Stability of Boolean Dynamics

Fakhteh Ghanbarnejad
Konstantin Klemm

Professur für Bioinformatik
Institut für Informatik
Universität Leipzig

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Stability of Boolean Dynamics
Outline

Motivations & Definitions

Stability of BN

Common Notion of Stability?

Results

Motivations & Definitions

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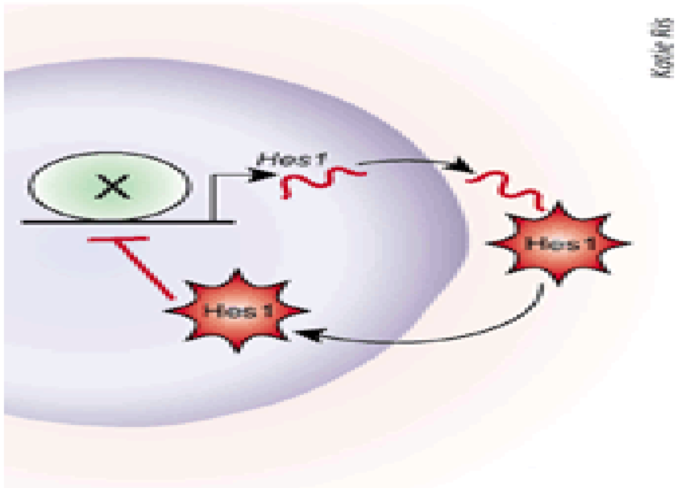
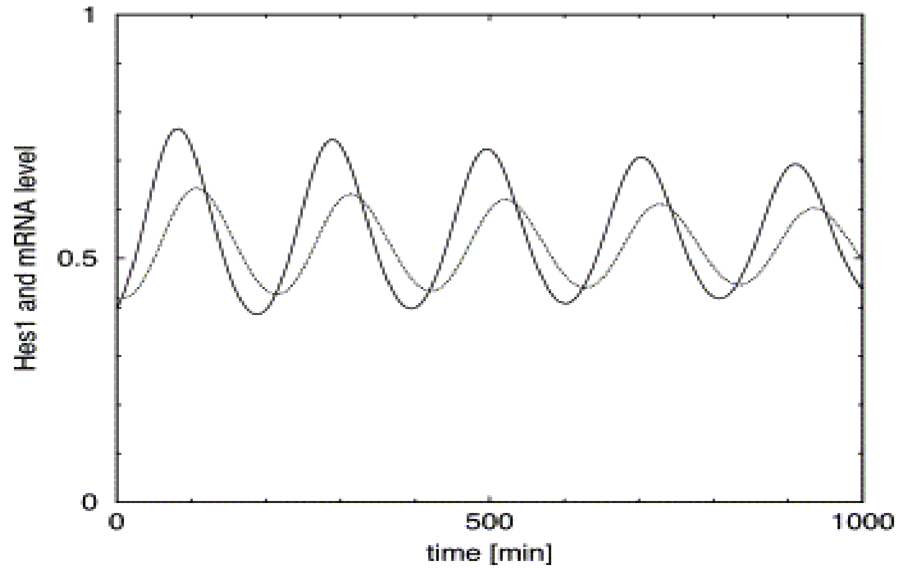
Regulatory Dynamics





$$\frac{[\mathrm{mRNA}]}{\mathrm{t}} = \frac{\alpha k^h}{k^h + [\mathrm{Hes1}(t-\tau)]} - \frac{[\mathrm{mRNA}(t)]}{\tau_{\mathbf{rna}}}$$

$$\frac{[\mathrm{Hes1}]}{\mathrm{t}} = \beta[\mathrm{mRNA}(t)] - \frac{[\mathrm{Hes1}(t)]}{\tau_{\mathbf{hes1}}}$$

M.H. Jensen, K. Sneppen, G. Tiana, FEBS Letters 541, 176 (2003)

Regulatory Dynamics

$$\frac{[\mathrm{mRNA}]}{\mathrm{t}} = \frac{\alpha k^h}{k^h + [\mathrm{Hes1}(t-\tau)]} - \frac{[\mathrm{mRNA}(t)]}{\tau_{\mathbf{rna}}}$$

$$\frac{[\mathrm{Hes1}]}{\mathrm{t}} = \beta[\mathrm{mRNA}(t)] - \frac{[\mathrm{Hes1}(t)]}{\tau_{\mathbf{hes1}}}$$

- 6 parameters for a single gene
  production delay τ, production rates α, β, binding threshold k^h, characteristic degradation times τ_{Hes1}, τ_{rna}
- larger systems??

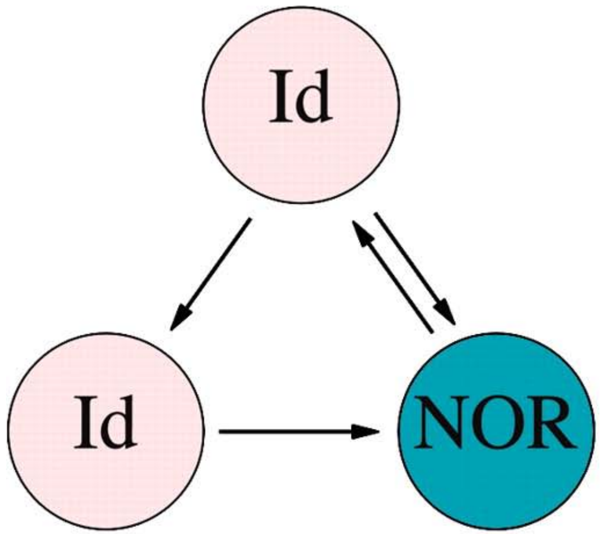
Discrete Abstract Model

- ON/OFF-States
- Interactions-Network
- Dynamics (time evolution of states):

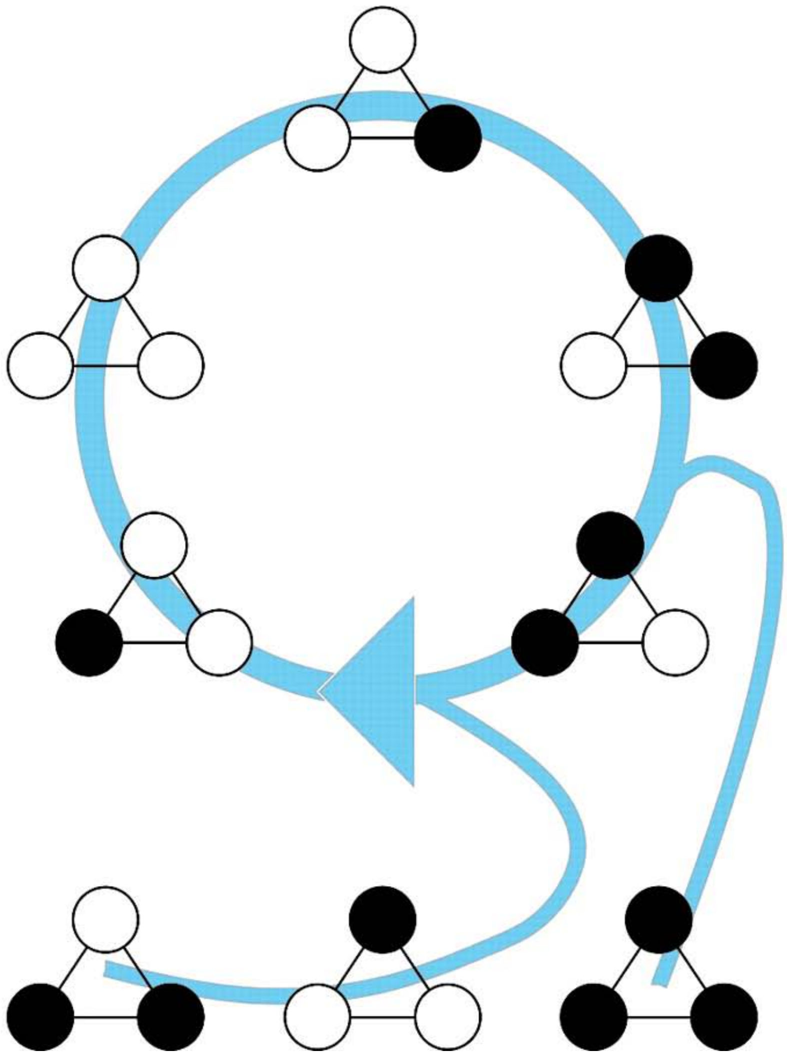
  Boolean functions

Boolean Networks

An Example of a Boolean Network



1	1	0
1	0	0
0	1	0
0	0	1



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Stability?

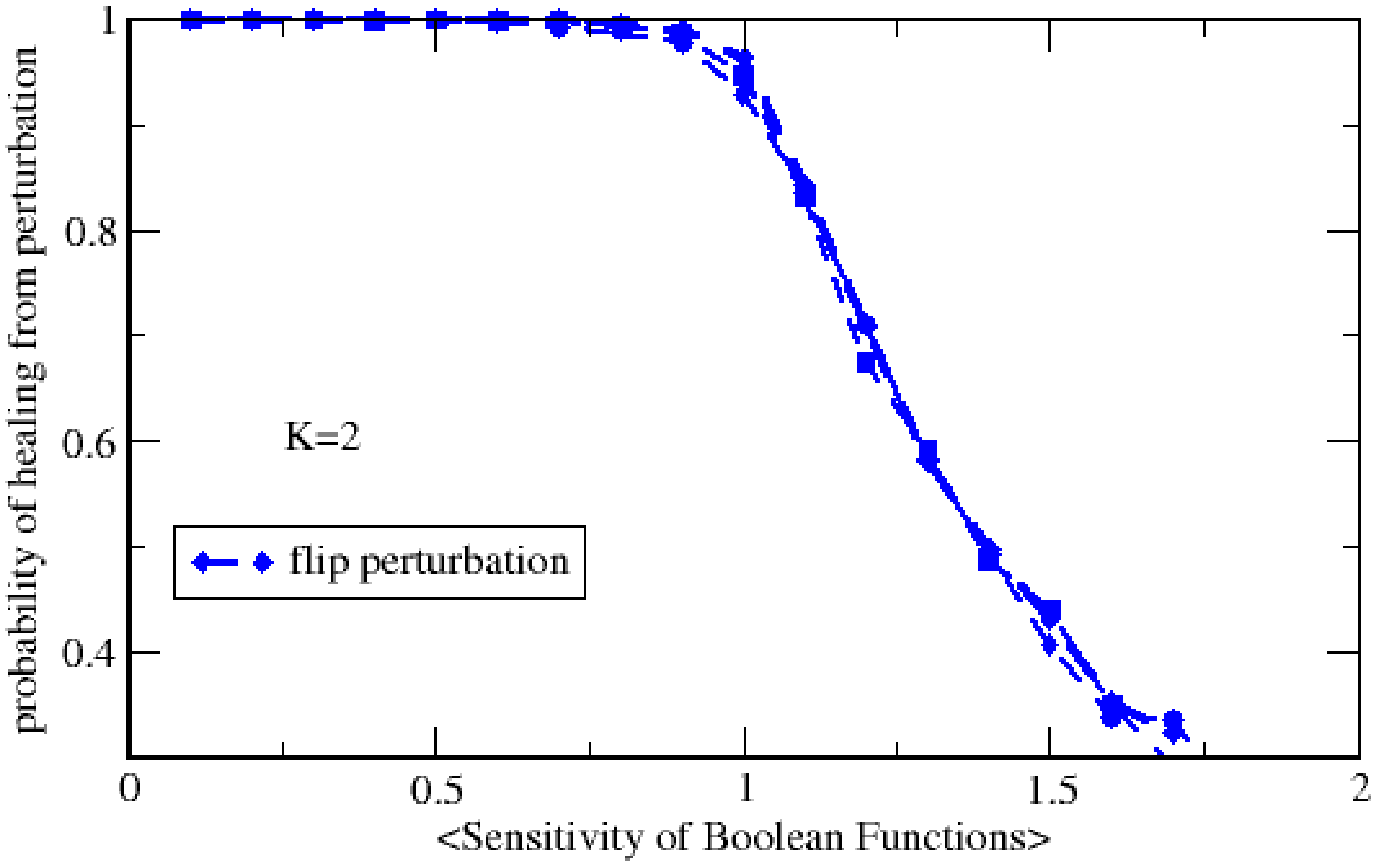
- small perturbation?
- known noise in BN:
  - Stochastic asynchronous update
  - Flip perturbations: damage spreading

Flip Perturbation

two initialized states that differ at one node only

→ reach the same trajectory.

Flip Perturbation



Common Notion of Stability?

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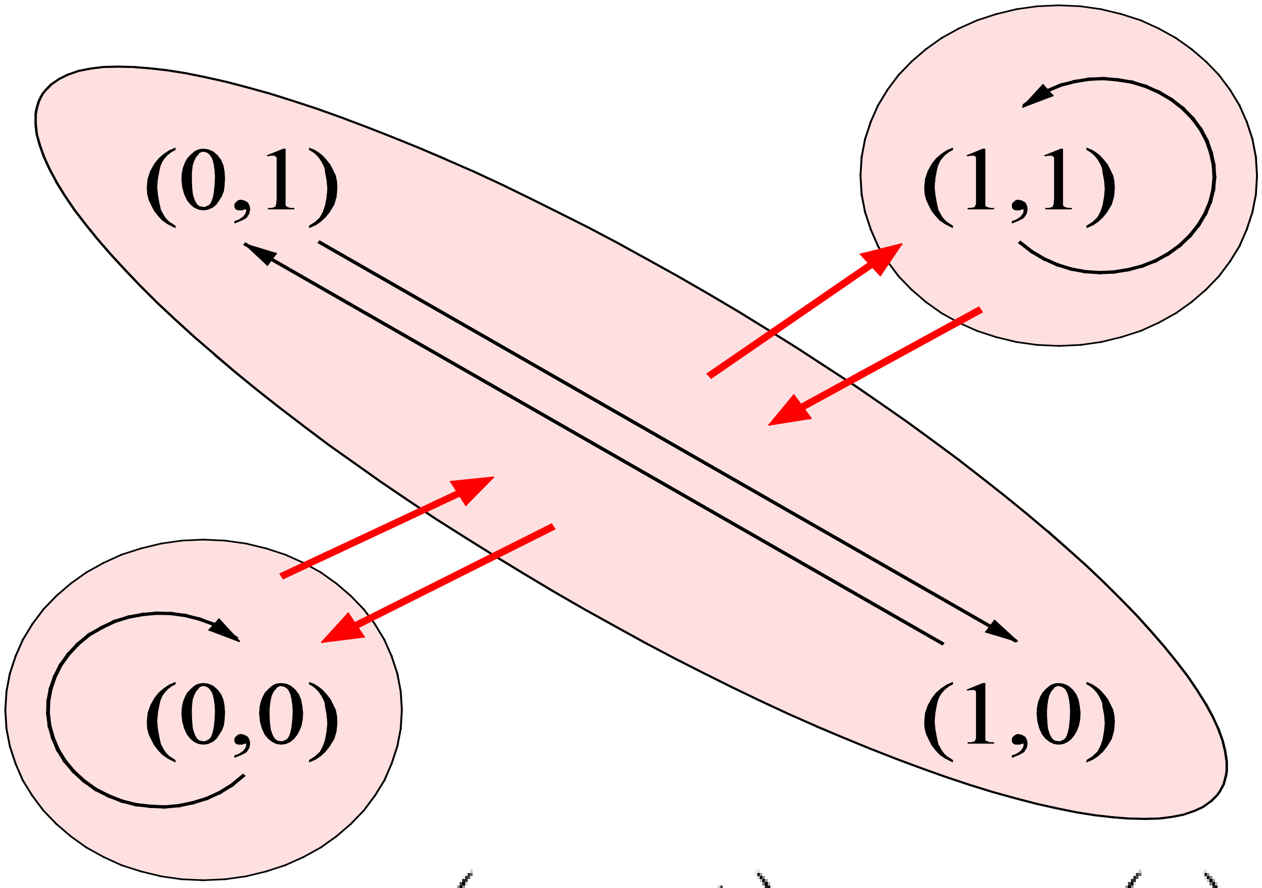
Stability?

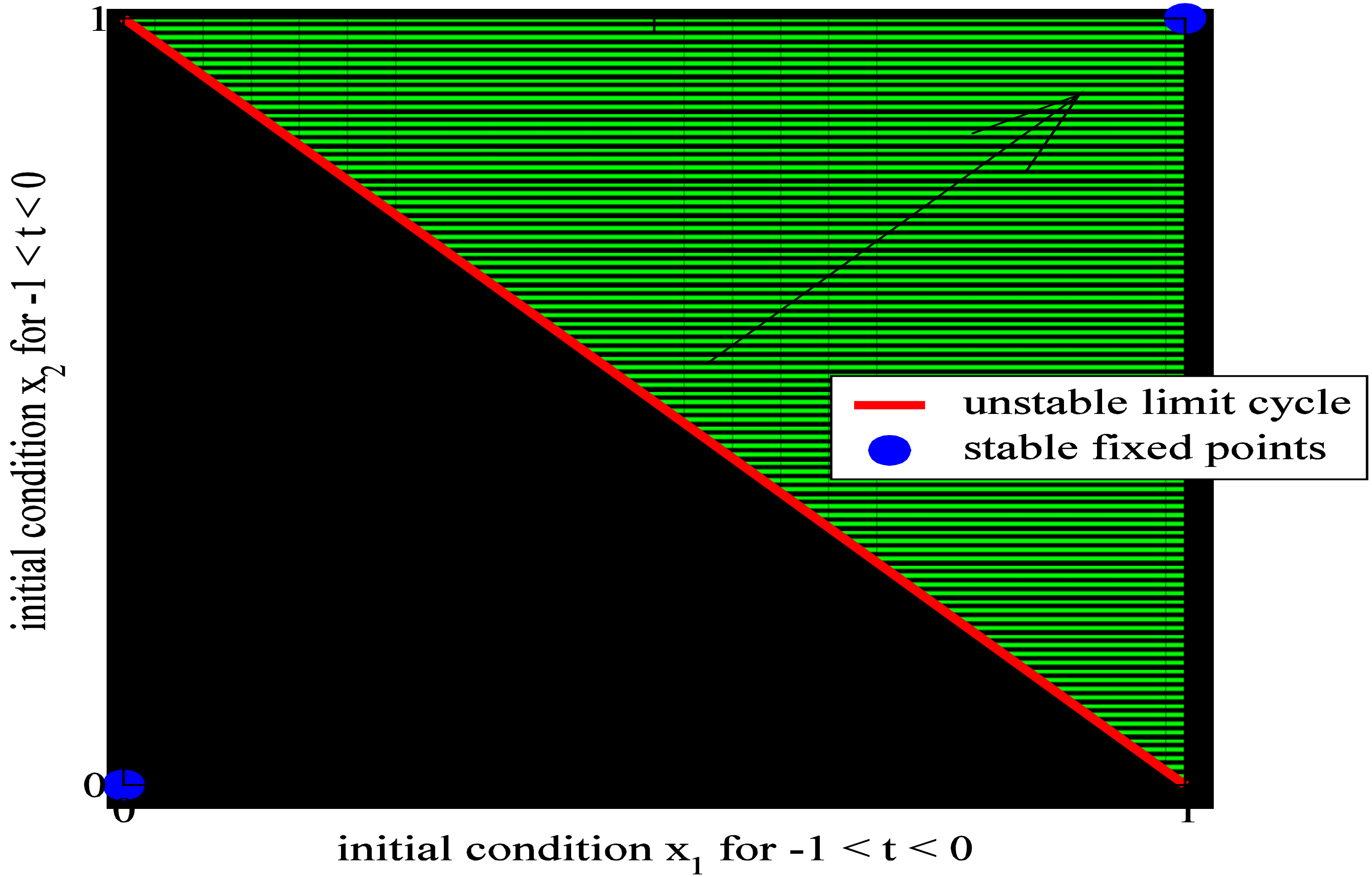
for all *perturbed initial conditions in a sufficiently small neighborhood*, the same trajectory obtains asymptotically

A minimal example



$$x_1(t+1) = x_2(t)$$

$$x_2(t+1) = x_1(t)$$



state-continuous dynamical system

$$\dot{x}_1(t+1) = \alpha\Theta\left(x_2(t) - x_1(t+1)\right)$$

$$\dot{x}_2(t+1) = \alpha\Theta\left(x_1(t) - x_2(t+1)\right)$$

Results

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Methods

- K=2
- DDE

$$y_i(t+1) = \begin{cases} \alpha h_i(t) & \text{if } y_i(t) \neq h_i(t) \\ 0 & \text{otherwise} \end{cases}$$

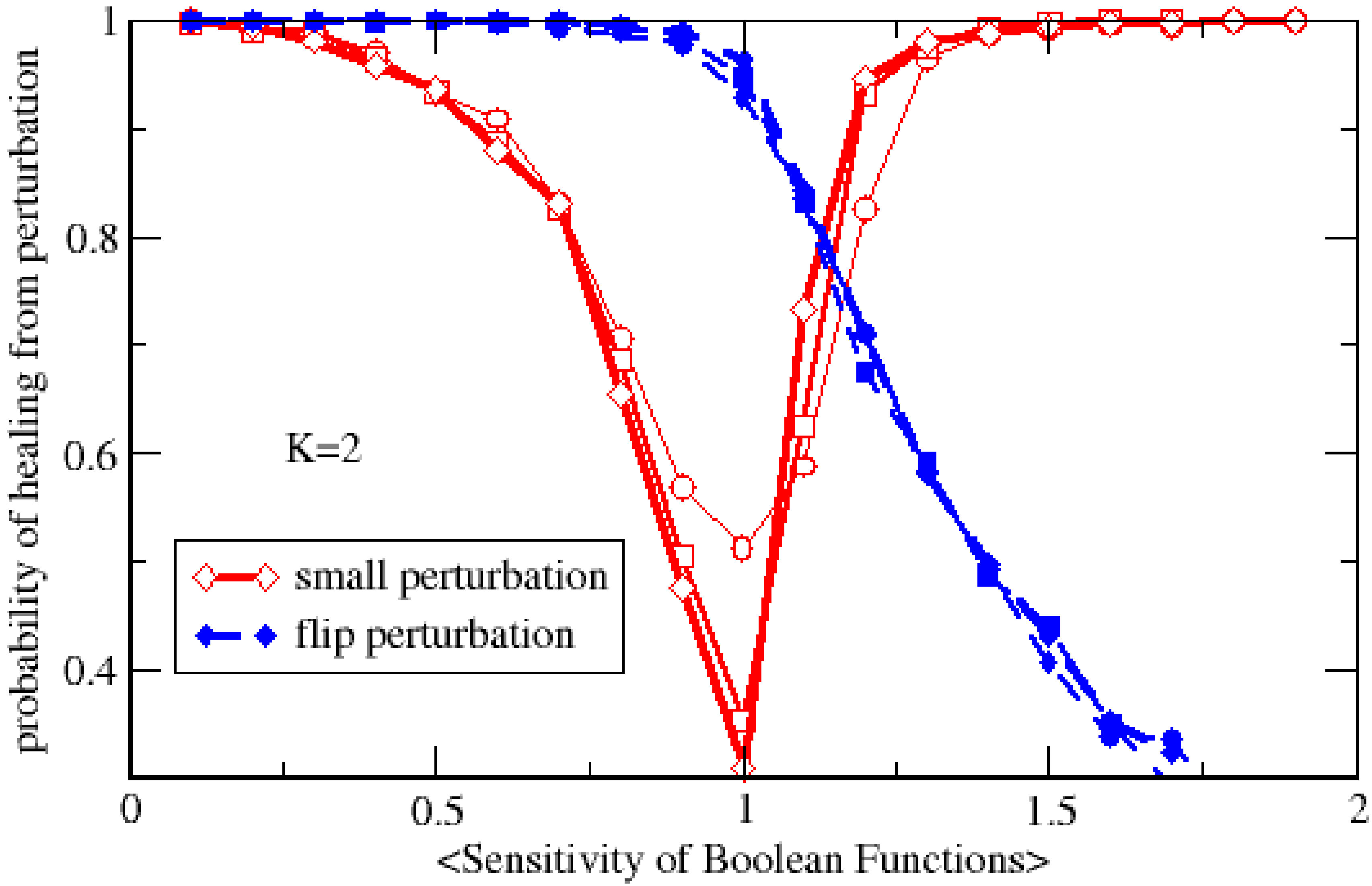
$$with \quad h_i(t) = f_i(\Theta(y_{j(i)}(t)), \Theta(y_{k(i)}(t)))$$

- Boolean function *f* with

$$P(f) \propto \exp(\lambda s(f))$$

- Perturbation

Flip vs. Small Timing Perturbation



Summary & Outlook

Flip perturbation:
disagreement with definition of stability in corresponding continuous case.
INCONSISTANCY!

Timing perturbation:
Fresh look at Order-Disorder Transition in Boolean networks

Summary & Outlook

Flip perturbation:
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Evolution?


Thanks!
to be continued ...

An Example of a DDE

$$\dot{s}_i(t) = \alpha[f(h_i(t-1)) - s_i(t)]$$

$$f(x) = \begin{cases} 0 & x < 0.5 \\ 1 & x >= 0.5 \end{cases}$$

$$h_i(t) = ax_1(t)x_2(t) + b_1x_1(t) + b_2x_2(t) + c$$

	a	b1	b2	c
and	1	0	0	0
nor	1	-1	-1	1