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# Non-equilibrium phase transition in an exactly solvable driven Ising model with friction


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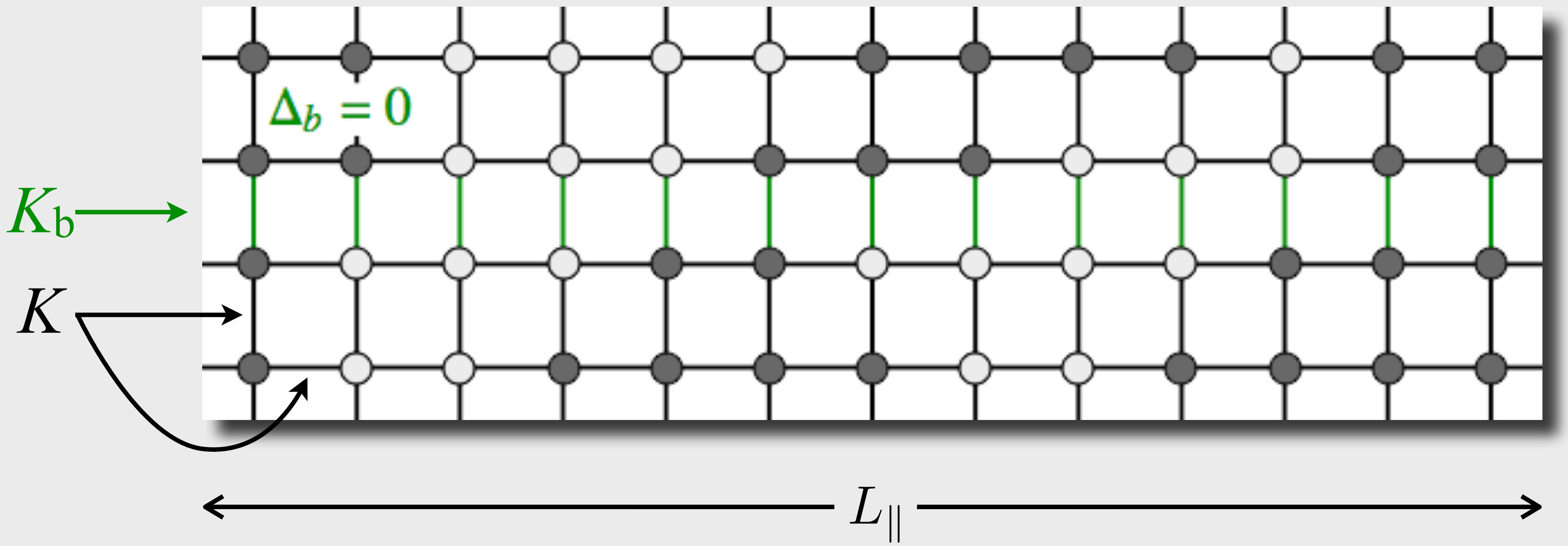
# Fluctuation induced friction at surfaces

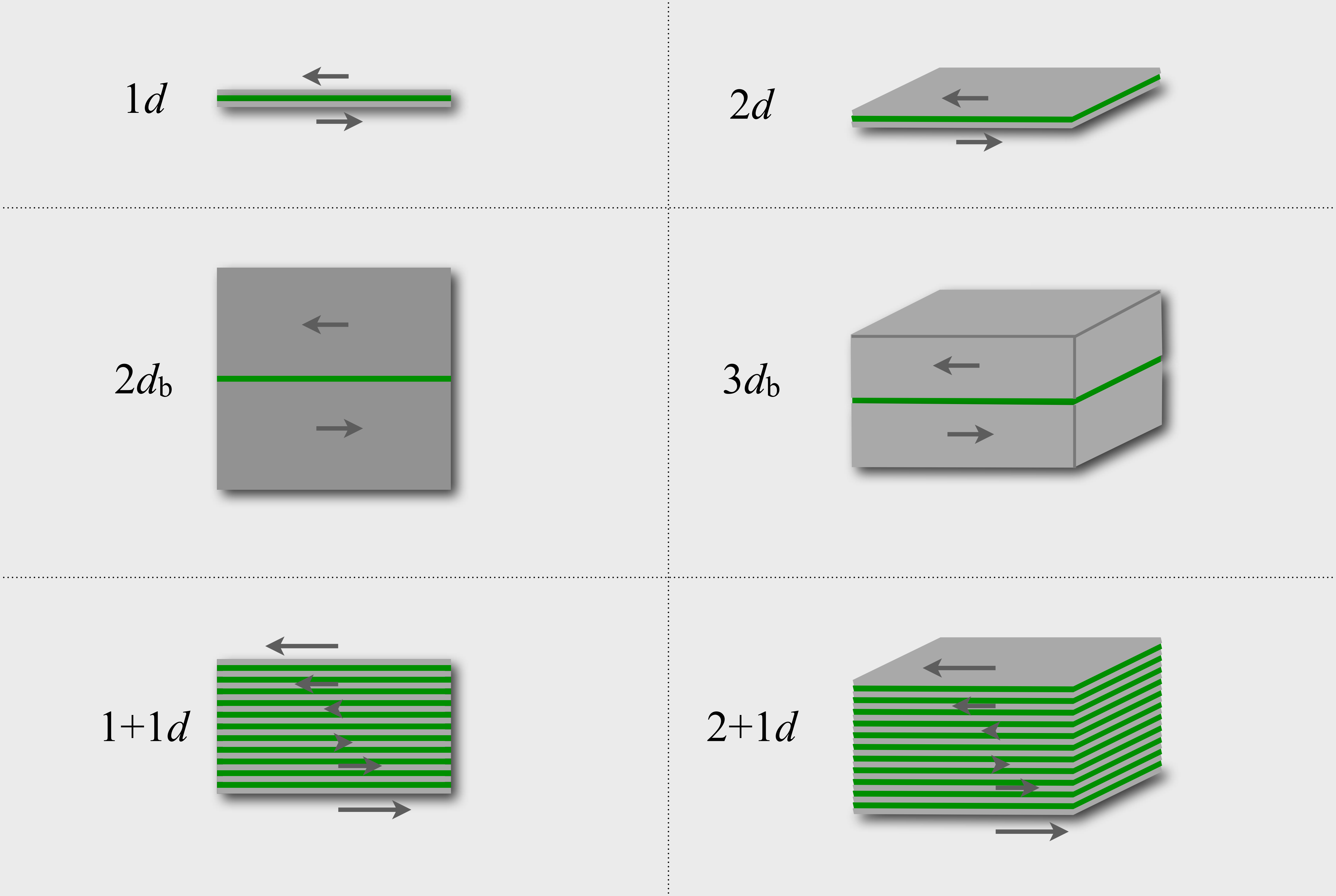



- Time–dependent Hamiltonian
  $\beta = 1/k_B T$, $K = \beta J$
- $\Delta_b(t) = vt$
- $v = 1 \quad \widehat{=} \quad 1\,\mathrm{cm/s}$

  $a_0 \approx 10^{-10}$ m, $t_0 \approx 10^{-8}$ s

$$\beta\mathcal{H}(t) = -K \sum_{\langle ij \rangle} \sigma_i \sigma_j - K_b \sum_{\langle ij \rangle_b(t)} \sigma_i \sigma_j$$

# Geometries for surface friction



# Non–equilibrium quantities: Monte Carlo results

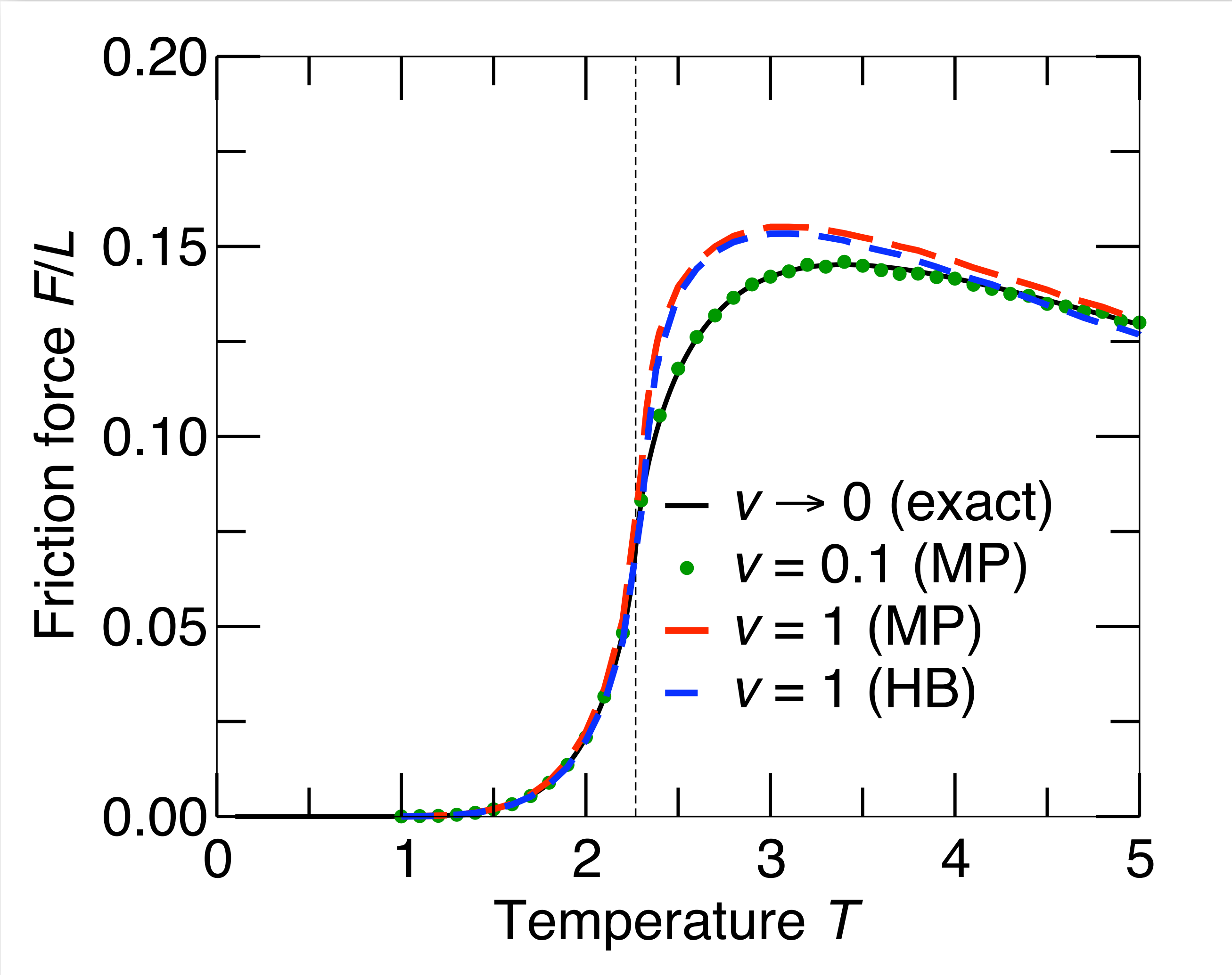
- Energy dissipation rate

$$P(v) = \frac{\langle \Delta E_{\mathrm{b}} \rangle}{\Delta t}$$

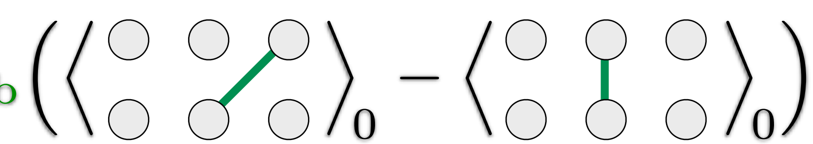
- Friction force

$$F(v) = \frac{P(v)}{v}$$

- Exact solution for small $v$



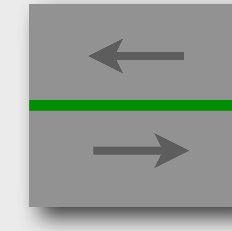

$$F(v \to 0) = \frac{\langle E_{\mathrm{b}}^{(f)} - E_{\mathrm{b}}^{(i)} \rangle}{\Delta s} = -L J_{\mathrm{b}} \Big( \langle \cdot \rangle_0 - \langle \cdot \rangle_0 \Big)$$



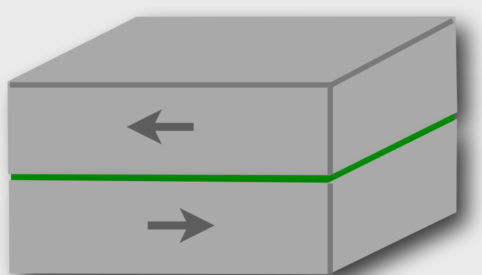
$$= -L J_{\mathrm{b}} \big( \langle \sigma_{0,0} \sigma_{1,1} \rangle_0 - \langle \sigma_{0,0} \sigma_{0,1} \rangle_0 \big)$$

D. Kadau, A. H. & D. E. Wolf, Phys. Rev. Lett. 101, 137205 (2008)

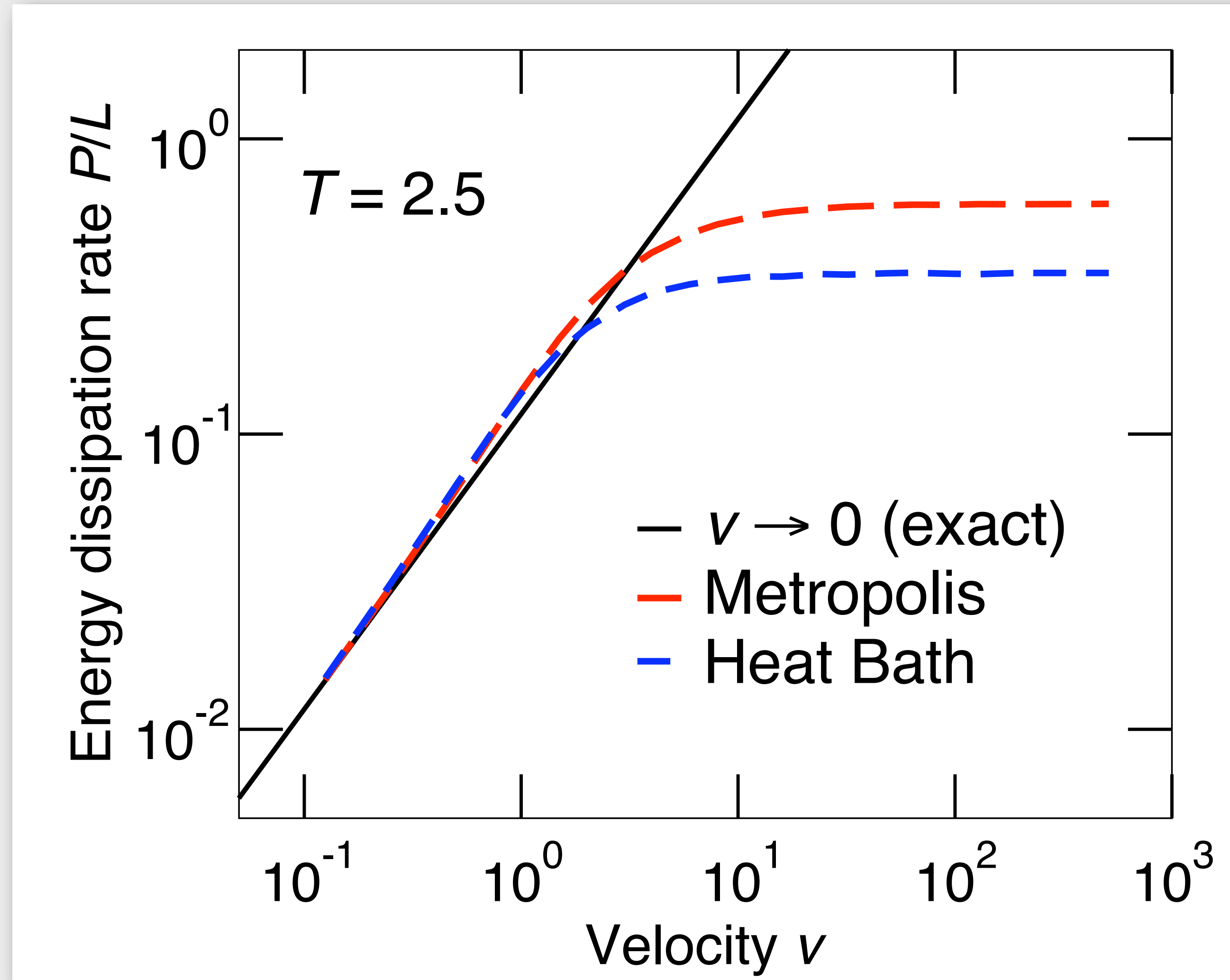
# Influence of finite driving velocity *v*



- Dissipation $P$ saturates at high velocities $v \gg 1$
- Quantities depend on Monte Carlo algorithm
- $3d_\mathrm{b}$ case (ideal contact):



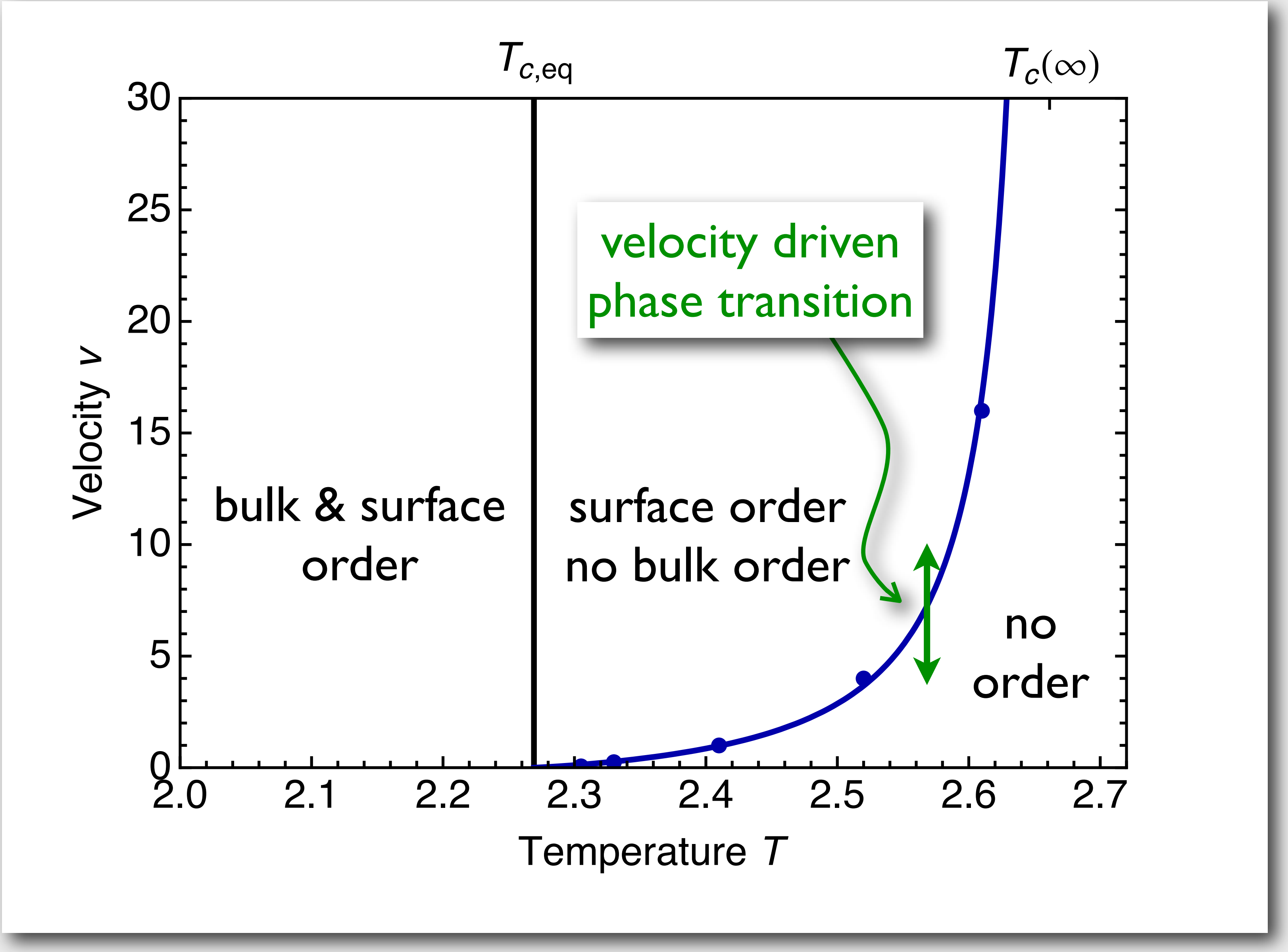
$$P(v \gg 1) \approx 100\,\mathrm{W cm^{-2}}$$



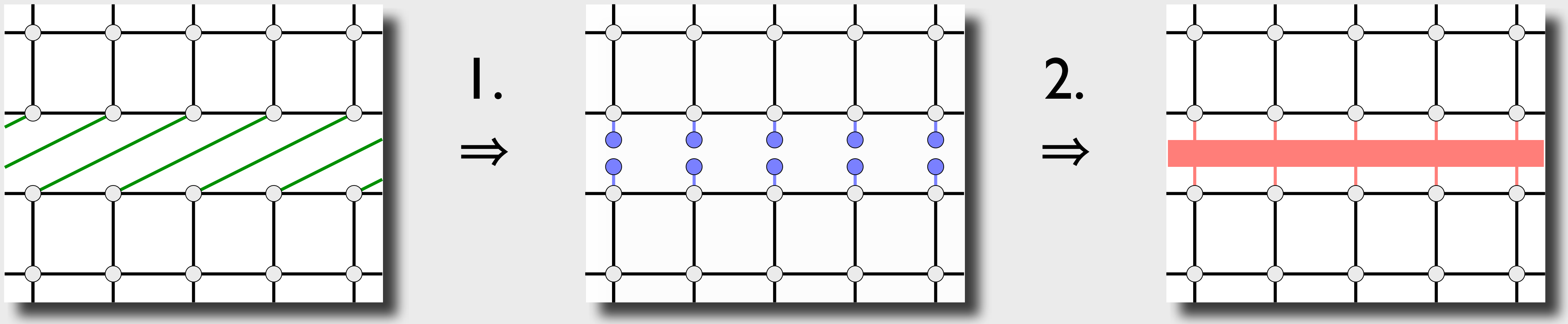

D. Kadau, A. H. & D. E. Wolf, Phys. Rev. Lett. 101, 137205 (2008)

# The phase diagram

- $T_c$ depends on velocity $v$
- 3 phases:
  - bulk order
  - surface order
  - no order
- Velocity driven surface phase transition



# High velocities: Exact solution in 3 steps




0. Start with driven system

1. Map boundary couplings $K_b$ to fluctuating fields $\mu_i$ with average $\langle \mu_i \rangle = m_b$

2. Replace fluctuating fields $\mu_i$ with static field $h_b(m_b)$

$$\tanh h_b = m_b \tanh K_b$$

3. Solve self-consistency condition

$$m_{b,eq}(T, h_b(m_b)) = m_b \quad \Rightarrow \quad m_b(T)$$

# Application to $1d$ model at high velocities

- Solution of equilibrium model

$$m_{\mathrm{eq}}(K,h) = \frac{\sinh h}{\sqrt{e^{-4K} + \sinh^2 h}}$$

$$\chi^{(0)}_{\mathrm{eq}}(K) = e^{2K}$$

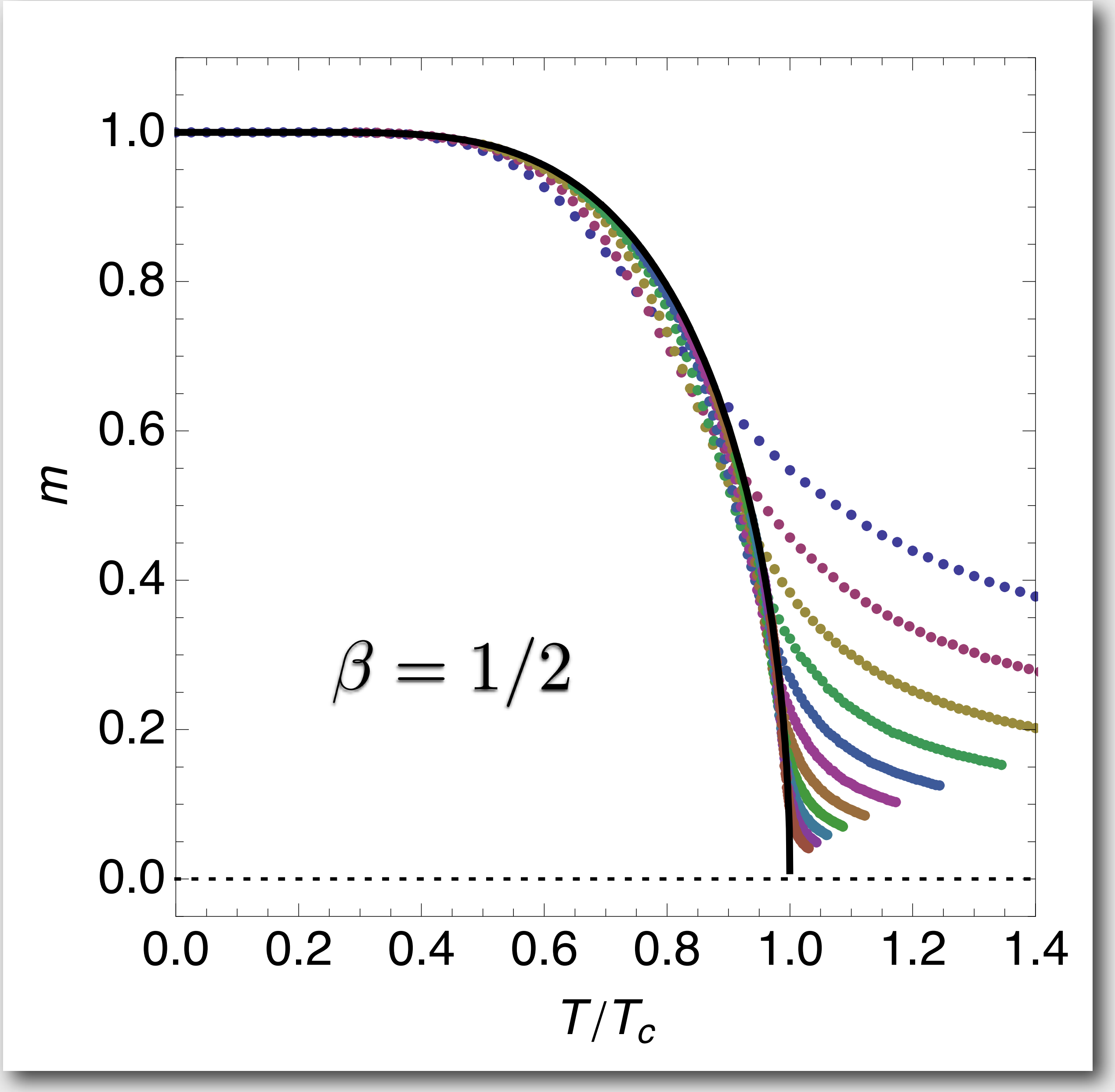
- Critical point $T_{\mathrm{c}}$:

$$\chi^{(0)}_{\mathrm{eq}}(K_c)\,\tanh K_{\mathrm{b},c} \overset{!}{=} 1$$

$$T_{\mathrm{c}} = 2/\log(\sqrt{2}+1) = 2.26918\ldots$$

- Order parameter $m$:

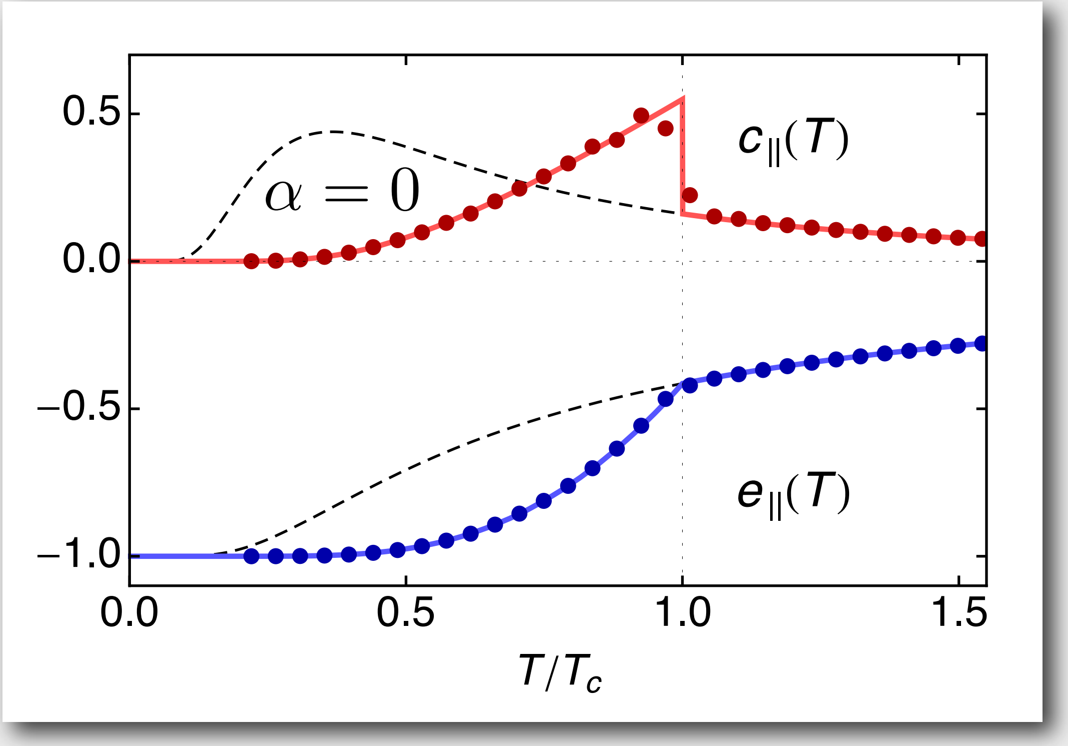
$$m(K,K_{\mathrm{b}}) = \sqrt{\frac{\cosh 2K_{\mathrm{b}} - \coth 2K}{\cosh 2K_{\mathrm{b}} - 1}}$$

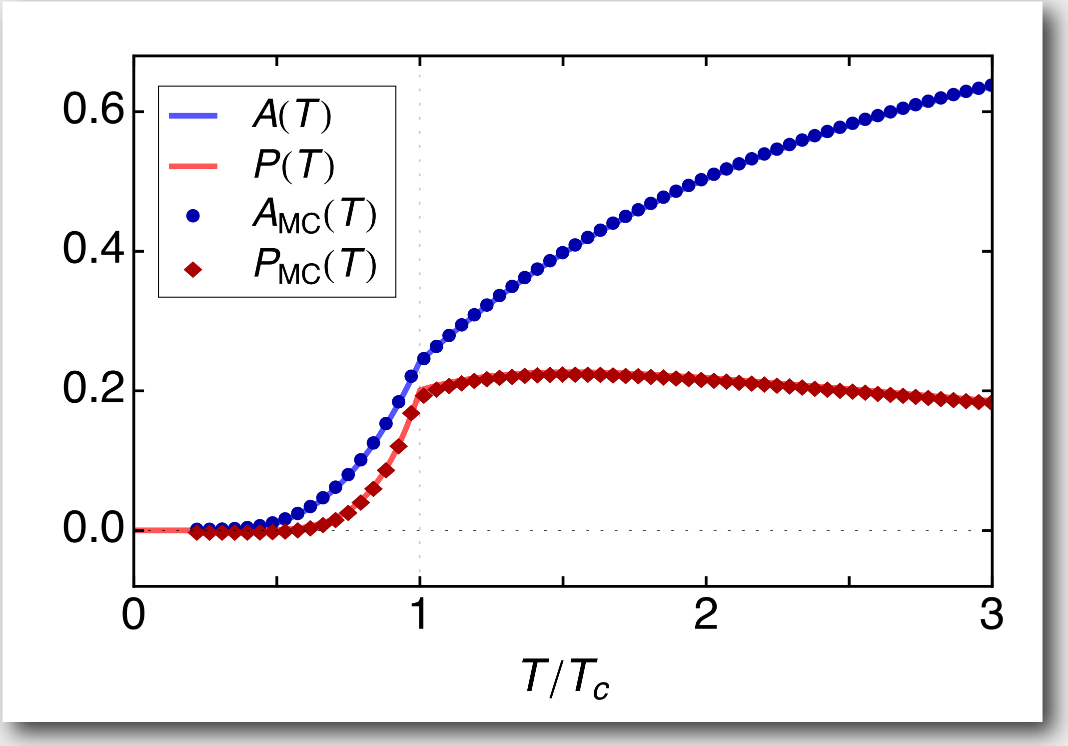



- **Note**: Identical to surface magnetization of $2d$ Ising model

# Other quantities in $1d$

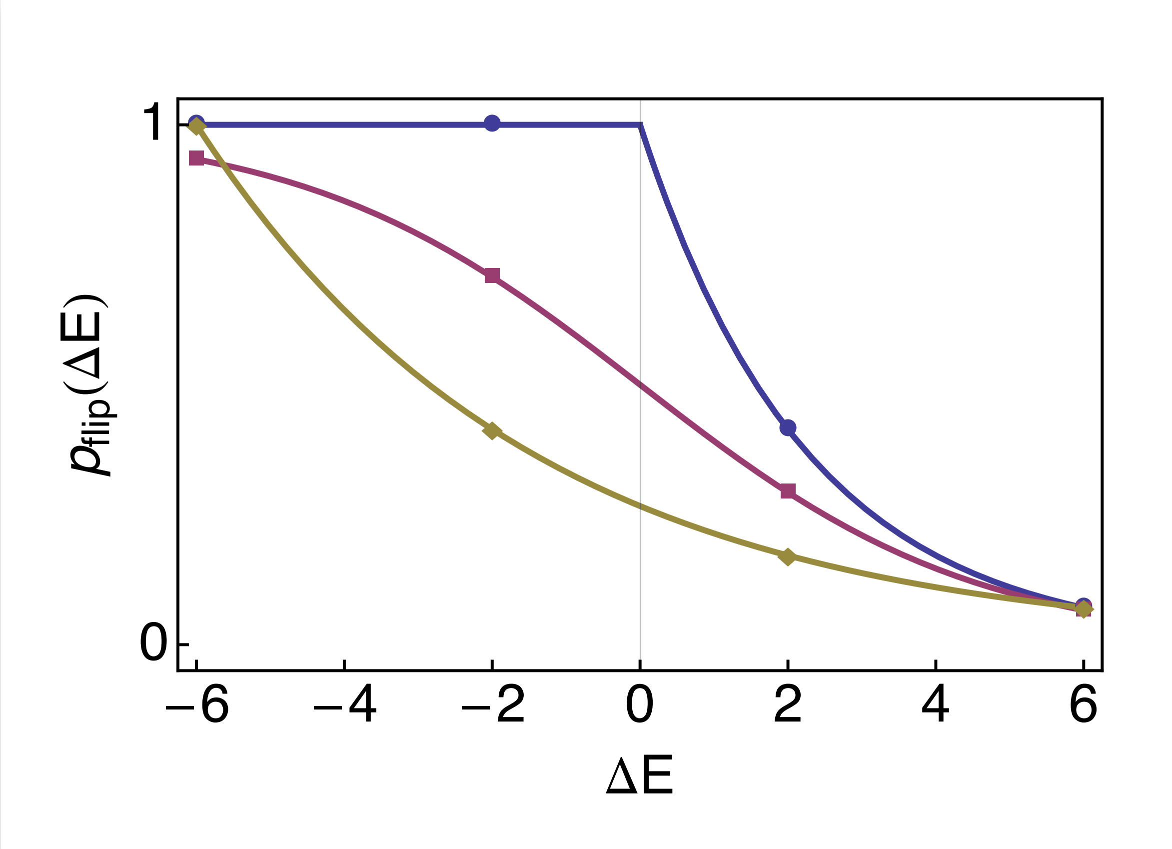
- Static properties:
  - internal energy $e_{||}(T)$
  - specific heat $c_{||}(T)$
  - correlation functions $G(r)$
  - ...
- Dynamic properties:
  - spin flip acceptance rate $A(T)$
  - energy dissipation rate $P(T)$



# An integrable Monte Carlo algorithm

- Critical temperature $T_\mathrm{c}$ depends on MC algorithm
- Example: $1d$ case with $J_\mathrm{b} = J = 1$

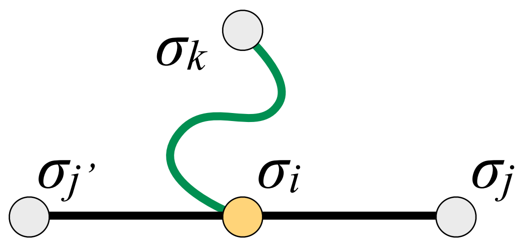



Acceptance rate $A = \langle p_\mathrm{flip} \rangle$

| | | | |
|---|---|---|---|
| Metropolis | $p_\mathrm{flip}^\mathrm{MP}(\Delta E) = \min(1, e^{-\beta\,\Delta E})$ | $T_\mathrm{c}^\mathrm{MP} = 1.910(2)$ | $A_\mathrm{c}^\mathrm{MP} = 0.476(2)$ |
| Heat-Bath | $p_\mathrm{flip}^\mathrm{HB}(\Delta E) = \dfrac{1}{1 + e^{\beta\,\Delta E}}$ | $T_\mathrm{c}^\mathrm{HB} = 2.031(2)$ | $A_\mathrm{c}^\mathrm{HB} = 0.366(2)$ |
| Multiplicative | $p_\mathrm{flip}^*(\Delta E) = e^{-\frac{\beta}{2}(\Delta E - \Delta E_\mathrm{min})}$ | $T_\mathrm{c}^* = 2.269(1)$ | $A_\mathrm{c}^* = 0.242(2)$ |
| Exact solution | | $T_\mathrm{c} = 2.2692\ldots$ | $A_\mathrm{c} = 0.24264\ldots$ |

# An integrable Monte Carlo algorithm

- Consider MC update of boundary spin $\sigma_i$




$$\Delta E = \underbrace{2J\sigma_i \sum_{\langle j \rangle} \sigma_j}_{\Delta E_1} + \underbrace{2J_\mathrm{b}\sigma_i\sigma_k}_{\Delta E_2}$$

- Usual MC algorithms introduce correlations over boundary: Influence on $\sigma_k$ depends on $\sigma_j$

$$\sigma_i = -\sigma_j \rightarrow \Delta E_1 = -4J \rightarrow p_\mathrm{flip} = 1$$
$$\sigma_i = +\sigma_j \rightarrow \Delta E_1 = +4J \rightarrow p_\mathrm{flip}(\sigma_k)$$

- Solution: Use algorithm which
  - fulfills detailed balance
  - is multiplicative

$$\frac{p_\mathrm{flip}(\Delta E)}{p_\mathrm{flip}(-\Delta E)} = e^{-\beta \Delta E}$$

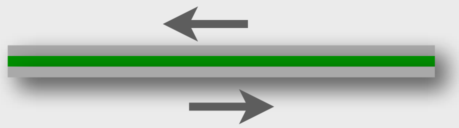
$$p_\mathrm{flip}(\Delta E_1 + \Delta E_2) = p_\mathrm{flip}(\Delta E_1)\, p_\mathrm{flip}(\Delta E_2)$$

- Result:

$$p^*_\mathrm{flip}(\Delta E) = e^{-\frac{\beta}{2}(\Delta E - \Delta E_\mathrm{min})}$$

# Results for other geometries (sc lattice)

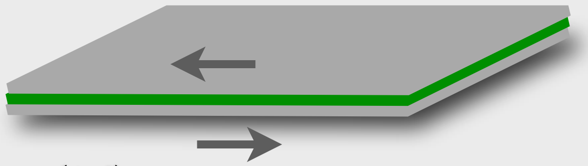
$1d$



$$\chi_{\mathrm{eq}}^{(1d)}(K_{\mathrm{c}}) \tanh K_{\mathrm{b,c}} = 1$$

$$T_{\mathrm{c}} = 2/\log(\sqrt{2}+1) = 2.26918\ldots$$

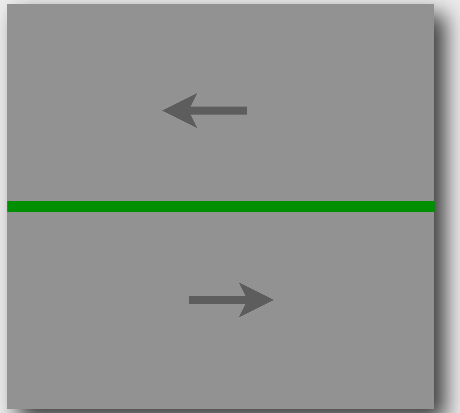
$2d$



$$\chi_{\mathrm{eq}}^{(2d)}(K_{\mathrm{c}}) \tanh K_{\mathrm{b,c}} = 1$$

$$T_{\mathrm{c}} = 4.0587824231379800009877504068 0\ldots$$

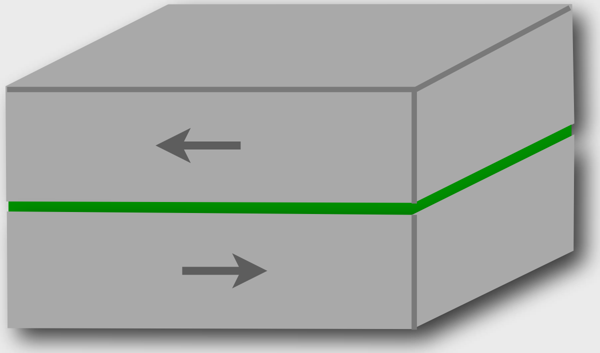
$2d_{\mathrm{b}}$



$$\chi_{\mathrm{b,eq}}^{(2d)}(K_{\mathrm{c}}) \tanh K_{\mathrm{b,c}} = 1$$

$$T_{\mathrm{c}} = 2.6614725655752\ldots$$

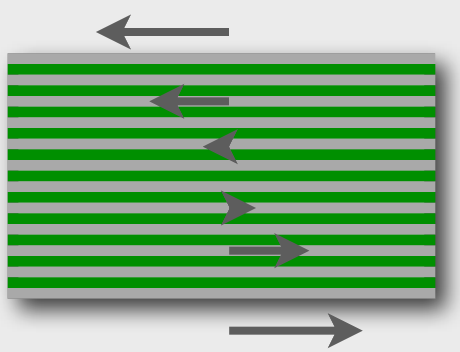
$3d_{\mathrm{b}}$



$$\chi_{\mathrm{b,eq}}^{(3d)}(K_{\mathrm{c}}) \tanh K_{\mathrm{b,c}} = 1$$

$$T_{\mathrm{c}} = 4.8(1)$$

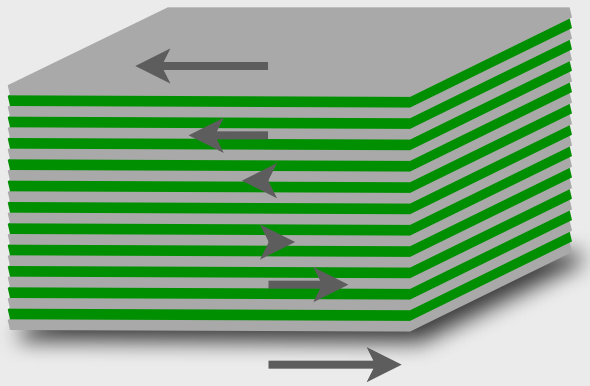
$1{+}1d$



$$2\,\chi_{\mathrm{eq}}^{(1d)}(K_{\mathrm{c}}) \tanh K_{\mathrm{b,c}} = 1$$

$$T_{\mathrm{c}} = 1/\log\left(\tfrac{1}{2}\sqrt{3+\sqrt{17}}\right) = 3.46591\ldots$$

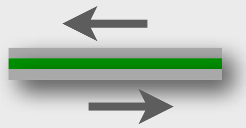
$2{+}1d$

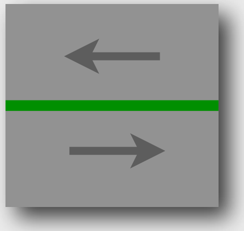


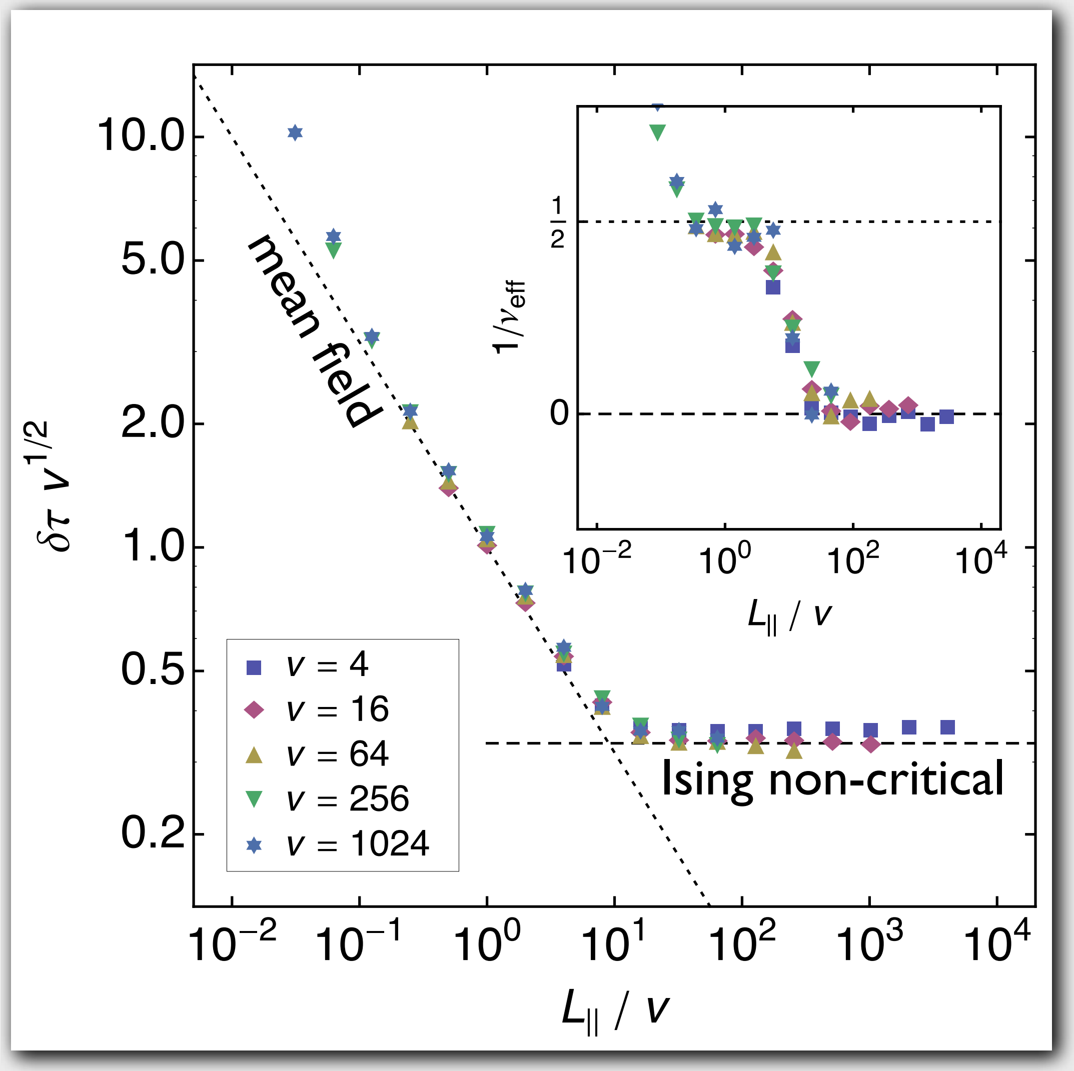
$$2\,\chi_{\mathrm{eq}}^{(2d)}(K_{\mathrm{c}}) \tanh K_{\mathrm{b,c}} = 1$$

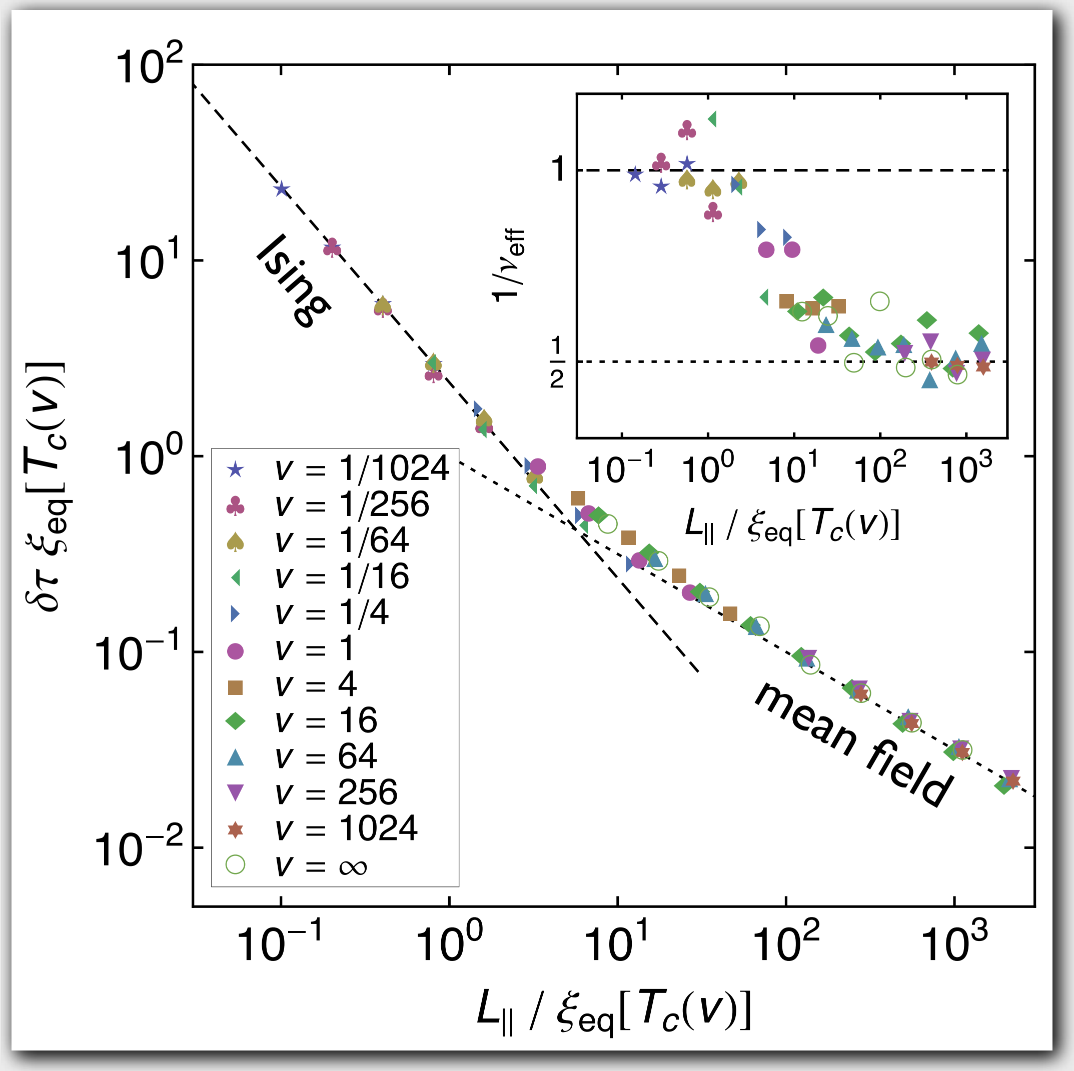
$$T_{\mathrm{c}} = 5.2647504145147435505980172034 24\ldots$$

# Finite *v*: Cross-over scaling of critical width δτ








A.H., Phys. Rev. E 80 (in press), arXiv:0909.0533

# Conclusions & Outlook

- New contribution to friction from spin correlations
- Driven model shows non–equilibrium phase transition
- Integrable *multiplicative* MC rate for non-equilibrium systems
- Exactly solvable for high velocities ($v \to \infty$) in many geometries
- Phase transition in mean–field class for $d > 1$ and $v > 0$
- Strongly anisotropic phase transition in sheared systems, with $\theta = \nu_{||} / \nu_{\perp} = 3$