Wilson loops in very high order lattice perturbation theory

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CompPhys09, Leipzig, November 2009

Outline

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   - Langevin equation for lattice QCD
   - Perturbative Langevin equation
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Motivation I

- 1978: Shifman, Vainshtein and Zakharov introduced the non-perturbative gluon condensate $\langle \frac{\alpha}{\pi} G\, G \rangle$
- Lattice gauge theory provides a promising tool to calculate it from Wilson loops.
- In the early 80th first computations : Plaquette(1981 Banks et al., DiGiacomo and Rossi), larger Wilson loops(1981/1982 Kripfganz et al., Ilgenfritz et al.)

$\langle \frac{\alpha}{\pi} G\, G \rangle$ is conventionally derived using the plaquette P from the relation

$$P_{MC} = P_{pert} - a^4 \frac{\pi^2}{36} \left[\frac{-b_0\, g^2}{\beta(g)} \right] \langle \frac{\alpha}{\pi} GG \rangle \,,$$

$\rightarrow$ P_{pert} **is needed to very high order in LPT!**

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- General interest in the behavior of perturbative series in QCD:

$$Q(n^\star) \sim \sum_n^{n^\star} a_n \lambda^n \,,$$

- Series are asymptotic, and assumed that for large n the leading growth of the coefficients a_n can be parametrized

$$a_n \sim \; C_1 \, (C_2)^n \, \Gamma(n + C_3) \,,$$

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- Standard diagrammatic approach in LPT is restricted essentially to two-loop
- Di Renzo et al. formulated the so-called Numerical Stochastic Perturbation Theory (NSPT)
- Based on the Langevin quantization method of Parisi/Wu
- NSPT drops the concept of Feynman diagrams - uses the action with the corresponding perturbative expansion of fields
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Langevin equation for lattice QCD

Use Euclidean lattice Langevin equation with “time” t

$$\frac{\partial}{\partial t} U_{x,\mu}(t;\eta) = \mathrm{i}\left(\nabla_{x,\mu} S_G[U] - \eta_{x,\mu}(t)\right)\; U_{x,\mu}(t;\eta)$$

$\eta = \eta^a T^a$ random field with Gaussian distribution
$\nabla_{x,\mu}$ left Lie derivative on the group

For $t \to \infty$ link gauge fields U are distributed according to measure $\exp(-S_G[U])$

Discretise $t = n\epsilon$
Get solution at next time step $n+1$ in the Euler scheme

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$$F_{x,\mu}[U,\eta] = \mathrm{i}\left(\epsilon \nabla_{x,\mu} S_G[U] + \sqrt{\epsilon}\,\eta_{x,\mu}\right)$$

We use the Wilson plaquette gauge action S_G

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Perturbative Langevin equations I

Use that solution for perturbative expansion (DiRenzo et al.):
Rescale $\varepsilon = \beta\epsilon$ and expand gauge fields U (and "force" F)

$$U_{x,\mu}(n;\eta) \to 1 + \sum_{l>0} \beta^{-l/2} U^{(l)}_{x,\mu}(n;\eta)$$

Solution transforms to system of equations

$$\begin{aligned} U^{(1)}(n+1) &= U^{(1)}(n) - F^{(1)}(n) \\ U^{(2)}(n+1) &= U^{(2)}(n) - F^{(2)}(n) \\ &\quad + \frac{1}{2}(F^{(1)}(n))^2 - F^{(1)}(n)U^{(1)}(n) \\ &\dots \end{aligned}$$

Random noise field η enters only in $F^{(1)}$
Higher orders are stochastic via dependence on lower orders

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Wilson loops in NSPT

Wilson loop W_{NM} of size $N \times M$:

$$W_{NM}(n^\star) = \sum_{n=0}^{n^\star} W_{NM}^{(n)} g^n$$

($n^\star$ denotes the maximal order of LPT)
The coefficients $W_{NM}^{(n)}$ are obtained as

$$W_{NM}^{(n)} = \left\langle \sum_{n_i} \left(\prod_{j=1}^{2(N+M)} U_{\mu_j}^{(n_i)}(x_j) \right) \delta_{(\sum n_i),n} \right\rangle$$

It must be $W_{NM}^{(n)} = 0$ for $n =$odd.
We expand around the trivial vacuum $U_\mu^{(0)}(x) = 1$.

Computer implementation of NSPT

Computational framework:

- Quenched Wilson gauge action
- NSPT up to **order** $n = 20$ for Wilson loops W_{NM}
- Lattice sizes L^4 with $L = 4, ..., 12$
- $L = 12$ on a NEC SX-9 computer of RCNP at Osaka University
- The rest on Linux/HP-clusters at Leipzig university

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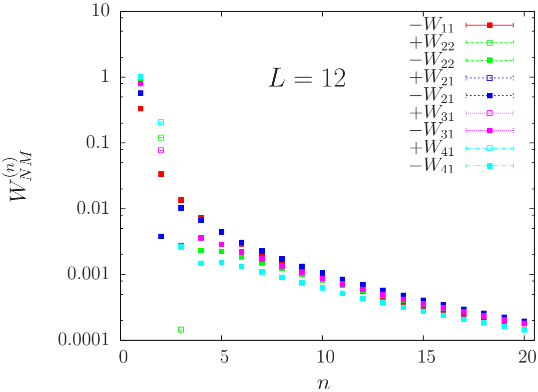
Perturbative coefficients for $\varepsilon \longrightarrow 0$

For $L = 12$ we get for some moderate Wilsonloop sizes



Questions

- In order to extract quantities like $\langle \frac{\alpha}{\pi} G\, G \rangle$ the perturbative series should be known as precise as possible.
- Is there a factorial behaviour of the perturbative coefficients?
- Up to now: calculations for Wilsonloops up to order $n = 10$ (Di Renzo et al.) and up to $n = 16$ (Rakow, plaquette)
- Our investigation: Order $n = 20$ of LPT for various sizes of W_{NM}
- Is $n = 20$ "sufficient" or do we need some kind of extrapolation formulae?
- Do we observe a deviation from the assumed a^4-behaviour of $\langle \frac{\alpha}{\pi} G\, G \rangle$?

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Extended HRS model

Question: Can one find a functional form $F(g)$ for the behaviour of the Wilsonloops?

$$F(g) = \sum_{n=1} c_n\, g^{2n} \quad \rightarrow \quad r_n = \frac{c_n}{c_{n-1}}$$

We found a hypergeometric functional form

$$W_{NM,pert} \sim {}_2F_1(1-\rho_1, 1-\rho_2; 1+s; u\, g^2)$$

Taylor epansion in g results in the ratio r_n

$$r_n = u\, \frac{(n-\rho_1)(n-\rho_2)}{n(n+s)}$$

generalization of an older HRS (Horsley, Rakow, Schierholz) formula

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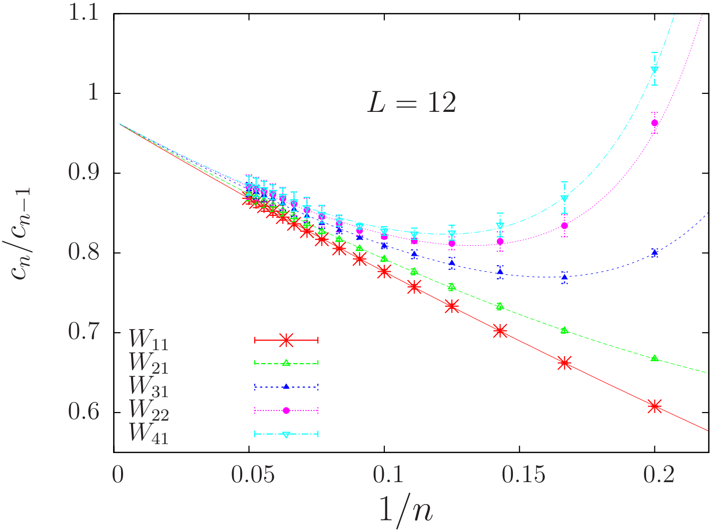
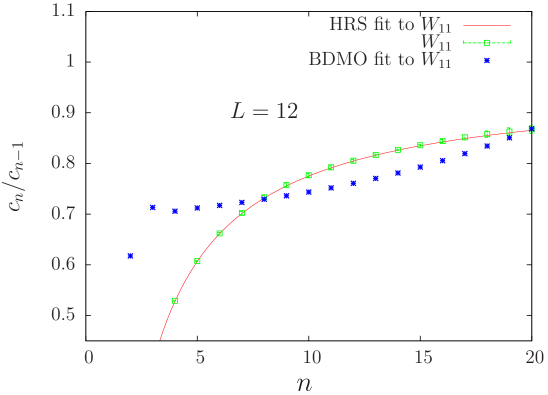
Domb-Sykes plot



$r_n(n)$ plot

Speculation of factorial behaviour based on renormalon inspired model (Burgio et al. (1998), see also Y. Meurice (2006))



We do not observe a factorial growth, at least in the region $n \leq 20$ and for our lattice sizes.

Boosted perturbation theory

- Bare coupling constant g is a bad expansion parameter
- Redefinition into boosted coupling g_b and rearrangement of series $\rightarrow$ better behaviour
- First application by Rakow (2005)

For the plaquette $P = W_{11}$ we define $g^2 \rightarrow g_b^2 = \frac{g^2}{P_{pert,b}}$ and transform

$$P_{pert}(g, n^\star) = 1 + \sum_{n=1}^{n^\star} W_{11}^{(n)}\, g^{2n} \rightarrow P_{pert,b}(g_b, n^\star) = 1 + \sum_{n=1}^{n^\star} W_{b,11}^{(n)}\, g_b^{2n}$$

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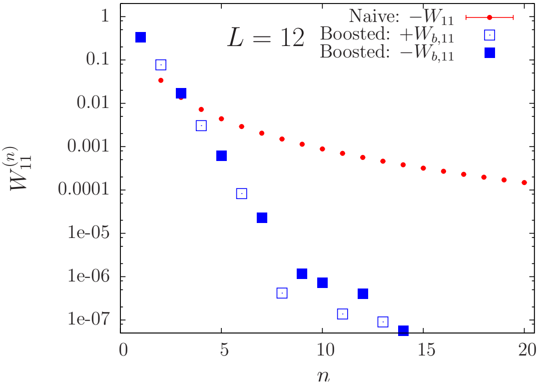
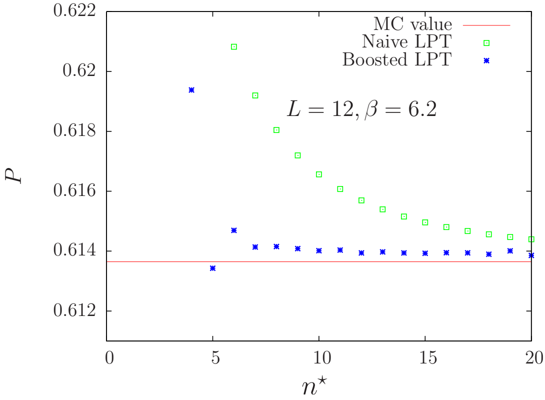
Coefficients for "naive" and boosted LPT



$W_{11}^{(n)}$ oscillate and show a very sharp decrease with n

P for "naive" and boosted LPT

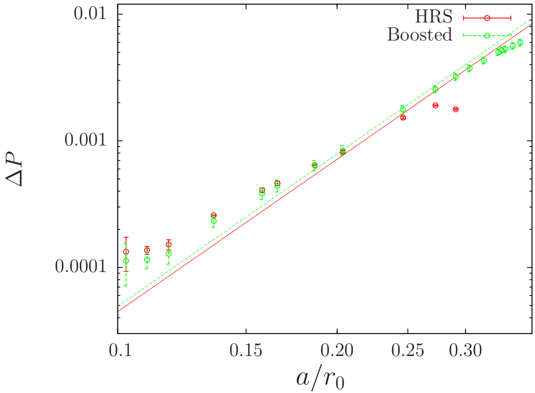
Summed series for P at $\beta = 6.2$ and for $L = 12$ as function of maximal order $n^{\star}$, MC data from QCDSF



$P_{pert,b}$ show a superior convergence behavior.

Gluon condensate

- $\Delta P = (P_{pert} - P_{MC}) \sim a^2$ or $\sim a^4$?
- Check: plot of ΔP versus a/r_0 (r_0 - Sommer scale) together with fit curves $\sim (a/r_0)^4$
- MC data taken from Boyd et al. and QCDSF



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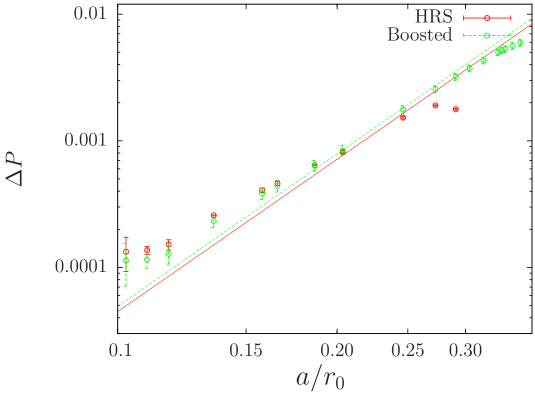

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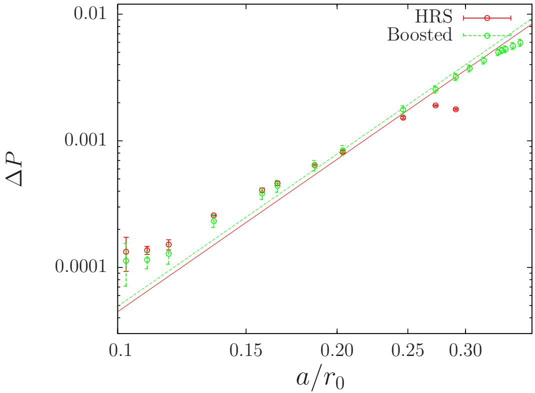

Extracting a value for the gluon condensate

$\Delta P = a^4 \frac{\pi^2}{36} \left[\frac{-b_0\, g^2}{\beta(g)} \right] \langle \frac{\alpha}{\pi} GG \rangle$

- Ansatz: $\Delta P(a/r_0) = C\,(a/r_0)^4$ and $\left(\frac{-b_0\, g^2}{\beta(g)} \right) \sim 1$.
- Fitting C in the range $0.1 \leq a/r_0 \leq 0.25$
- $r_0^4 \, \langle \frac{\alpha}{\pi} G\, G \rangle_{HRS} = 1.63(9), \quad r_0^4 \, \langle \frac{\alpha}{\pi} G\, G \rangle_{boosted} = 1.80(5)$.
- For $r_0 = 0.5$ fm we obtain

$$\langle \frac{\alpha}{\pi} G\, G \rangle_{HRS} = 0.039(2)\, GeV^4, \quad \langle \frac{\alpha}{\pi} G\, G \rangle_{boosted} = 0.043(2)\, GeV^4 .$$

- Fit the more general ansatz $\Delta P = C\,(a/r_0)^\delta \to \delta = 3.5 \pm 0.1$

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