Critical Loop Gases & the Worm Algorithm

Adriaan M.J. Schakel

w/ Thomas Neuhaus & Wolfhard Janke

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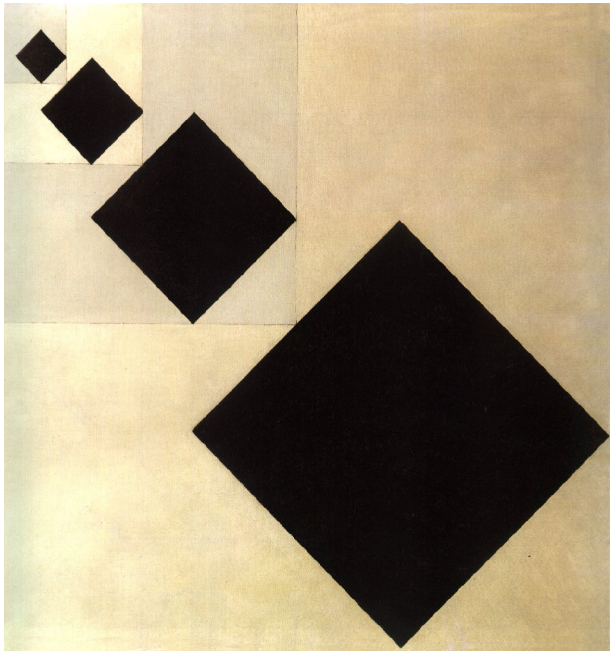


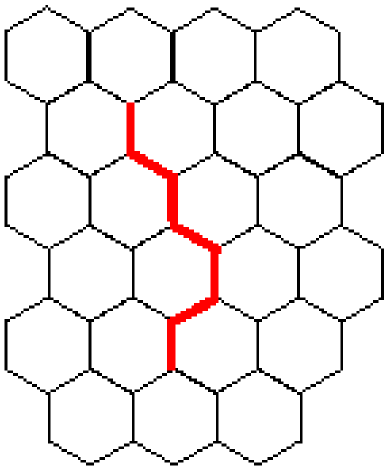
Theo van Doesburg, 1930

Particle-Field Duality

- 1940ies: Feynman, Schwinger, Tomonaga, and others formulated QED
- Feynman: (intuitive) spacetime approach
- Schwinger & Tomonaga: (formal) quantum field theory (QFT)
- Dyson: showed the equivalence and Feynman diagrams became tools of QFT
- Schwinger: Feynman diagrams had "brought computation to the masses”
- From my perspective: "Dyson killed Feynman's spacetime approach"
- Statistical physics: field vs. high-temperature (HT) representation of spin models
- Efficient algorithms to simulate spin models as field theory: cluster updates
- Worm algorithm by Prokof'ev and Svistunov (2001): simulates HT representation
- It calls for new observables

Worm Algorithm

2D Ising model:



- randomly choose either endpoint of worm
- randomly choose any of the links attached
- update bond $b_l \longrightarrow b_l' = 1 - b_l \quad (b_l = 0, 1)$ w/

$$P_{\text{accept}} = \min\left(1, K^{1-2b_l}\right)$$

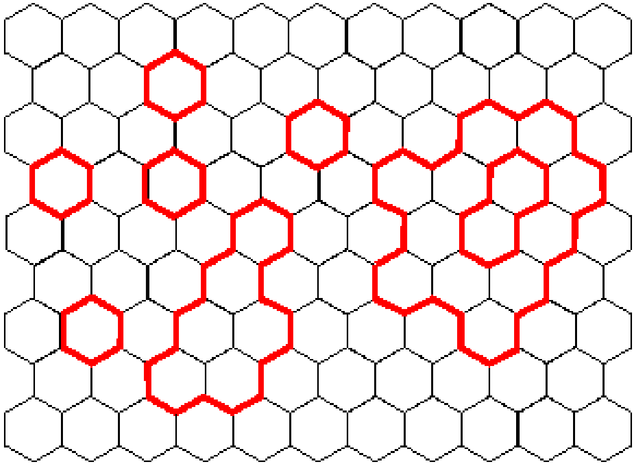
K : bond fugacity

- if worm closes: choose link at random and update

Observables

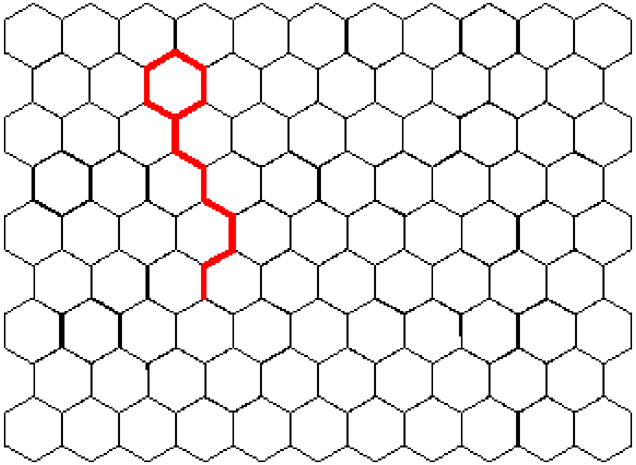
SAW:

- Loop length distribution
- Radius of gyration (closed & open chains)
- End-to-end distance



Percolation:

- Winding threshold
- Average length of winding loops



Simulations:

- On honeycomb lattices of size up to 325
- Advantage: loops cannot intersect,
  but open chains can

Loop length distribution

$$\ell_n = \frac{1}{\mathcal{N}} \sum_m \delta_{n_m, n} \sim n^{-d/D-1} \mathrm{e}^{-\theta n}, \quad \theta \propto (K - K_\mathrm{c})^{1/\sigma}$$

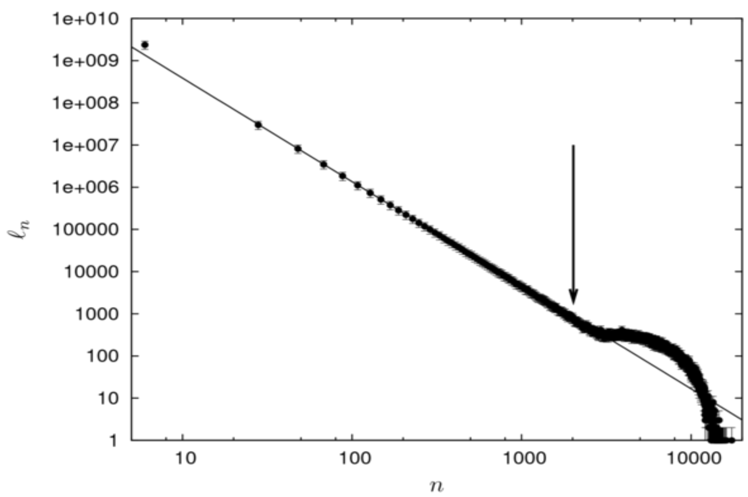
Configurational entropy

Boltzmann factor

Loops proliferate when

$$\theta \to 0$$

K : bond fugacity



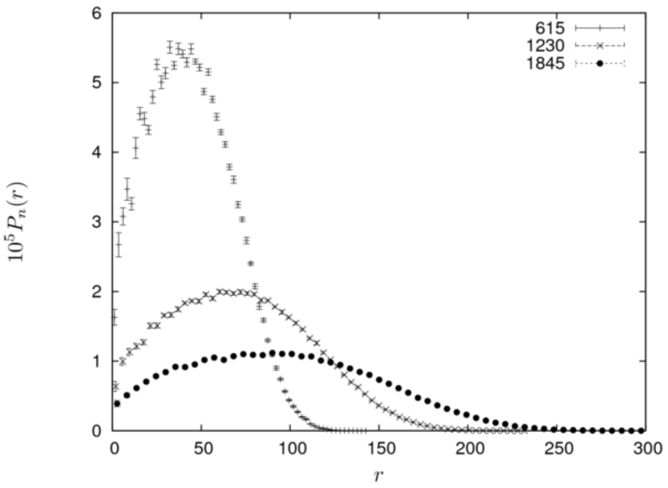
Scaling

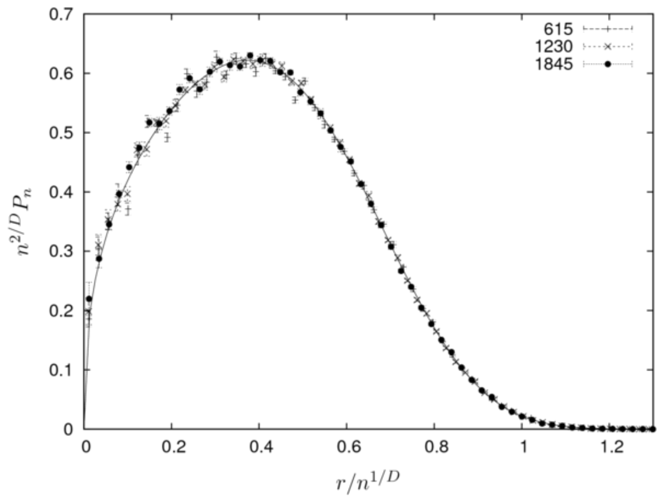
Probability for finding a worm connecting two sites in n steps:

$$P_n(x_i, x_j) \sim n^{-d/D}\,\mathcal{P}(r/n^{1/D}),$$

w/ scaling function (Fisher, 1966):

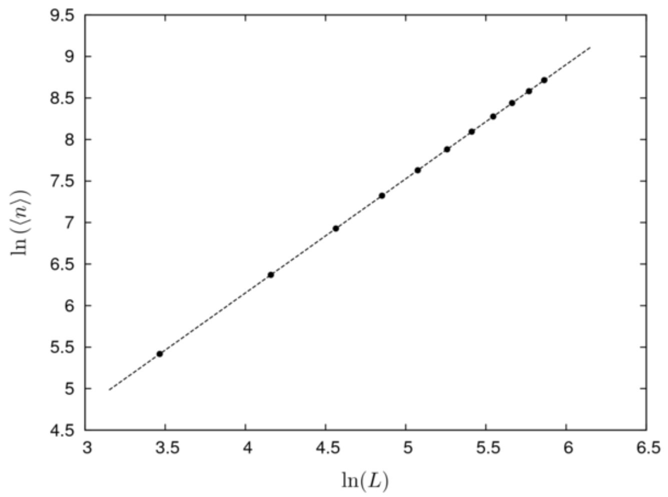
$$\mathcal{P}(t) = a t^{\vartheta} \exp\left(-b t^{\delta}\right), \quad 1/\delta = 1 - 1/D$$





Winding loops

Average length of loops winding the lattice (w/ periodic BC):



$$\langle n \rangle \sim L^D \qquad D = 11/8$$