# Phase Transitions in Spin Glasses

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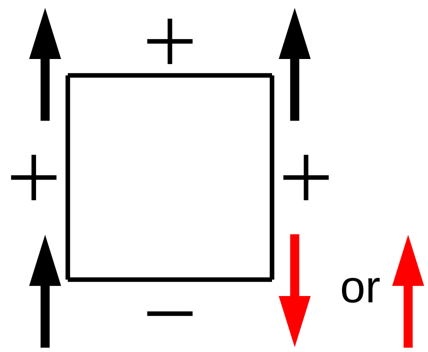
Can be downloaded from http://physics.ucsc.edu/~peter/talks/leipzig.pdf

# Overview

- Basic Introduction
  - What is a spin glass? experiments, theory
  - Finite size scaling for the correlation length
- Try to answer two long-standing questions for spin glasses
  - Is there a phase transition in an isotropic vector (XY or Heisenberg) spin glass?
  - Is there a transition in an Ising spin glass in a magnetic field (Almeida-Thouless line)?

# What is a spin glass?

A system with disorder and frustration.




Most theory uses the simplest model with these ingredients: the Edwards-Anderson Model:

$$\boxed{\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \; - \sum_{i} \mathbf{h}_i \cdot \mathbf{S}_i \, .}$$

$$[J_{ij}]_{\mathrm{av}} = 0; \quad [J_{ij}^2]_{\mathrm{av}}^{1/2} = J \, (= 1)$$

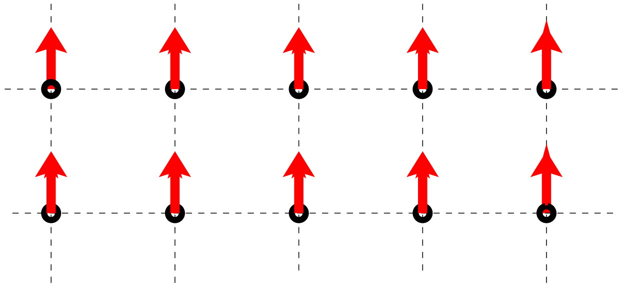
The $\mathbf{S}_i$ have $m$–components:

$$\begin{aligned} \mathbf{m} &= \mathbf{1} \quad \text{(Ising)} \\ \mathbf{m} &= \mathbf{2} \quad \text{(XY)} \\ \mathbf{m} &= \mathbf{3} \quad \text{(Heisenberg).} \end{aligned}$$

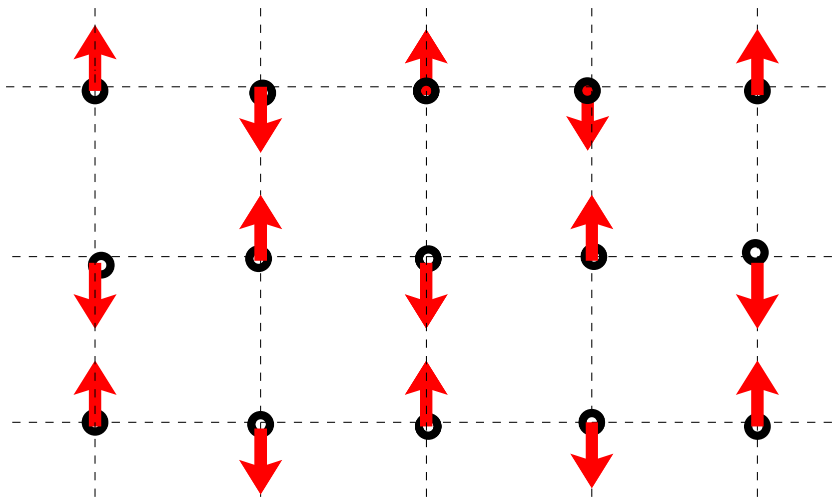
Will take a **Gaussian** distribution for the $\mathbf{J_{ij}}$.

# Compare with simpler systems

- Ferromagnet: All $J_{ij} > 0$. Ground state: All spins parallel. i.e. Ground state is trivial.



- Antiferromagnet: All (nearest neighbor) $J_{ij} < 0$ on a bipartite lattice. Ground state: Neighboring spins anti-parallel. i.e. Ground state is trivial.



- Spin Glass: Random mixture of ferro and anti-ferro. Because of frustration and disorder even the determining the ground state is a non-trivial optimization problem.

# Spin Glass Systems

- Metals:
  Diluted magnetic atoms, e.g. Mn, in non-magnetic metal, e.g. Cu.
  RKKY interaction:

$$J_{ij} \sim \frac{\cos(2k_F R_{ij})}{R_{ij}^3}$$

  Random in magnitude and sign, which gives frustration.
  Note: Mn (S-state ion) has little anisotropy; $\rightarrow$ Heisenberg spin glass.

# Spin Glass Phase Transition

Phase transition at $T = T_{SG}$.

For $T < T_{SG}$ the spin freeze in some random-looking orientation.

As $T \to T_{SG}^{+}$, the correlation length $\xi_{SG}$ diverges.

The correlation $\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle$ becomes significant for $R_{ij} < \xi_{SG}$, though the **sign** is random. A quantity which diverges is the spin glass susceptibility

$$\boxed{\chi_{SG} = \frac{1}{N} \sum_{i,j} [\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle^2]_{\mathrm{av}} \,,}$$

(notice the square) which is accessible in simulations. It is also essentially the same as the non-linear susceptibiliity, $\chi_{nl}$, defined by

$$\boxed{m = \chi h - \chi_{nl} h^3 + \cdots}$$

($m$ is magnetization, $h$ is field), which can be measured experimentally.

For the EA model $T^3 \chi_{nl} = \chi_{SG} - \frac{2}{3}$ .

# Non-linear susceptibility

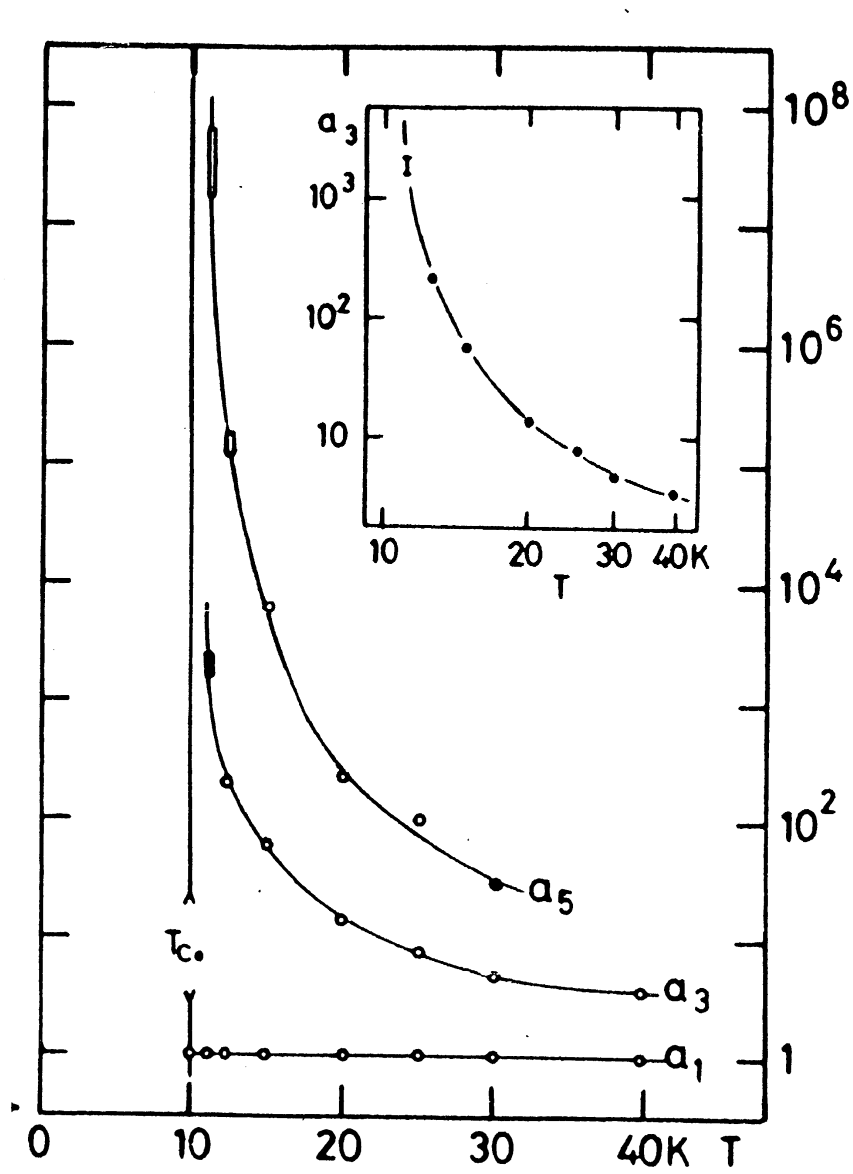
## Non-linear susceptibility

$\chi_{nl}$, is defined by

$$\boxed{m = \chi h - \chi_{nl} h^3 + \cdots}$$

Find: $\chi_{nl} \sim (T - T_{SG})$, with $\gamma$ generally in the range 2.5–3.5; i.e. there is a finite temperature spin glass transition.

e.g. results of Omari et al, (1983) for $CuMn_{1\%}$.



# Theory 1

Mean Field Theory (Edwards-Anderson, Sherrington-Kirkpatrick, Parisi). Exact solution of an infinite range (SK) model. Finite $T_{SG}$.

The Parisi solution is a "tour-de-force". Has an infinite number of order parameters.

More than a quarter of a century after it was obtained using the replica trick, (needed "replica symmetry breaking" (RSB)) Tallegrand proved rigorously! that the Parisi free energy is exact for the SK model.

# Theory 2

- Short-range (EA) models. Simulations on Ising systems also indicate a finite $T_{SG}$ (see later) in $d = 3$. Vector spin glasses? (See later.)

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- Equilibrium state below $T_{SG}$. Two main scenarios:

"Replica Symmetry Breaking" (RSB), (Parisi).

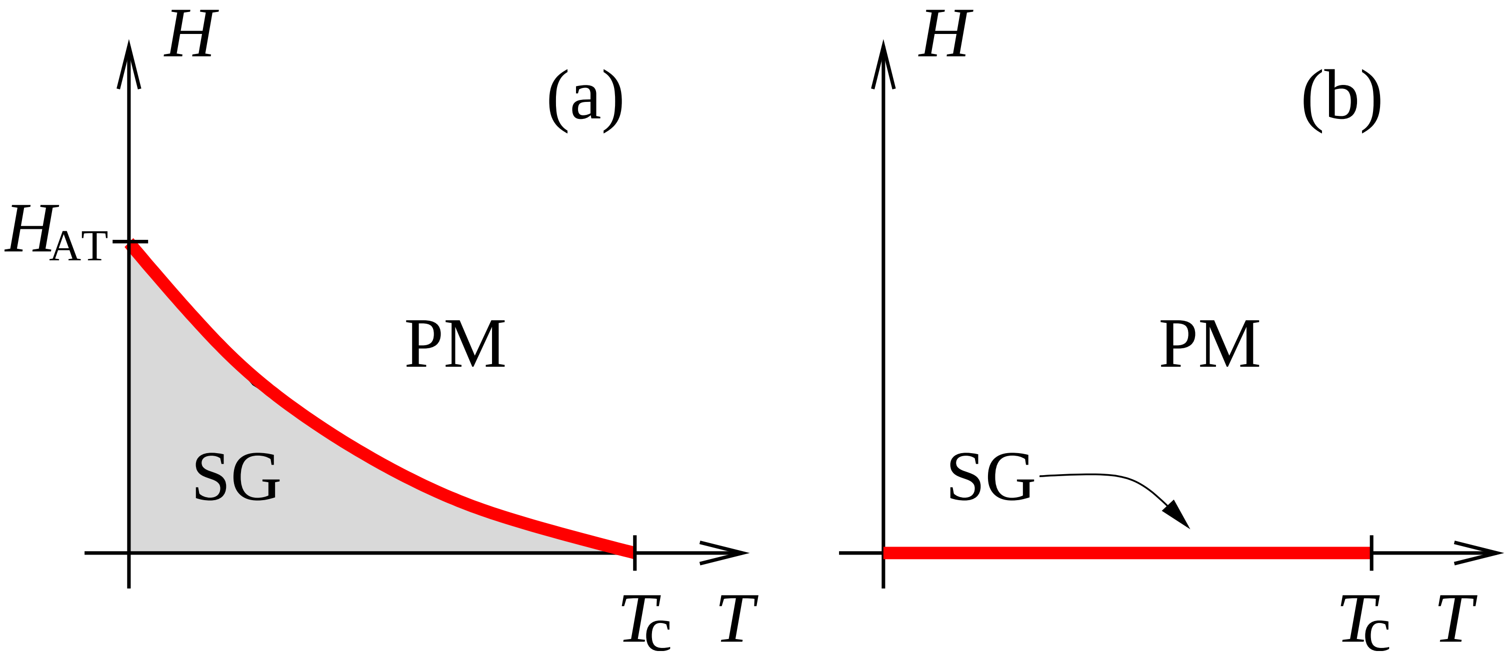
Assume short-range is similar to infinite-range. There is an AT line.

"Droplet picture" (DP) (Fisher and Huse, also Bray and Moore, and McMillan).

Focus on the geometrical aspects of the low-energy excitations.
No AT line.

# Is there an AT line? (Ising)

In MFT there's a transition in a field for an Ising spin glass, the Almeida Thouless (AT) line, from a spin glass (divergent relaxation times, RSB) to a paramagnetic (finite relaxation times, "replica symmetric") phase.
The AT line is a ergodic-non ergodic transition with no symmetry change.




Does an AT line occur in short range systems?

- RSB: yes (see (a)) DP: no (see (b))
- Experiments (dynamics) (Uppsala group): no
- Theory: conflicting claims.

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# Correlation Length

Spin glass correlation Function

$$\chi_{SG}(\mathbf{k}) = \frac{1}{N}\sum_{i,j}[\langle \mathbf{S}_i \cdot \mathbf{S}_j\rangle^2]_{\mathrm{av}} e^{i\mathbf{k}\cdot(\mathbf{R}_i - \mathbf{R}_j)}$$

Note: $\chi_{nl} \sim \chi_{SG}(\mathbf{k}=0)$, which is essentially the “correlation volume”.

Determine the finite-size spin glass correlation length $\xi_L$ from the Ornstein Zernicke equation:

$$\chi_{SG}(\mathbf{k}) = \frac{\chi_{SG}(0)}{1+\xi_L^2\mathbf{k}^2+\ldots},$$

by fitting to $\mathbf{k}=0$ and $\mathbf{k}=\mathbf{k}_{\min}=\frac{2\pi}{L}(1,0,0)$.

# Finite size scaling

Assumption: size dependence comes from the ratio $L/\xi_{\rm bulk}$ where

$$\xi_{\rm bulk} \sim (T - T_{SG})^{-\nu}$$

is the bulk correlation length.
In particular, the finite-size correlation length varies as

$$\boxed{\frac{\xi_L}{L} = X\left(L^{1/\nu}(T - T_{SG})\right),}$$

since $\xi_L/L$ is dimensionless (and so has no power of $L$ multiplying the scaling function $X$).

Hence data for $\xi_L/L$ for different sizes should intersect at $T_{SG}$ and splay out below $T_{SG}$.

Let's first see how this works for the Ising SG ...

# Results: Ising

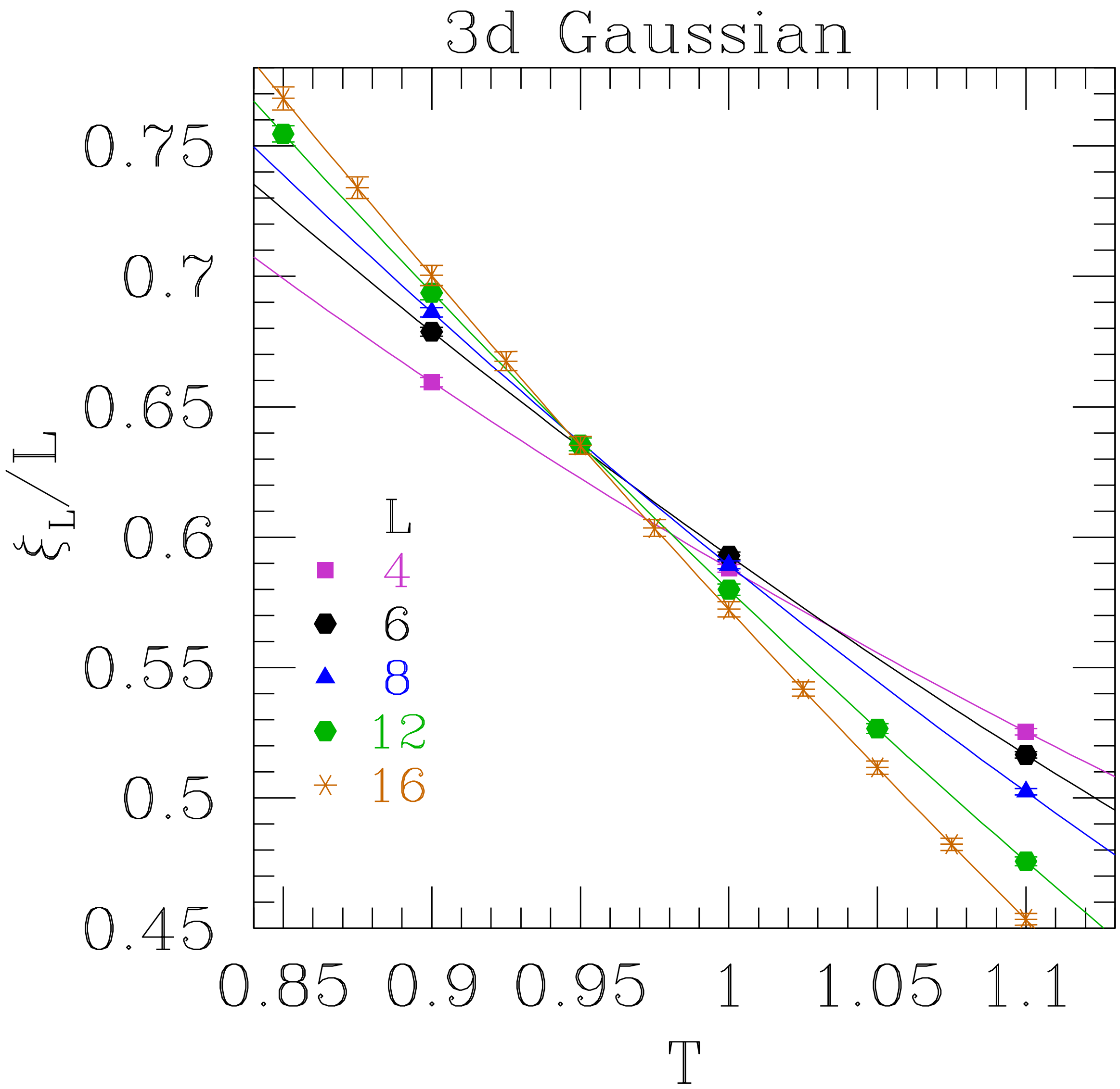
FSS of the correlation length for the Ising spin glass. (from Katzgraber, Körner and APY Phys. Rev. B **73**, 224432 (2006).)

Method first used for SG by Ballesteros et al. but for the $\pm J$ distribution.

The clean intersections (corrections to FSS visible for $L = 4$) imply

$$\boldsymbol{T_{SG} \simeq 0.96.}$$

Prevously Marinari et al. found $T_{SG} = 0.95 \pm 0.04$ using a different analysis



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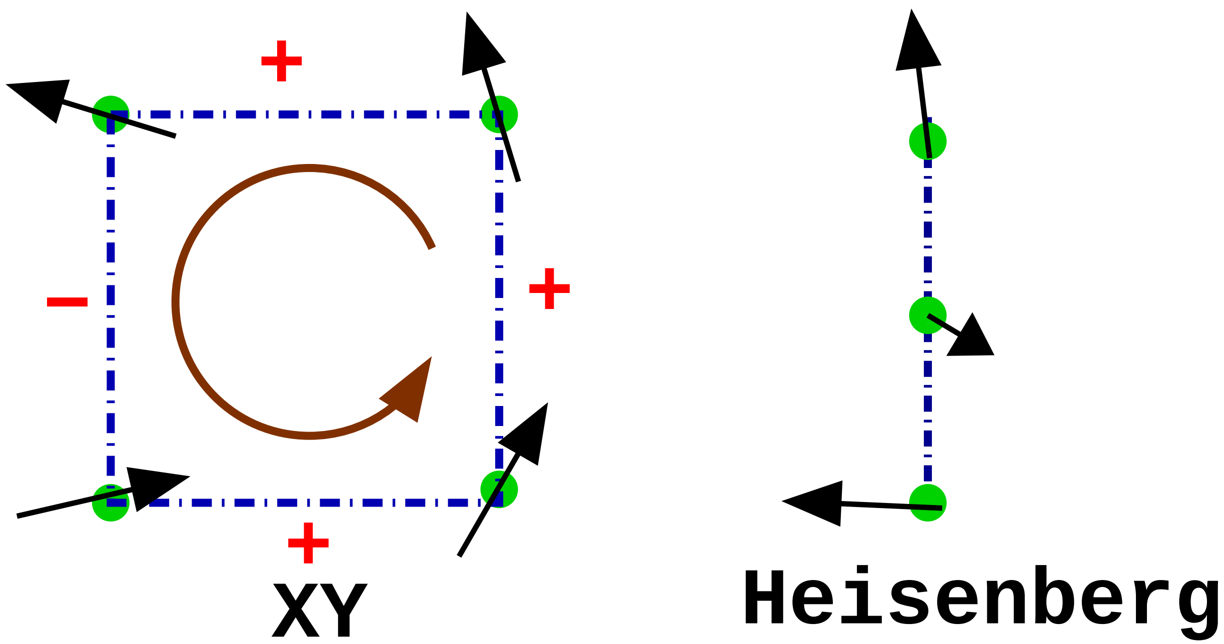
  - Is there a phase transition in a vector spin glass?

    Martin-Mayor, Tarancon, Fernandez, Gaviro, and APY (unpublished)
  - Is there a transition in an Ising spin glass in a magnetic field (Almeida-Thouless line)?

# Chirality

- Unfrustrated: Thermally activated chiralities (vortices) drive the Kosterlitz-Thouless-Berezinskii transition in the 2d XY ferromagnet.
- Frustrated: Chiralities are quenched in by the disorder at low-T because the ground state is non-collinear.
  Define chirality by: (Kawamura)

$$\kappa_i^\mu = \begin{cases} \frac{1}{2\sqrt{2}} \sum'_{\langle l,m\rangle} \mathrm{sgn}(J_{lm}) \sin(\theta_l - \theta_m), & \text{XY } (\mu \perp \text{square}) \\ \mathbf{S}_{i+\hat{\mu}} \cdot \mathbf{S}_i \times \mathbf{S}_{i-\hat{\mu}}, & \text{Heisenberg} \end{cases}$$



# Motivation for vector model

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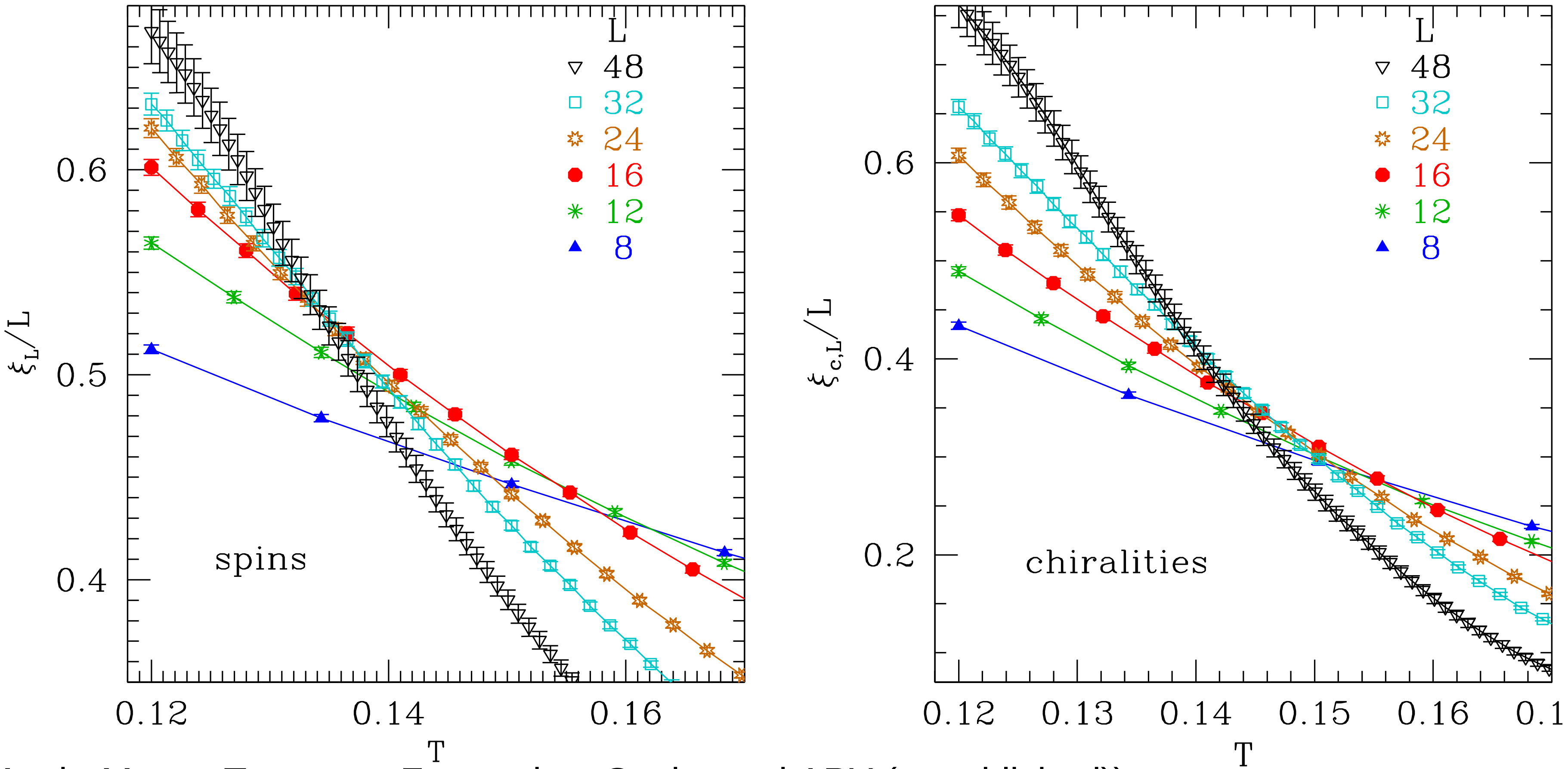
  - this was the most successful approach for the Ising spin glass.
  - treat spins and chiralities on equal footing.

# Heisenberg Spin Glass




(Martin-Mayor, Tarancon, Fernandez, Gaviro and APY (unpublished)).

**Note**: **much larger sizes** than for Ising (barriers smaller).

Spins and chiralities behave **very similarly (but not identically)** for this range of sizes.

Are there two very close but distinct transitions? Viet and Kawamura, $L \leq 32$, claim $T_{CG} = 0.145, T_{SG} = 0.120$. From our data, the difference seems less than this or zero. The apparent small difference in transition temperatures may be due to corrections to scaling.

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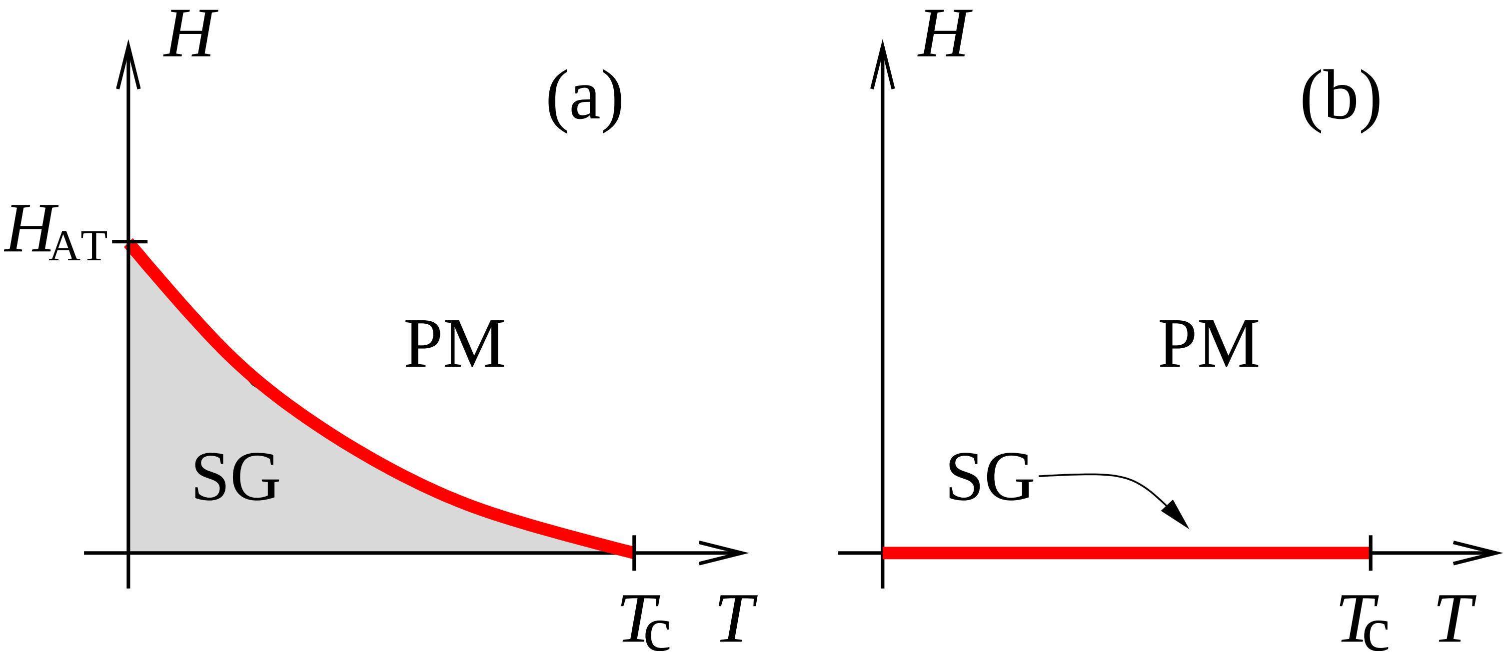



Does an AT line occur in short range systems?

- RSB: yes (see (a)) DP: no (see (b))
- Experiments (dynamics) (Uppsala group): no
- Theory: conflicting claims.

# Is there an AT Line?: 2

Experiments:, no static divergent quantity; ($\chi_{nl}$ doesn't diverge in a field).

Simulations: According to RSB, $\chi_{SG}$ diverges in a field, where now

$$\chi_{SG}(\mathbf{k}) = \frac{1}{N} \sum_{i,j} [(\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle)^2]_{\mathrm{av}} e^{i\mathbf{k}\cdot(\mathbf{R}_i - \mathbf{R}_j)}.$$

Hence can use FSS of $\xi_L/L$ in the simulations to see if there is an AT line

$\xi_L$ behaves as for the zero field case; i.e. so we look for intersections.

# Long-Range model in $d = 1$

$$\boxed{[J_{ij}^2]_{av} \propto \frac{1}{r_{ij}^{2\sigma}},}.$$

For the short-range model:

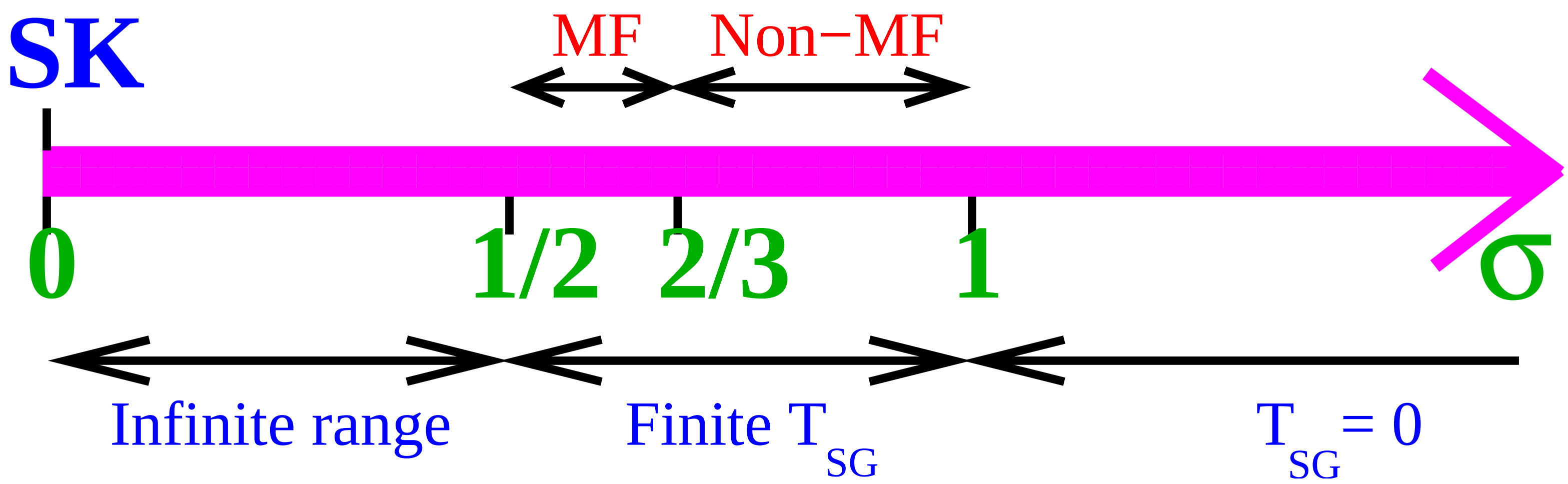
- there is a lower critical dimension $d_l$ (equal to about 2.5),
- and an upper critical dimension $d_u$ (equal to 6).

Make an analogy between varying $d$ for short-range, and varying $\sigma$ for long-range $d = 1$. (Works for ferromagnets, e.g. Fisher, Ma and Nickel.) In otherwords there is a $\sigma_l$ *above* which $T_{SG} = 0$, and a $\sigma_u$ *below* which the zero-field critical behavior is mean-field like, where

- $\sigma_l = 1,$ (Fisher and Huse)
- $\sigma_u = 2/3,$ (Kotliar, Stein and Anderson)
- The range $0 \le \sigma \le 1/2$ is infinite-range since $\sum_j [J_{ij}^2]_{\rm av}$ diverges.
- $\sigma \to \infty$ is the nearest-neighbor model.

# Dependence on $\sigma$

The following figure summarizes the behavior of the $d = 1$ long-range model as a function of $\sigma$.



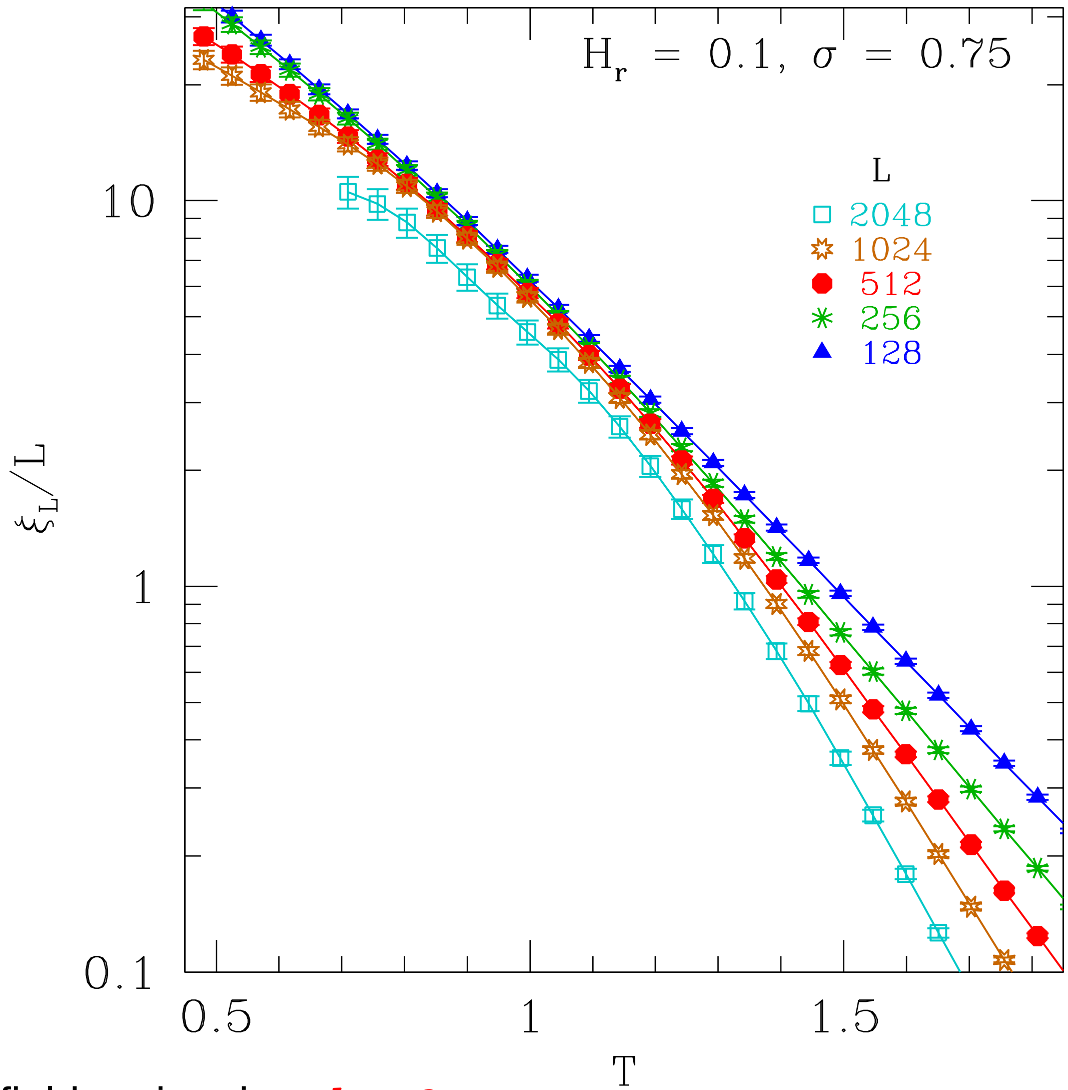

The interesting range is $\boxed{1/2 < \sigma < 1\,.}$

We use a (diluted) version of the model by Leuzzi et al (2008). The probability of a non-zero bond falls off like $1/r_{ij}^{2\sigma}$, but the strength of the bond does not fall off. We fix the mean coordination number to be 6.

# Results: AT-line

FSS of the correlation length for the Ising spin glass in a (Gaussian random) field of $H_r = 0.1$ with $\sigma = 0.75$ (non mean field region).

Lack of intersections implies no AT line down to this value of $H_r$.
Hence, if there is an AT line, it only occurs for extremely small fields
(Katzgraber, Larson and APY).



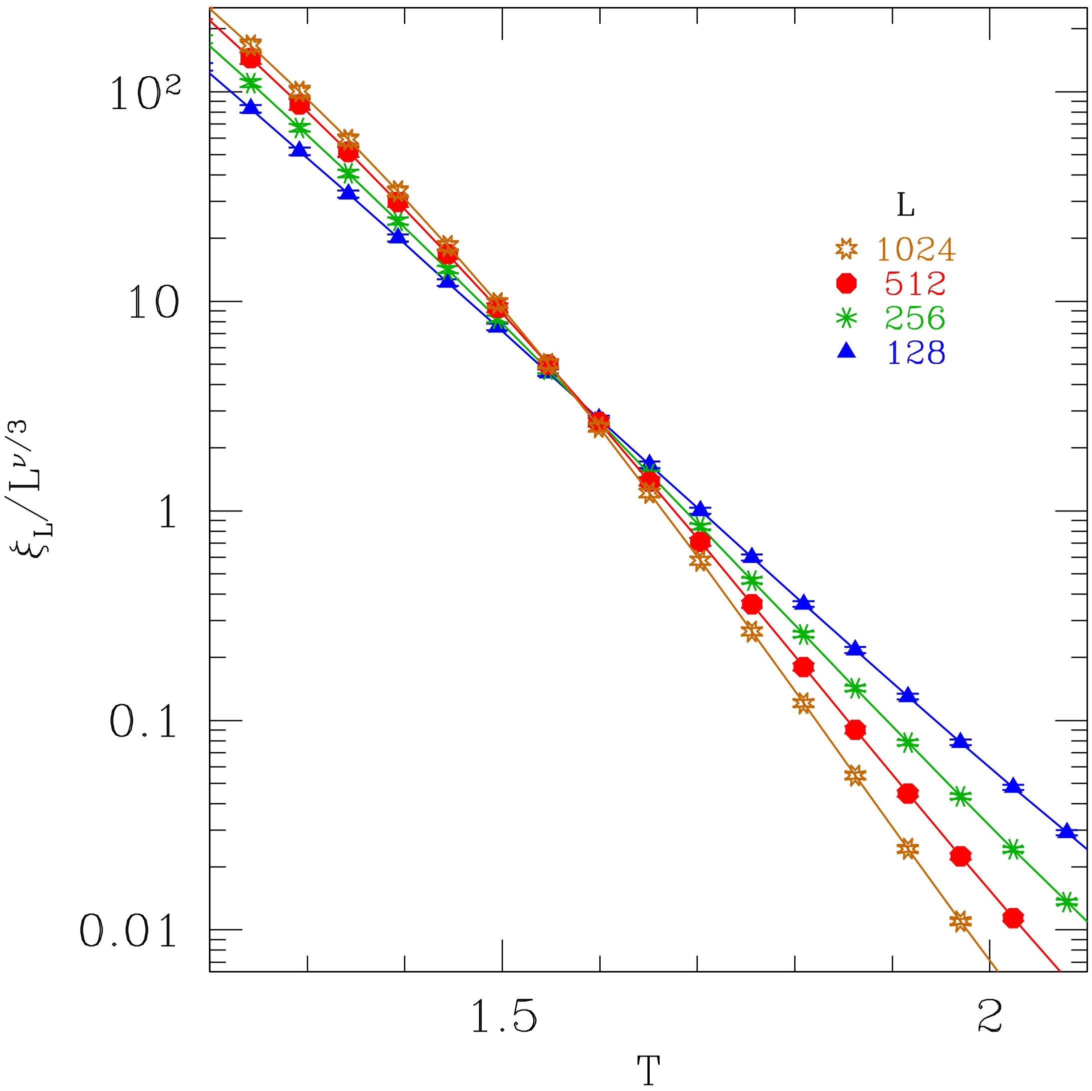

Hence expect **no AT line** in the non mean field region, i.e. $d < 6$.

However, for the same model, with same $\sigma$ and $H_r$, (but with $\pm 1$ bonds), Parisi et al., **arXiv:0811.3435**, find intersections and hence **claim an AT line.**

# Results: AT-line

FSS of the correlation length for the Ising spin glass in a (Gaussian random) field of $H_r = 0.1$ with $\sigma = 0.60$ (mean field region).
(Katzgraber, Larson and APY).

The intersection implies that there is an AT line in the mean field region



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Nature of the equilibrium state below $T_{SG}$. Absence of an AT line favors droplet theory, but other data (non-trivial order parameter distribution $P(q)$) favors RSB.

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Intermediate scenario "TNT", (Trivial, Non-Trivial),

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