The equilibrium low temperature phase of 3D Ising spin glasses: results from Janus

Víctor Martín-Mayor
in the name of the Janus Collaboration:

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Talk overview

1. Spin-glasses: basic facts.
2. The Janus computer.
3. The temperature random-walk.
4. Numerical results.
5. Conclusions.
6. Jobs with the Janus collaboration.

Spin glasses

The model

Observables

Spin glasses

The model

- $\mathcal{H} = -\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} J_{\boldsymbol{xy}}\sigma_{\boldsymbol{x}}\sigma_{\boldsymbol{y}}, \quad P(J_{\boldsymbol{xy}}) = \delta(J^2_{\boldsymbol{xy}} - 1).$
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- Link overlap (interfaces' density) $Q_\mathrm{link} = \frac{1}{V}\sum_{\boldsymbol{x}} q_{\boldsymbol{x}} q_{\boldsymbol{x}+\boldsymbol{r}}$, $r = 1$.
- Clustering correlation function: $C_4(\boldsymbol{r}|C) = \dfrac{\overline{\langle \delta(q-C)\, q_{\boldsymbol{x}} q_{\boldsymbol{x}+\boldsymbol{r}}\rangle}}{\overline{\langle \delta(q-C)\rangle}}$

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- TNT (Trivial-Non-Trivial):
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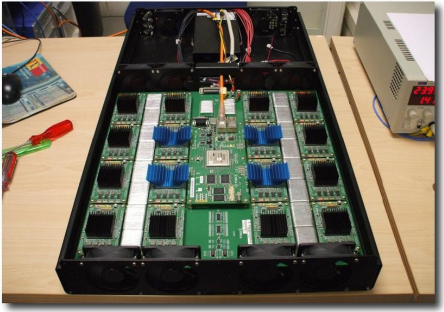
- Numerical work is needed to make these theories quantitative and to determine which one best describes the 3DSG phase.

The Janus Computer



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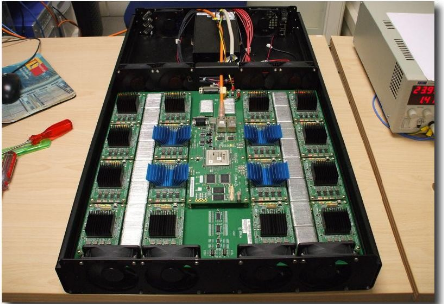

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70% of Janus time past 14 months: parallel tempering simulation of the 3D SG in large lattices at low temperatures.

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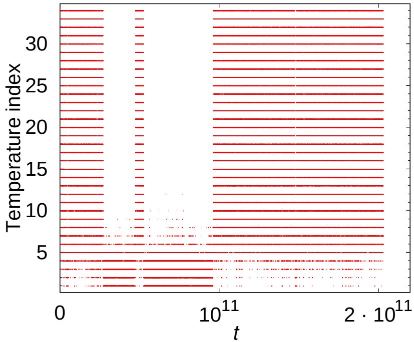
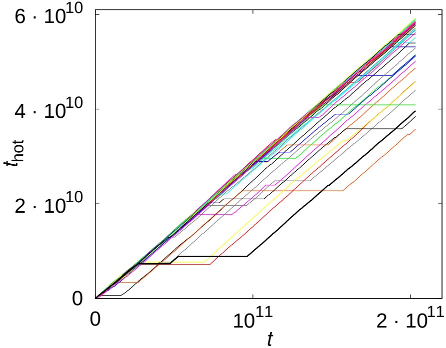
- Temperature swap attempt every 10 Heat-Bath.
- Dramatic sample dependence of thermalization time.
- Temperature chaos: T_{min} needs to grow with L.

L	T_{min}	T_{max}	N_T	$N_{\mathrm{MC}}^{\mathrm{min}}$	$N_{\mathrm{MC}}^{\mathrm{max}}$	$N_{\mathrm{MC}}^{\mathrm{med}}$	N_s
8	0.150	1.575	10	5.0×10^6	8.30×10^8	7.39×10^6	4000
8	0.245	1.575	8	1.0×10^6	6.48×10^8	2.30×10^6	4000
12	0.414	1.575	12	1.0×10^7	5.01×10^9	4.48×10^7	4000
16	0.479	1.575	16	4.0×10^8	2.72×10^{11}	9.37×10^8	4000
24	0.625	1.600	28	1.0×10^9	1.81×10^{12}	3.20×10^9	4000
32	0.703	1.549	34	4.0×10^9	7.68×10^{11}	1.08×10^{10}	1000
32	0.985	1.574	24	1.0×10^8	4.40×10^9	1.13×10^8	1000

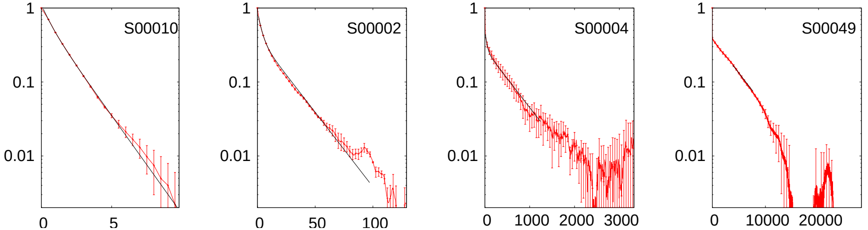
Follow the temperature of a single copy during the Parallel Tempering: the T-random walk is strongly non Markovian





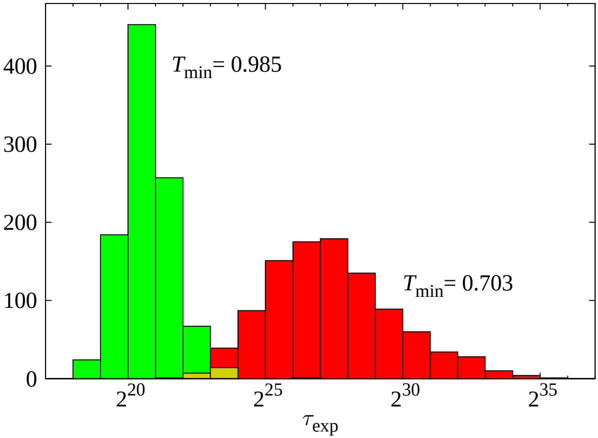
The random walk in T space (II)

- i_t temperature index at time t; pdf of i uniform in $\{1, 2, \ldots, N_T\}$
- $f(i)$, with $\sum_i f(i) = 0$ (i.e. $\langle f \rangle = 0$), changing sign only at i_c.
- $C(s) = \langle\, f(i_t) f(i_{t+s}\,)\rangle \longrightarrow \tau_{\mathbf{int}}, \tau_{\mathbf{exp}}$
- Automatize analysis: simulation length (at least) 12 $\tau_{\mathbf{exp}}$



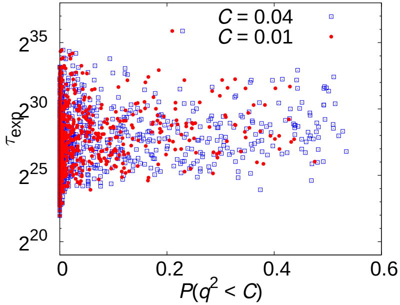
Sample to sample fluctuations in PT dynamics

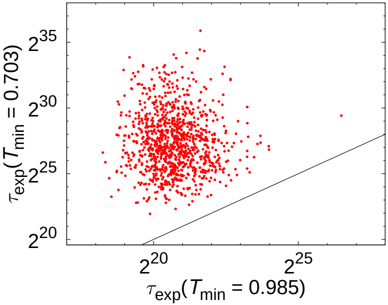
- τ_{exp} is a wildly oscillating variable, $\log \tau_{\mathrm{exp}}$ better behaved.



Sample to sample fluctuations in PT dynamics

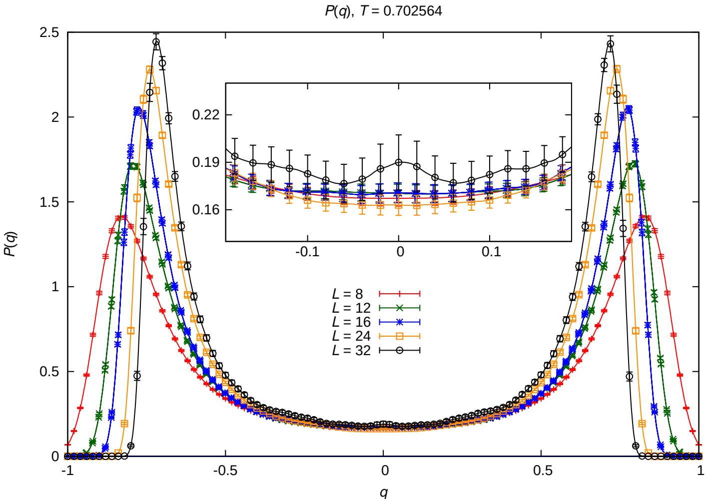
- $\tau_{\rm exp}$ is a wildly oscillating variable, $\log \tau_{\rm exp}$ better behaved.
- $\log \tau_{\rm exp}$ does not correlate with single T properties ($P(q)$). Failure of PT seems a genuine effect of temperature chaos.





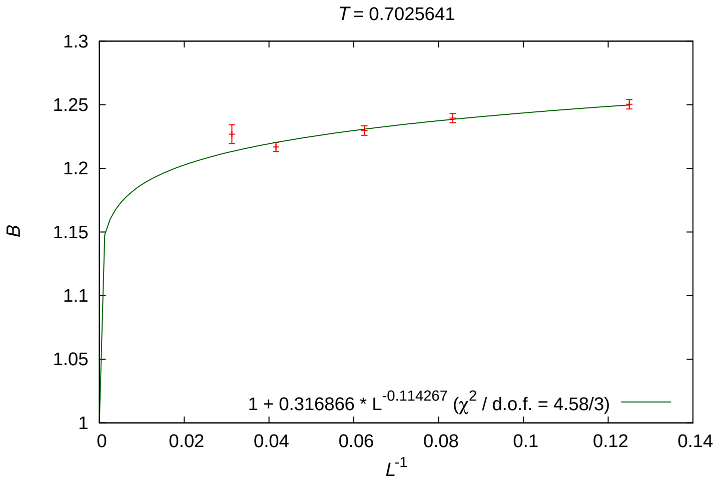
The $P(q)$ at low temperatures

$$P(q = C) \equiv \overline{\langle \delta(C - q) \rangle}, \; T < T_c$$



The $P(q)$ at low temperatures

$$B = \frac{\overline{\langle q^4 \rangle}}{\overline{\langle q^2 \rangle}^2}, \quad \text{droplet: } B^{(L=\infty)}_{T<T_c} = 1, \text{ RSB: } B^{(L=\infty)}_{T<T_c} > 1$$



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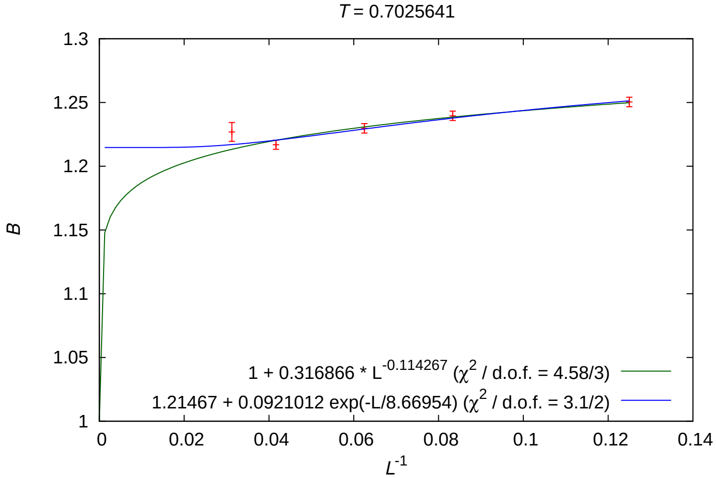

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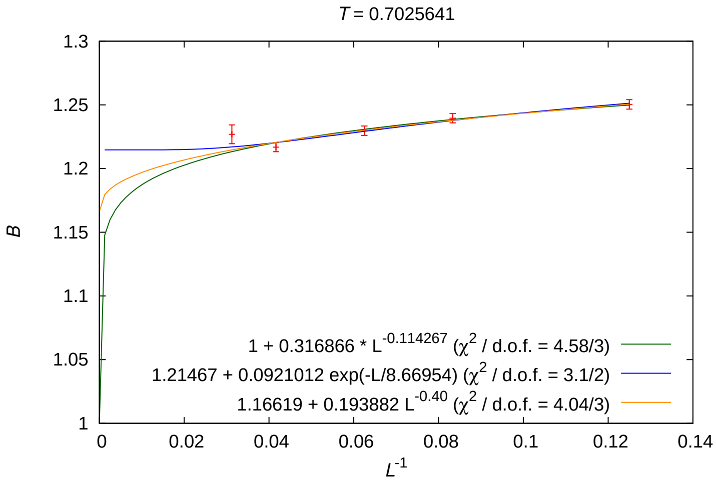
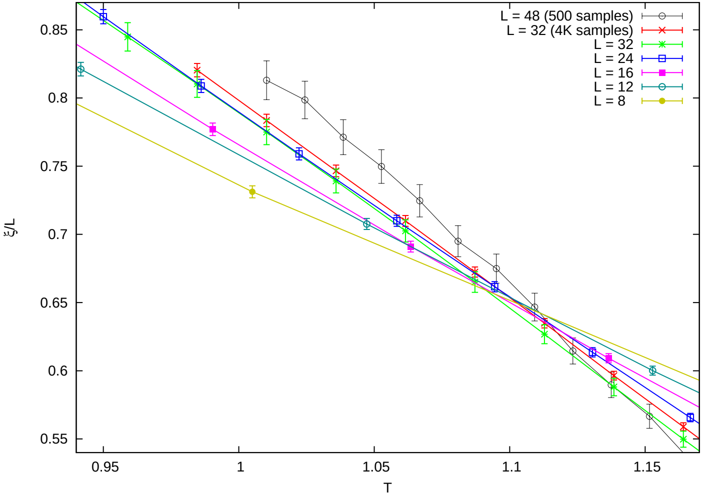


Distressing result: ξ/L KT-like?



Correlation functions (II)

A different interpretation:

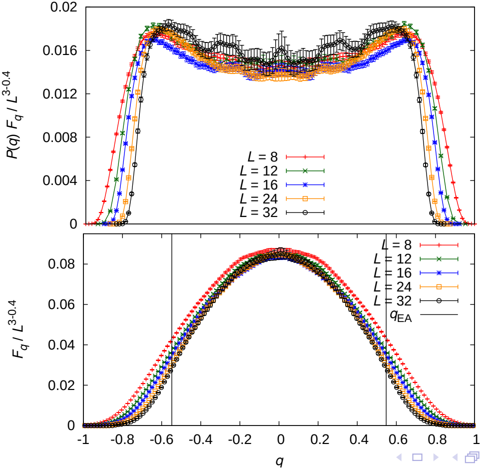
$$C_4(r) \sim \frac{1}{r^\theta} + \text{const.} \qquad \theta \sim 0.4$$

$$F = \hat{C}_4\left(\frac{2\pi}{L}\right) \sim L^{D-\theta} \qquad \chi = C_4(0) = L^D$$

$$\frac{\xi}{L} = \frac{1}{2L\sin\frac{\pi}{L}}\sqrt{\frac{\chi}{F} - 1} \sim L^{\theta/2} \qquad \left[\frac{32}{24}\right]^{0.4/2} \approx 1.06$$

Correlation functions (II)

It is best to study clustering correlation functions: $F = \int_{-1}^{1} \mathrm{d}q \; F_q P(q)$.
Nice scaling for $q \sim 0$.



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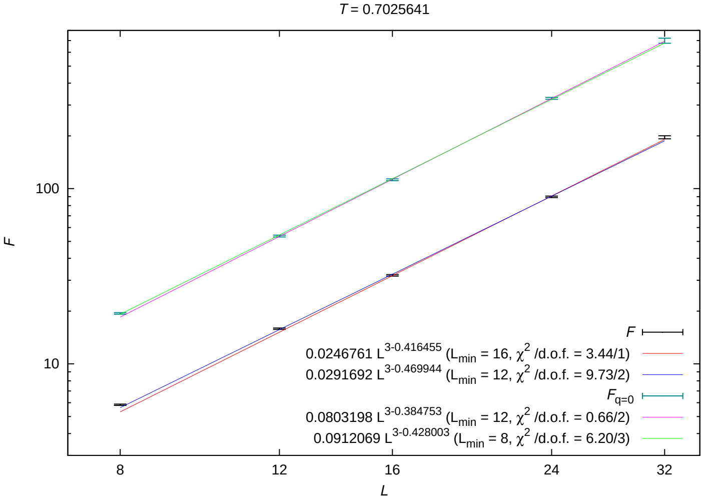
Computation of θ. Beware of finite-size effects:



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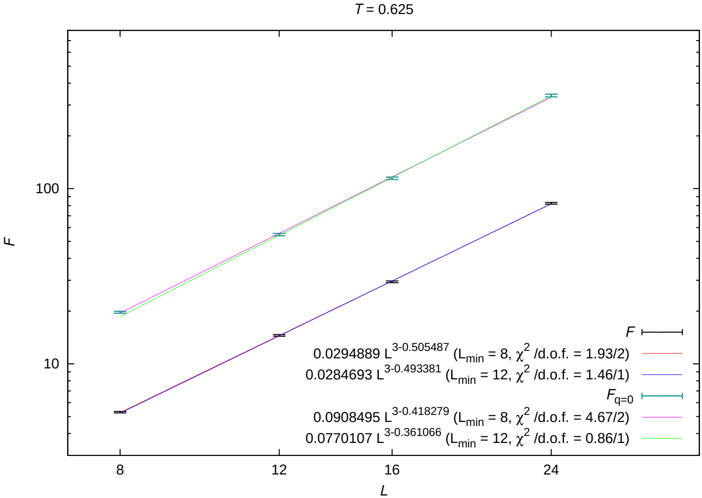
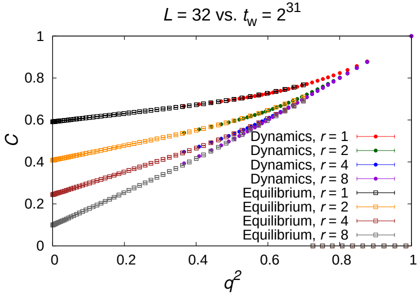
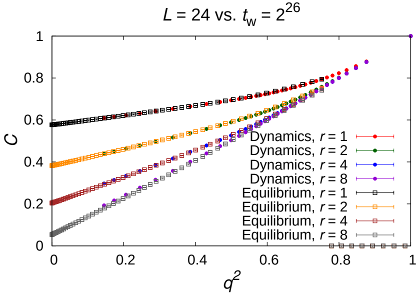


Equilibrium is relevant to *nonequilibrium* experiments

Surprise? Non-equilibrium computation of equilibrium $C_4(r|q)$
Equilibrium $L = 32$:0.002 experimental seconds!! ($L = 128$: 1 hour)





Qlink susceptibility

- Couple two real replicas through Q_{link}

$$H = H^{(1)} + H^{(2)} + \epsilon N Q_{\text{link}}$$

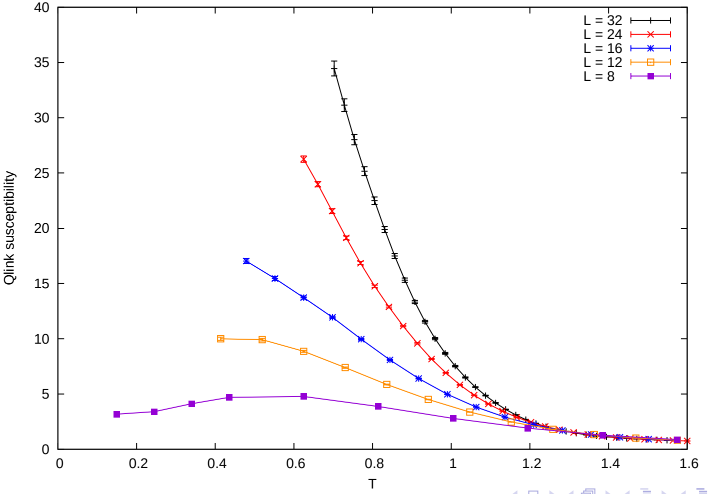
- RSB: discontinuity $\overline{\langle Q_{\text{link}} \rangle}_{\epsilon=0^+, L=\infty} - \overline{\langle Q_{\text{link}} \rangle}_{\epsilon=0^-, L=\infty} > 0$
- Droplects expect $\overline{\langle Q_{\text{link}} \rangle}_{\epsilon, L=\infty}$ differentiable at $\epsilon = 0$.
- Finite system:

$$\text{Does} \quad \frac{\mathrm{d}\overline{\langle Q_{\text{link}} \rangle}_{\epsilon, L}}{\mathrm{d}\epsilon} \propto L^D \quad ?$$

(mind expected violation of Chayes *et al.* bound).

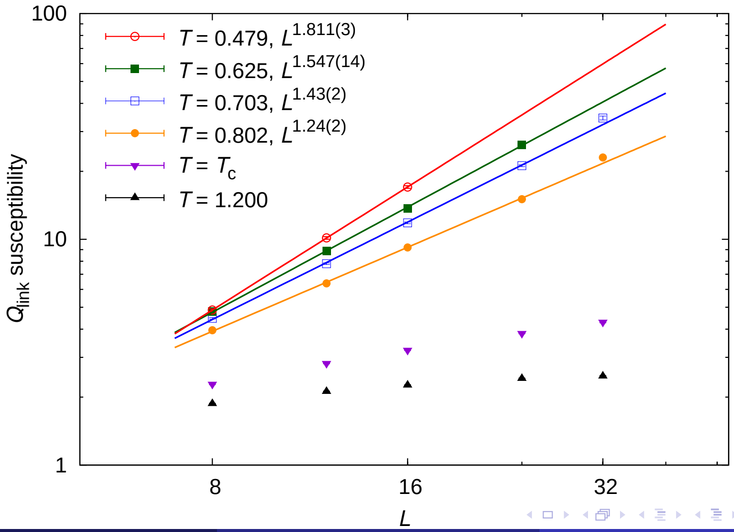
Qlink susceptibility

Certainly, not an effect of critical fluctuations ($T_c \approx 1.1$)



Qlink susceptibility

Effective exponents, already violate Chayes bound. Pre-asymptotic.



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- Large number of well thermalized samples, awaiting further analysis.

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  - A tenure tack through *Ramon y Cajal* program (Zaragoza, ask Alfonso Tarancon; competitive at a national level).