneberg, Diehl (2008)



Comparison with Monte Carlo simulations of the XY model:

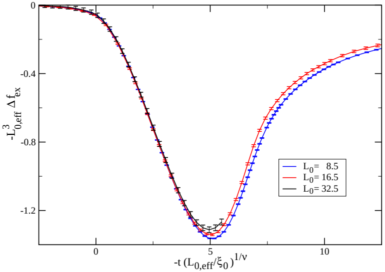
Our result for the minimum of $\theta(x)$:
$x_{min} = -1.20(5)$ and $\theta_{min} = -0.66(2)$

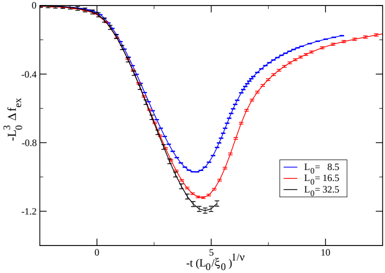
At the bulk critical point:
$\theta(0) = -0.60(2)$

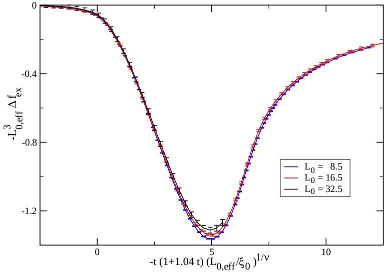
Vasilyev, Gambassi, Maciolek and Dietrich (2008):
Qualitative agreement of the curve with our curve
$x_{min} = -0.73(1)$ and $\theta_{min} = -0.633(1)$

$\theta(0) = -0.5986(14)$

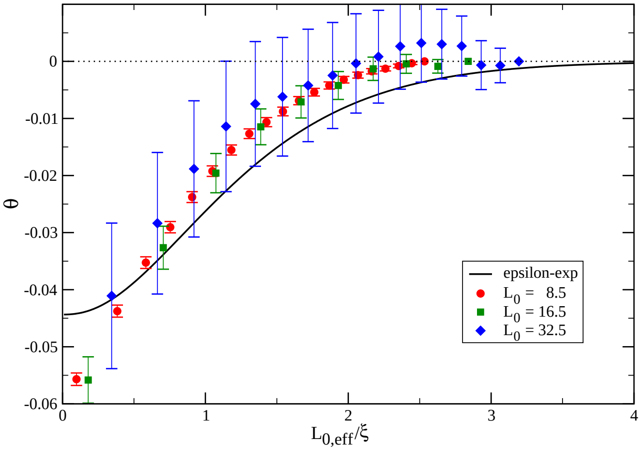
Free boundary conditions







ϵ-expansion: Krech and Dietrich (1992)



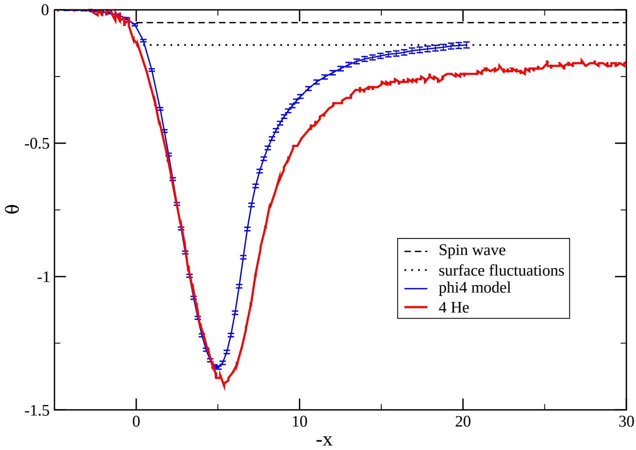
Comparison with Monte Carlo simulations of the XY model:

Our result for the minimum of θ:
$x_{min} = -4.95(3)$ and $\theta_{min} = -1.31(1)$

Hucht (2007)
Qualitative agreement of the curve with ours
$x_{min} = -5.3(1)$ and $\theta_{min} = -1.35(3)$

Vasilyev, Gambassi, Maciolek and Dietrich (2008)
Qualitative agreement of the curve with ours
$x_{min} = -5.43(2)$ and $\theta_{min} = -1.260(5)$

Comparison with experiment: Garcia, Chan (1999), Ganshin, Scheidemantel, Garcia, and Chan (2006)



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