Exact ground states of the random-field Ising magnet around the upper critical dimension

Björn Ahrens, Alexander K. Hartmann

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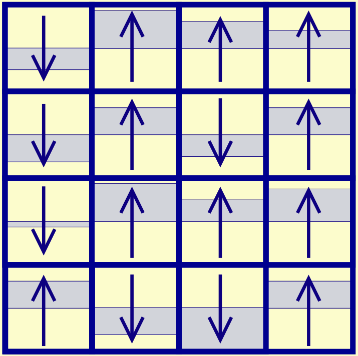
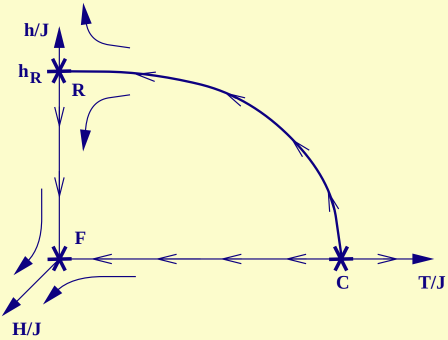
Random-field Ising magnets (RFIM)

$$\mathcal{H} = -J\sum_{\langle i,j\rangle} s_i s_j - h\sum_i \sigma_i s_i - \sum_i H s_i$$



- d dimensional hyper cubic lattice
- ferromagnetically coupled Ising spins $s_i = \pm 1$
- quenched random local field $h\sigma_i$ on each site
- small external field H
- phase transition at $h = h_c$
- upper critical dimension $d_u \geq 6$ [Tasaki, 1989]; mean-field behavior is believed

Phase diagram

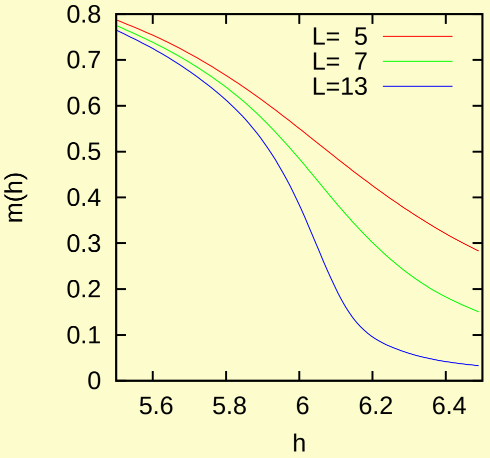


RNG flow in the phase diagram

[Nattermann, 1997]

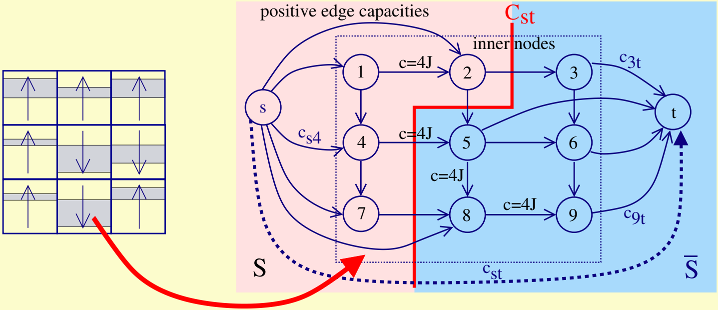
- mean field theory: second order phase transition along PB
- saddle point **R** is attractive to RNG flow on the phase boundary (PB) for $T > 0$
- critical behavior at $T = 0$ is equal to that along PB

Phase transition



magnetisation as function of disorder in $d = 5$

Method



- map random field to edge weights of a graph
- calculate maximum flow in polynomial time:
  $\sim \mathcal{O}(n^{2/3} m \log(n^2/m))$ [Goldberg and Rao, 1997]
- obtain exact groundstate

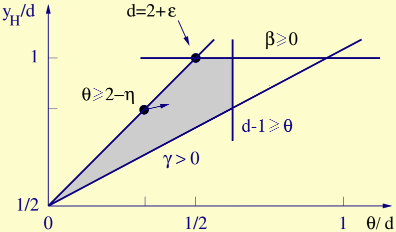
Definition of the critical exponents

Magnetization	$m \sim \|h-h_c\|^{\beta}$	$\beta = 1$
Suceptibility	$\chi \sim \|h-h_c\|^{-\gamma}$	$\gamma = 1$
Specific heat	$C \sim \|h-h_c\|^{-\alpha}$	$\alpha = 0$
Correlation length	$\xi \sim \|h-h_c\|^{-\nu}$	$\nu = 1/2$

Exponents obey inequalities $\Rightarrow$ domains of allowed values.

$$y_J = d - \frac{2-\alpha}{\nu}$$

$$\theta := d - \beta/\nu$$

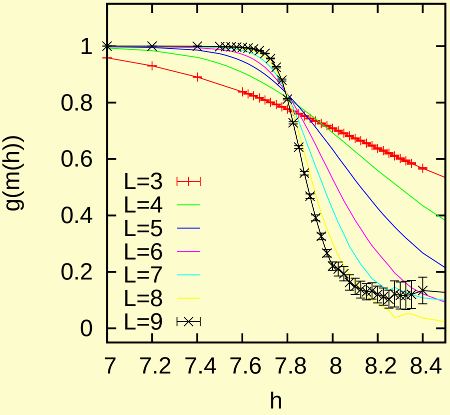


Binder parameter

Definition

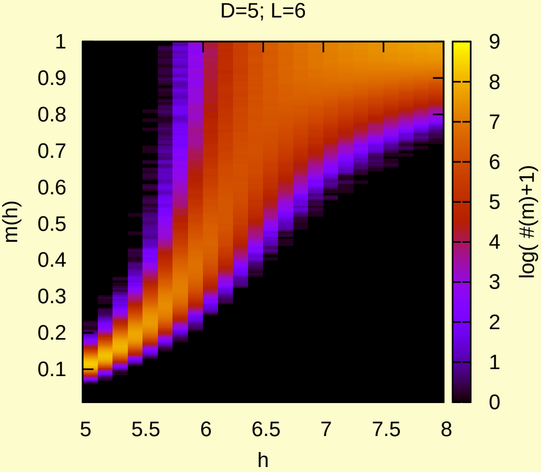
$$g(L,h) = \frac{1}{2}\left(3 - \frac{[\langle m^4\rangle]_h}{[\langle m^2\rangle]_h^2}\right)$$

- $g(L,h)$ governed by L/ξ, with correlation length ξ
- crossing for all $g(L,h)$ at $h = h_c$

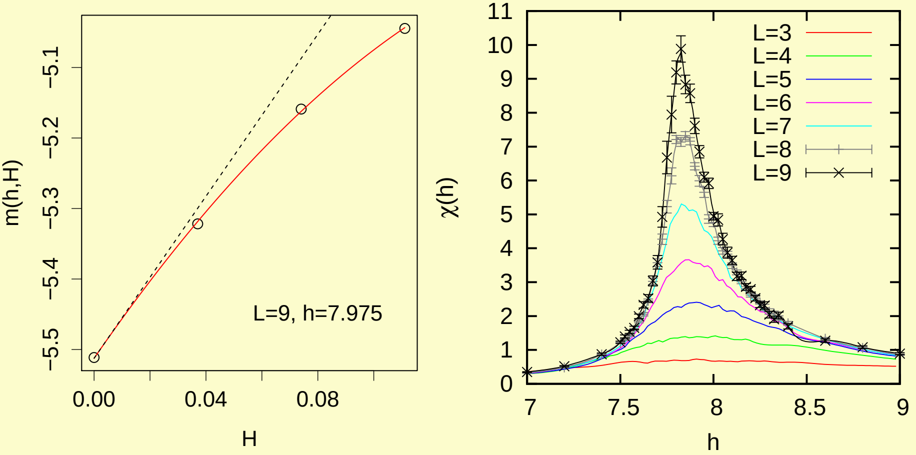


Magnetization Distribution

For special distributions, like double-delta the binder cumulant is negative



Susceptibility



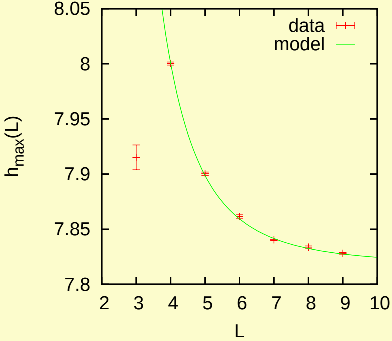
Definition: $\chi(h) = \frac{\mathrm{d}m}{\mathrm{d}H}\Big|_{H=0}$

therefore aditionally simulations at $H = \{0, H_1, 2H_1, 3H_1\}$

Peaks of susceptibility

Scaling law: $\chi(h,L) \sim L^{-\gamma/\nu}\tilde{\chi}\left((h-h_c)L^{-1/\nu}\right)$

scaling of the peak position

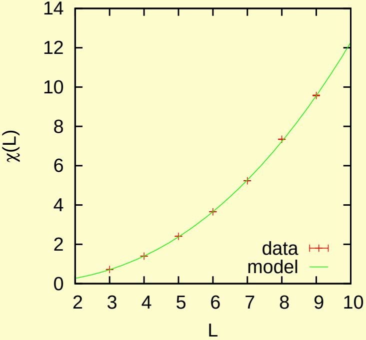


$h^*(L) = h_c + cL^{-1/\nu^*}$

$h_c = 7.817(1)$, $c = 21(4)$,

$\nu^* = -3.43(15)$

scaling of the peak height

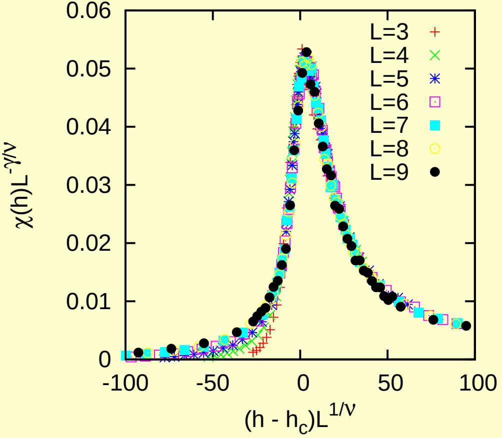


$\chi_{max}(L) = s_0 L^{\gamma/\nu}$

$s_0 = 0.052(1)$, $\gamma/\nu = 2.36(1)$

Scaling of susceptibility

data collapse with: $\nu/\gamma = 0.484/1.152 = 2.381$



Specific heat*

Specific heat at $T = 0$:
$\mathcal{C} = \frac{\partial}{\partial h}\left(\frac{\partial E}{\partial J}\right)$

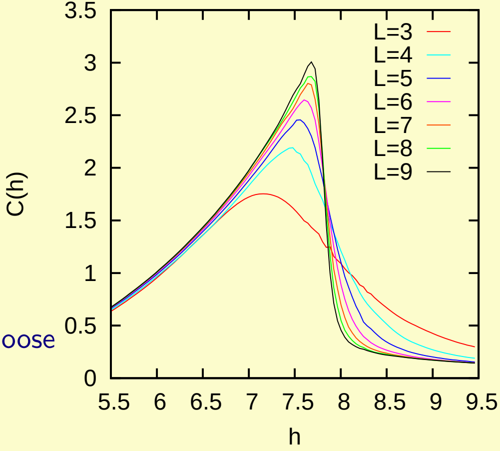
E is the total energy.
Using $\frac{\partial E}{\partial h} = E_h$ in

$$\frac{\partial E}{\partial h} = J\frac{\partial E_J}{\partial h} + h\frac{\partial E_h}{\partial h} + E_h$$

$$J\frac{\partial E_J}{\partial h} + h\frac{\partial E_h}{\partial h} = 0$$

same scaling in both terms: choose the first, so

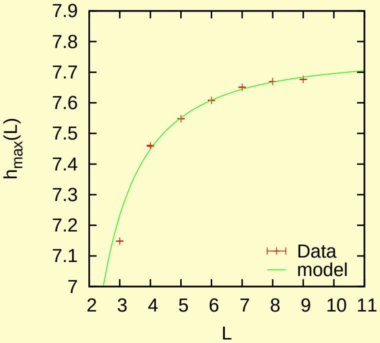
$$C\left(\frac{h_1+h_2}{2}\right) = \frac{[E_J(h_1)]_h - [E_J(h_2)]_h}{h_1 - h_2}$$



Peaks of specific heat

Scaling law: $C^*_{max}(h, L) \sim \log(L/L0)^{e_2}\,\tilde{C}\left((h - h_c)L^{-1/\nu}\right)$

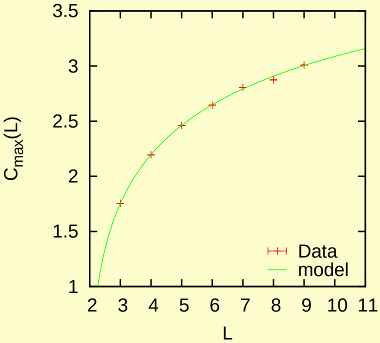
scaling of the peak position



$h_{max}(L) = h_\infty + AL^{-1/\nu}$

$\nu = 0.53(12)$, $h_\infty = 7.75(3)$, $A = -4.0(2.2)$

scaling of the peak height

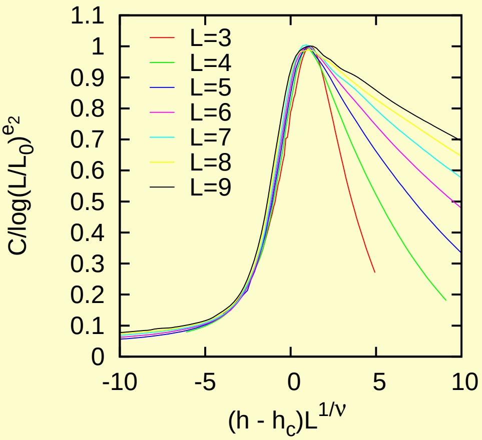


$C_{max}(L) = C_0 log(L/L0)^{e_2}$

$e_2 = 0.39(2)$, $C_0 = 2.58(3)$, $L_0 = 2.07(7)$

Scaling of specific heat

data collapse with log-corrections does not work so far:



Summary

- $\alpha = 0$ for $d = 5, 6, 7$
- $\nu = 0.48...0.53$ fits well to the MF theory
- γ is close to MF predictions
- scaling of peaks does not collapse the data
- corrections to scaling still unknown