Tuning the shape of the condensate in spontaneous symmetry breaking

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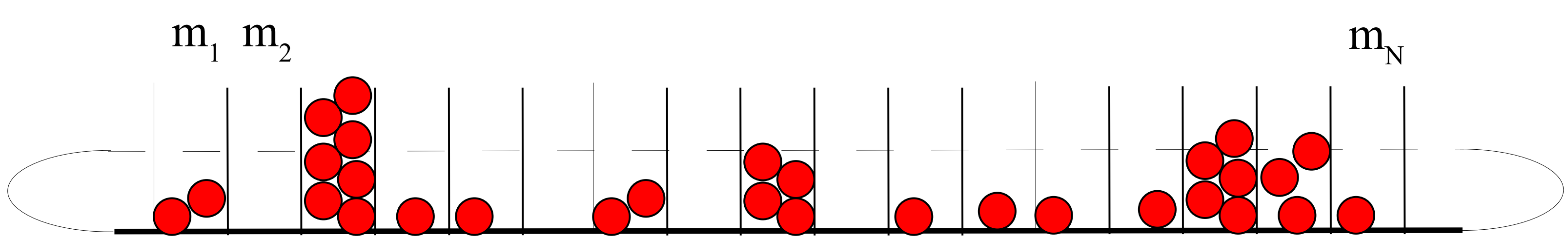
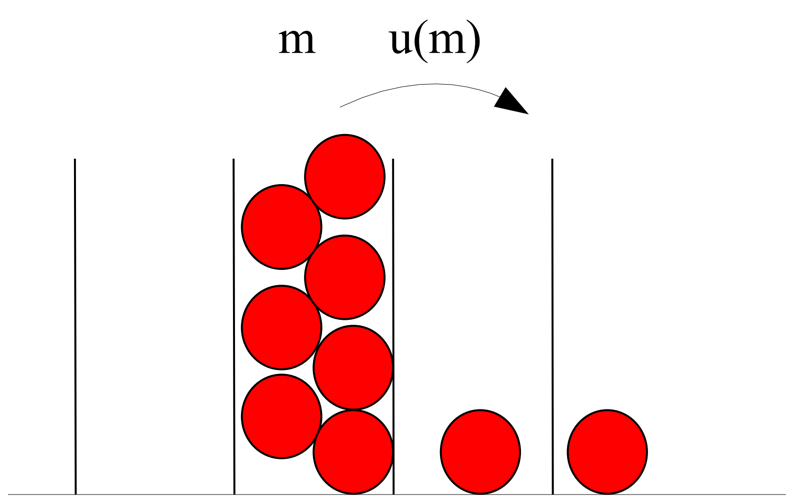
Far-from-equilibrium models:
- defined by dynamics and not by hamiltonian
- probability of a microstate unknown
- traditional methods of Stat. Phys. do not work

Exception (one of few)

Zero-range process





Hopping rate u(m) depends only on the occupation of departure site

Steady state factorizes over sites:

$$P(m_1, \ldots, m_N) = \frac{1}{Z(N,M)} p(m_1) \cdots p(m_N)\, \delta_{m_1 + \cdots + m_N - M}$$

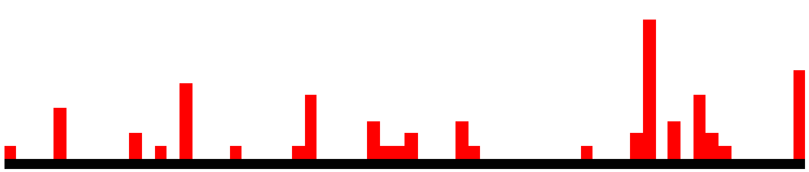
$$p(m) = \begin{cases} \prod_{k=1}^{m} \frac{1}{u(k)} & \text{for } m > 0 \\ 1 & \text{for } m = 0 \end{cases}$$

Example

$$p(m) \cong \frac{\Gamma(b+1)}{m^b}$$

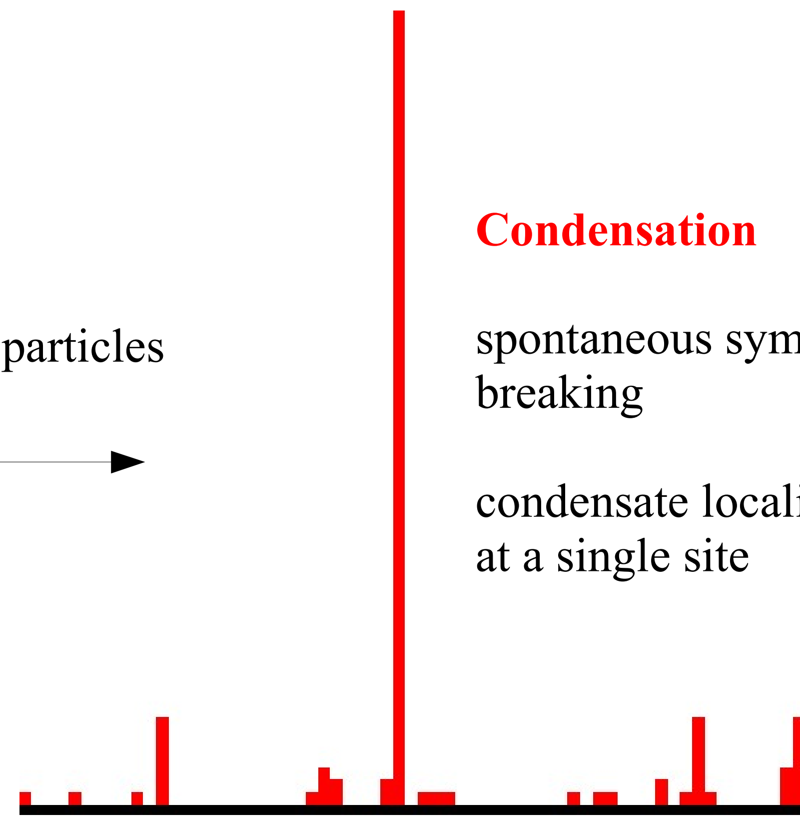
$$\Updownarrow$$

$$u(m) \xrightarrow{m \to \infty} 1 + b/m$$



density of particles

$\rho > \rho_{critical}$ →



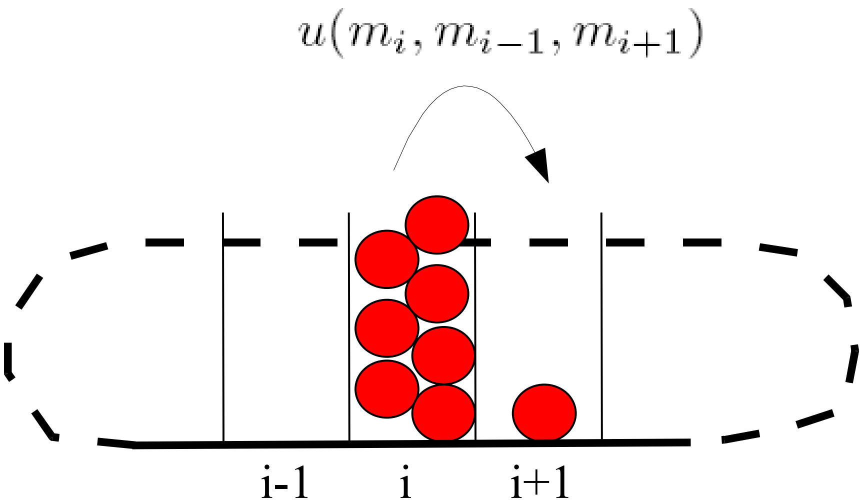
Condensation

spontaneous symmetry breaking

condensate localized at a single site

Pair-factorized steady state (PFSS)

Evans, Hanney, Majumdar,
PRL 97, 010602 (2006)



Factorization over pairs

$$P(m_1, \ldots, m_N) = \prod_i g(m_i, m_{i+1})$$

Sufficient condition: $$u(m_i, m_{i-1}, m_{i+1}) = \frac{g(m_i - 1, m_{i+1})g(m_i - 1, m_{i-1})}{g(m_i, m_{i+1})g(m_i, m_{i-1})}$$

- the role of interactions?

$$g(m, n) = K(|m - n|)\sqrt{p(m)p(n)}$$

"surface stiffness" K(x) "on-site" potential p(m)

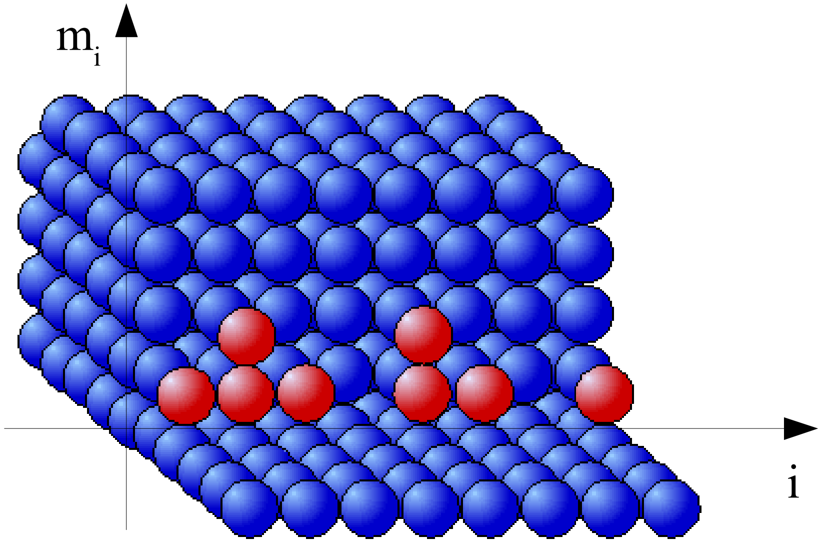
“Solid-state” example

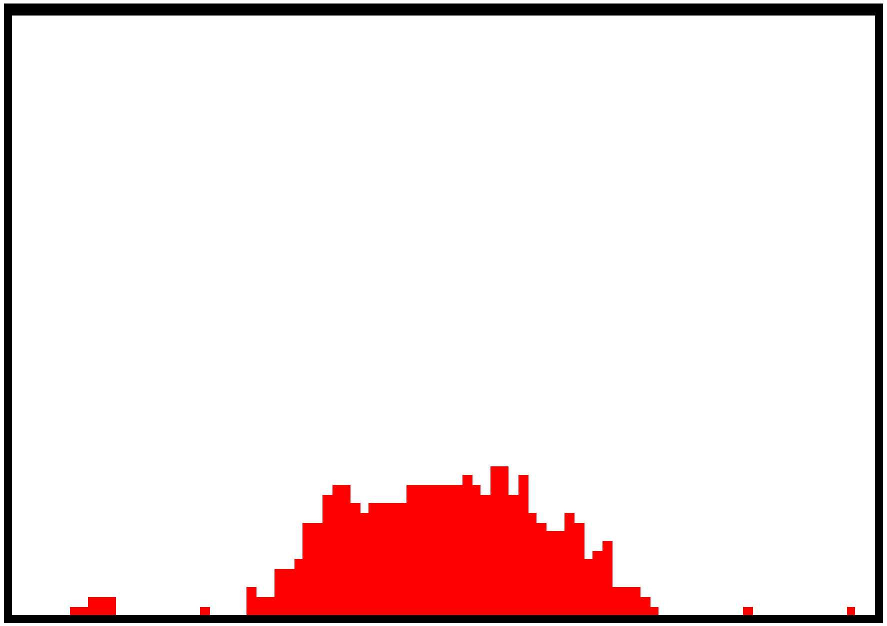
Solid-On-Solid (SOS) model

Energy

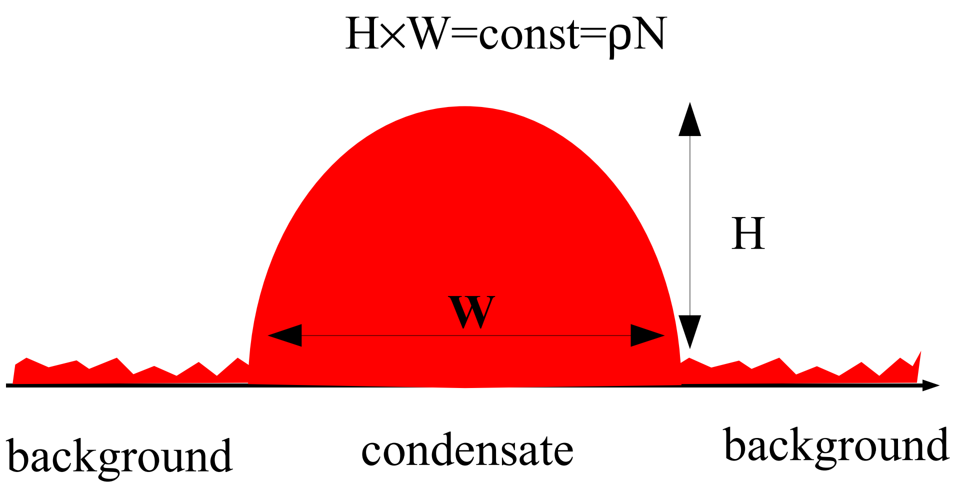
$$E = J \sum_i |m_i - m_{i-1}| + U \sum_i \delta_{m,0}$$

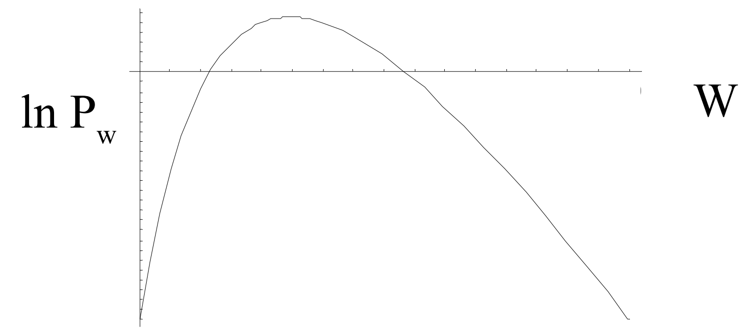
$$\begin{cases} K(x) = e^{-Jx} \\ p(m) = e^{U\delta_{m,0}} \end{cases}$$





Width of the condensate

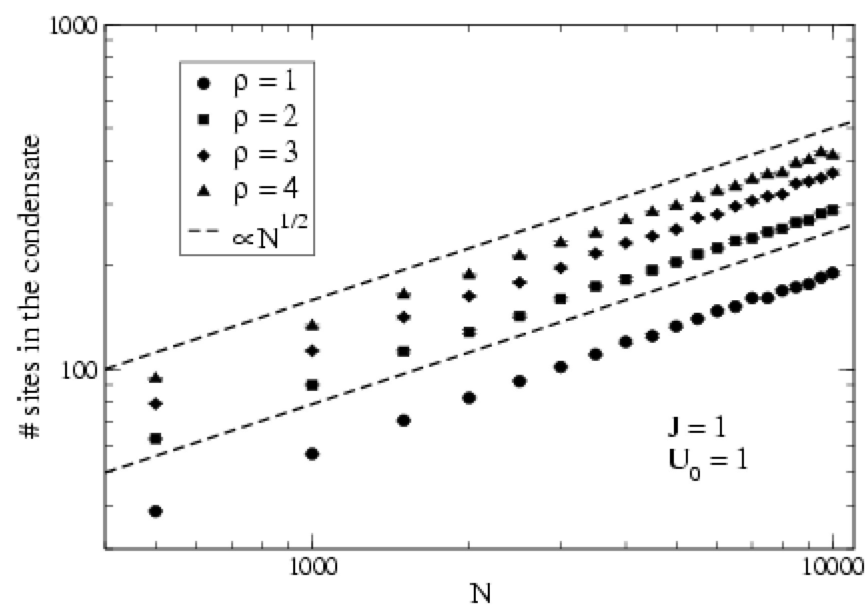




$$P(m_1, \ldots, m_N) = \exp\left[\sum_i -J|m_i - m_{i+1}| + U_0 \delta_{m_i,0}\right]$$

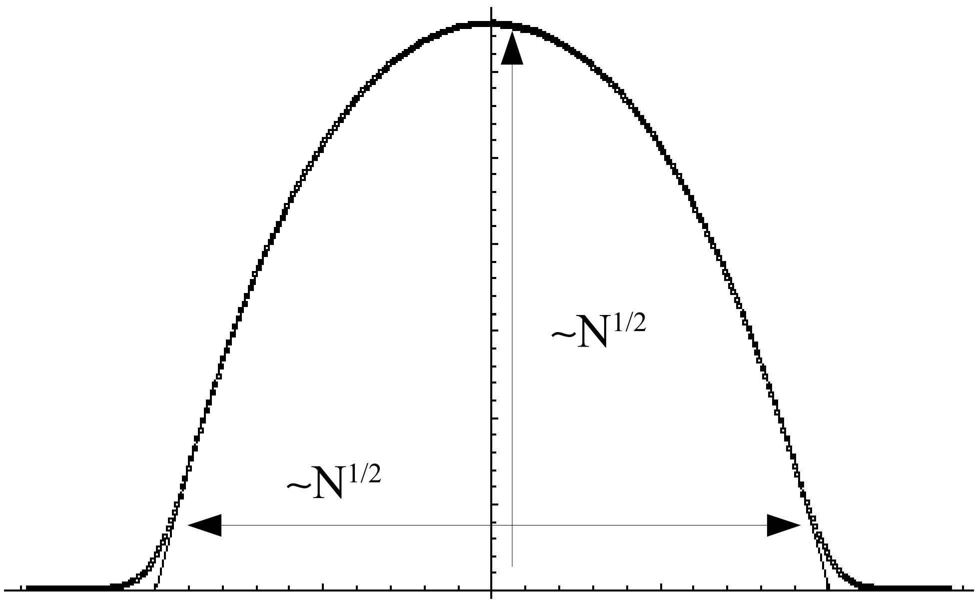
$$P_W \sim \exp(c_1(N-W) - 2JH)$$
$$\approx \exp(c_1(N-W) - 2J\rho N/W)$$

has maximum for $W \sim N^{1/2}$

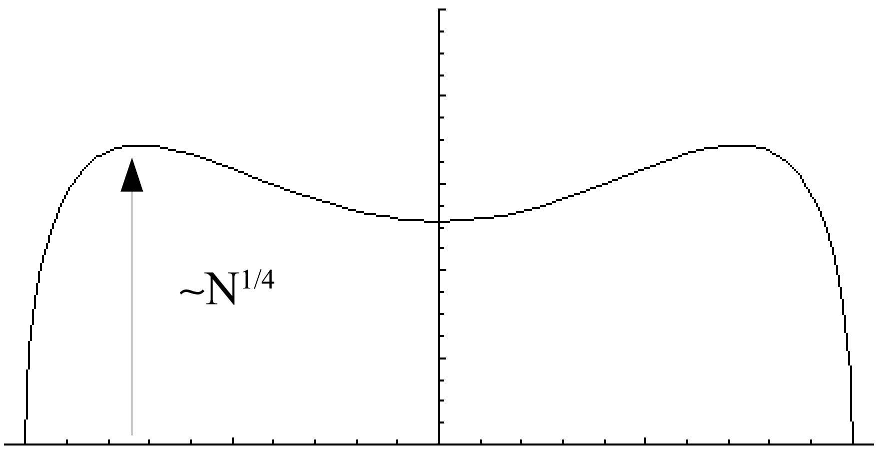


and more...

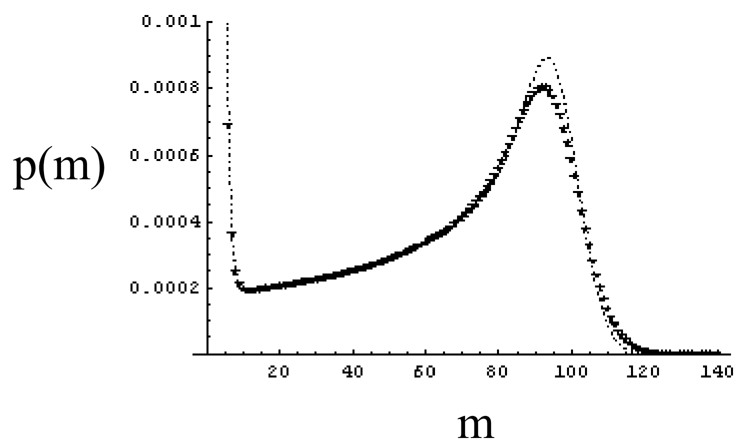
- the shape of the condensate



- Fluctuations indeed small

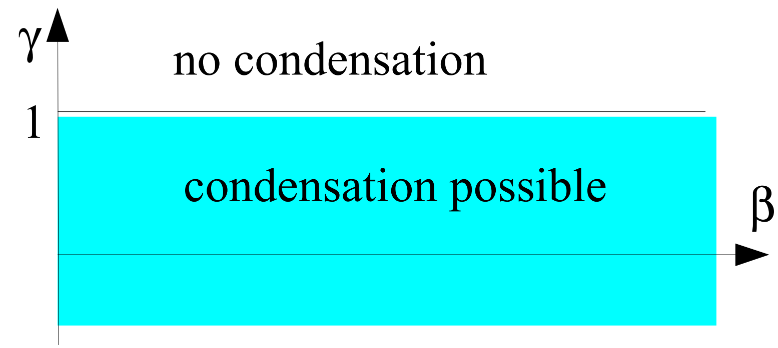


- distribution of particles



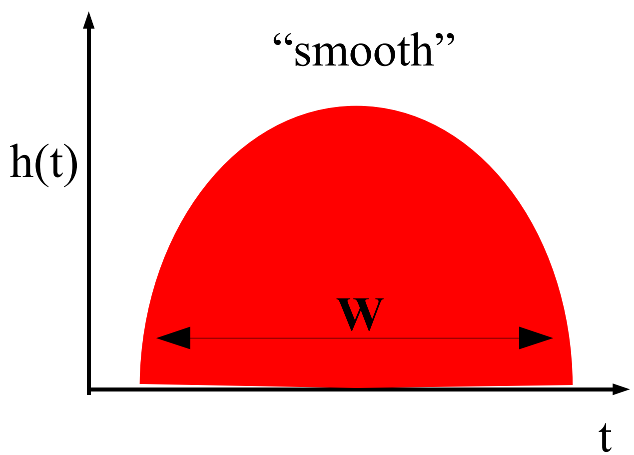
2nd example

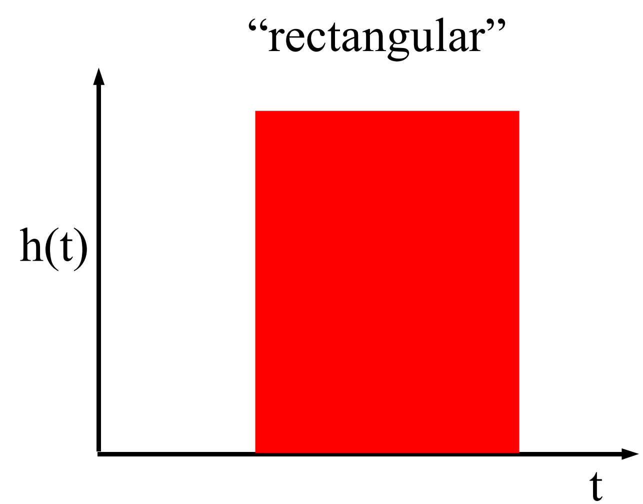
$$K(x) = \exp(-|x|^{\beta}), \quad p(m) = \exp(-m^{\gamma})$$



Estimating the extension:

- only two generic shapes:

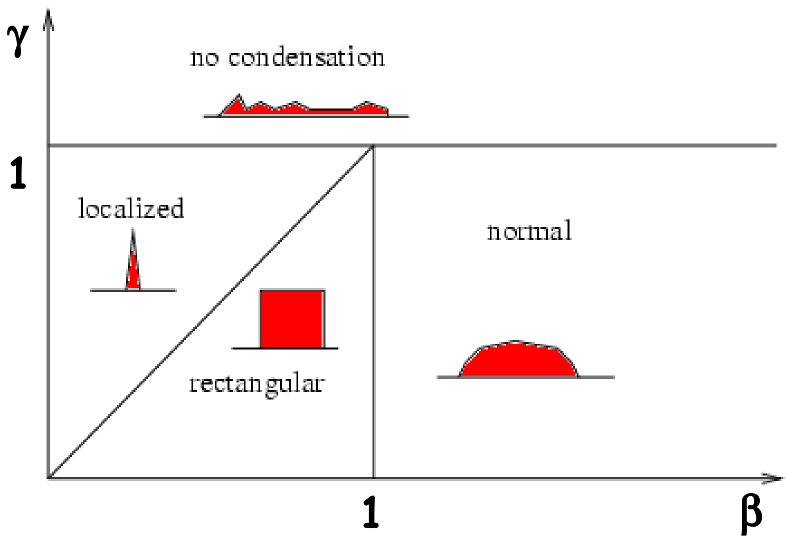


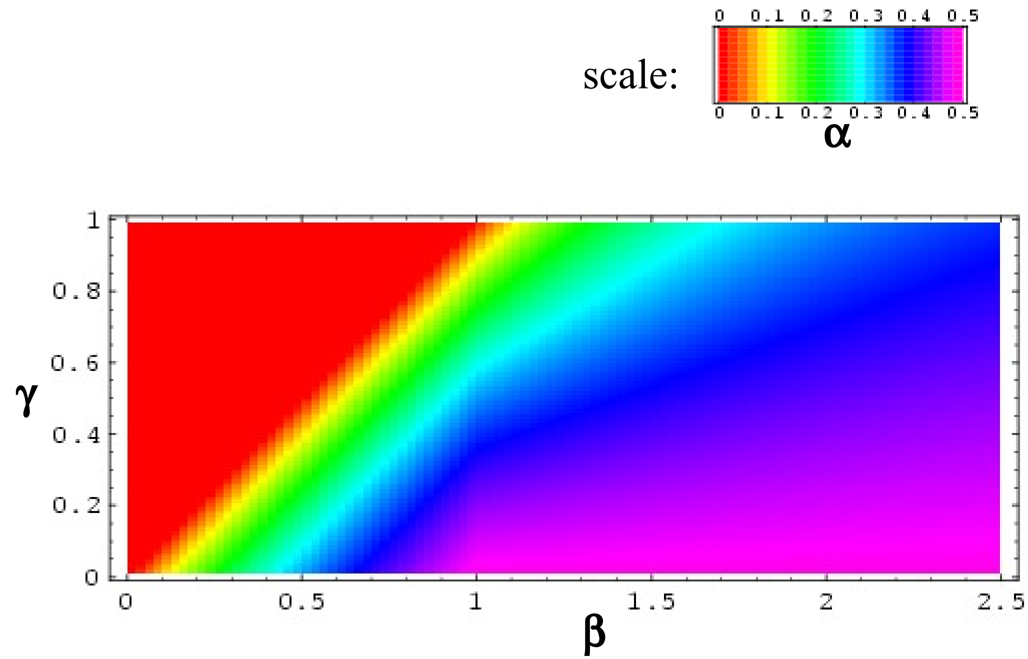


$$\ln P(W) \cong (N-W)c + W \ln p(N/W) + W \int \ln K(2Nh'(t)/W^2)dt$$

$$\ln P(W) \cong (N-W)c + W \ln p(N/W) + 2 \ln K(N/W)$$

We search for max. of both P(W) and choose the larger one





The extension $W \sim N^{\alpha}$ is tunable

Conclusion

- node-node interactions make the condensate extended (but not always)
- the extension is tunable
- the shape is non-universal

Thanks: J. Sopik (Toulouse), W. Janke (Leipzig), H. Meyer-Ortmanns (Bremen)

Paper:

B.W, J. Sopik, W. Janke and H. Meyer-Ortmanns, Phys. Rev. Lett. **103** (2009) 080602