Nonequilibrium $3D$ spin glass dynamics with Janus

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[1] F. Belletti, A. Cruz, L.A. Fernandez, A. Gordillo-Guerrero, M. Guidetti, A. Maiorano, F. Mantovani, E. Marinari, V. Martin-Mayor, J. Monforte, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, J.J. Ruiz-Lorenzo, S.F. Schifano, D. Sciretti, A. Tarancon, R. Tripiccione and D. Yllanes

Introduction

Spin glasses: experiments vs. simulations

- Experiments focus on nonequilibrium dynamics.
- The system is rapidly cooled to a subcritical temperature, $T < T_c$, let to equilibrate a time t_w and probed at $t + t_w$.
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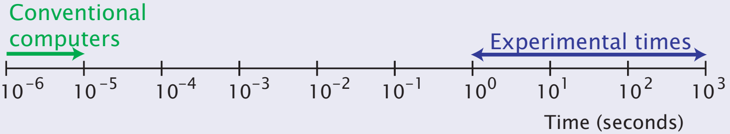

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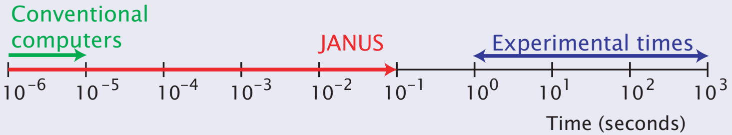

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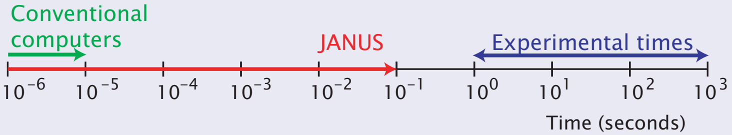
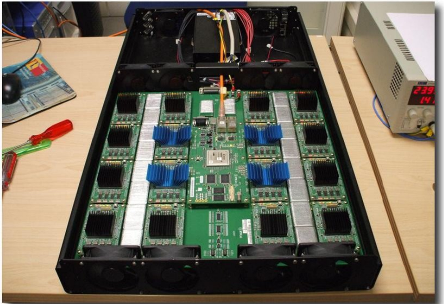


The Janus computer

- Janus is a custom built computing system:
  - Massively parallel
  - Made of FPGAs
  - Made of modules
- Designed with spin glasses in mind, but reconfigurable.

Our simulations



Our implementation

- 8×16 computing cores.
- 20 ps per spin update.
- Simulation time
  - PC: > 200 CPU years, wall clock > 3 years.
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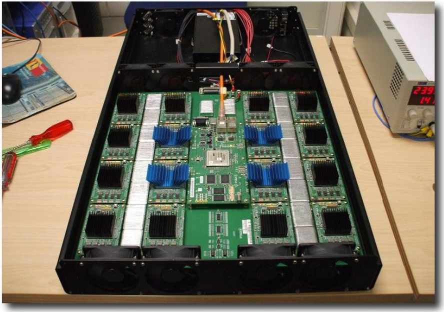

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The model and our parameters

- $\mathcal{H} = -\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} J_{\boldsymbol{xy}}\sigma_{\boldsymbol{x}}\sigma_{\boldsymbol{y}}, \quad P(J_{\boldsymbol{xy}}) = \delta(J_{\boldsymbol{xy}}^2 - 1)$.
- SG transition at $T_c = 1.109(10)$ (Hasenbusch et al. 2008).
- $L = 80$ systems for ~ 100 samples ($T = T_c, 0.8, 0.7, 0.6$).
- We follow the system for 10^{11} Monte Carlo steps (~ 0.1 s).

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- TNT (Trivial-Non-Trivial):
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- Numerical work is needed to make these theories quantitative and to determine which one best describes the SG phase.

The coherence length: definition

- We consider the correlation function of the replica field

$$C_4(\boldsymbol{x}, t_\mathrm{w}) = \overline{L^{-3} \sum_{\boldsymbol{x}} q_{\boldsymbol{x}}(t_\mathrm{w}) q_{\boldsymbol{x}+\boldsymbol{r}}(t_\mathrm{w})}, \quad q_{\boldsymbol{x}}(t_\mathrm{w}) = \sigma_{\boldsymbol{x}}^{(1)}(t_\mathrm{w})\sigma_{\boldsymbol{x}}^{(2)}(t_\mathrm{w}).$$

$$T < T_\mathrm{c} \quad \Longrightarrow \quad C_4(\boldsymbol{x}, t_\mathrm{w}) \simeq r^{-a} \mathrm{e}^{-[r/\xi(t_\mathrm{w})]^b}$$

- At $T < T_\mathrm{c}$ the value of a matters:
    - Coarsening dynamics: $a = 0$.
    - RSB: $a > 0$.
- T_c: $a = 1 + \eta = 0.625(10)$ (Hasenbusch et al 2008).

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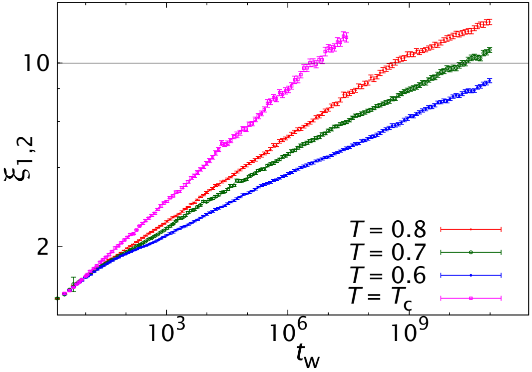
- We would like an Ansatz-independent determination of ξ and a:

$$I_k(t_{\mathrm{w}}) = \int_0^\infty \mathrm{d}r \; r^k C_4(r, t_{\mathrm{w}})$$

then, if $C_4 \simeq r^{-a} f(r/\xi)$,

$$\xi_{k,k+1}(t_{\mathrm{w}}) = I_{k+1}(t_{\mathrm{w}})/I_k(t_{\mathrm{w}}) \propto \xi(t_{\mathrm{w}}), \quad (\text{for } \xi \ll L)$$

The coherence length: our results (I)



Fair fits to

$$\xi(t_w) = A(T)\, t_w^{1/z(T)}$$

$$z(T_c) = 6.86(16)$$
$$z(0.8) = 9.42(15)$$
$$z(0.7) = 11.8(2)$$
$$z(0.6) = 14.1(3)$$
$$z(T) \simeq z(T_c) T_c / T$$

- We checked for finite size effects using $L = 24, 40$.
- Our measurements are safe for $3 \leq \xi(t_w) \leq 10$.

The coherence length: our results (II)

- $C_4(r, t_\mathrm{w}) \sim r^{-a} f(t/\xi(t_\mathrm{w})) \implies I_1(t_\mathrm{w}) \propto \xi_{k,k+1}^{2-a}(t_\mathrm{w})$.

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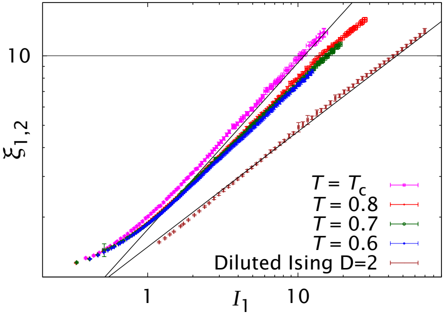

We plot $\xi_{1,2}$ vs. I_1:

$$a(0.8) = 0.442(11)$$
$$a(0.7) = 0.397(12)$$
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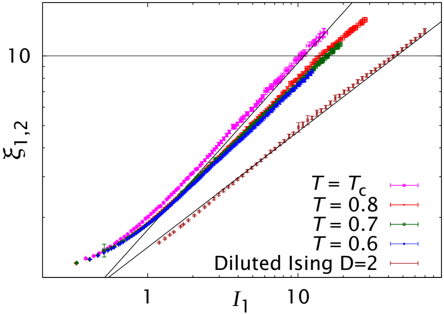

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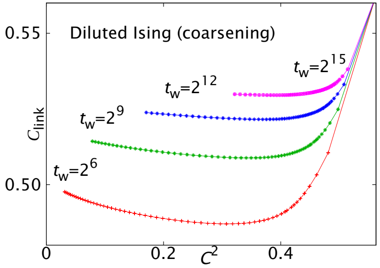
- $T < T_c$:
  - Incompatible with coarsening dynamics ($a = 0$).
  - Agreement with equilibrium results (Marinari & Parisi, 2001).
- $a(T_c)$ 2.5 SD below the FSS estimate $a = 0.625(10)$.

The link correlation function

- We are now interested in the spin and link correlation functions,

$$C(t,t_{\mathrm{w}}) = \overline{L^{-3}\sum_{\boldsymbol{x}} \sigma_{\boldsymbol{x}}^{t+t_{\mathrm{w}}}\sigma_{\boldsymbol{x}}^{t_{\mathrm{w}}}}, \quad C_{\mathrm{link}}(t,t_{\mathrm{w}}) = \overline{L^{-3}\sum_{\langle \boldsymbol{x},\boldsymbol{y}\rangle} \sigma_{\boldsymbol{x}}^{t+t_{\mathrm{w}}}\sigma_{\boldsymbol{x}}^{t_{\mathrm{w}}}\sigma_{\boldsymbol{y}}^{t+t_{\mathrm{w}}}\sigma_{\boldsymbol{y}}^{t_{\mathrm{w}}}}.$$

- We eliminate t as an independent variable and study $C_{\mathrm{link}}(C^2)$.
- Coarsening dynamics: $C^2 < q_{\mathrm{EA}}^2 \ \Rightarrow\ C_{\mathrm{link}}(C^2) = \text{constant}$.
- RSB: C_{link} is not constant (example: in SK $C_{\mathrm{link}} = C^2$)



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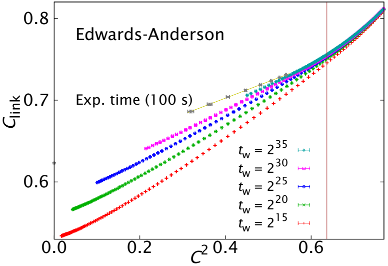

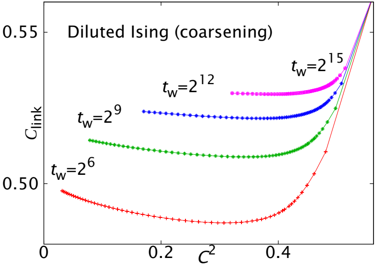

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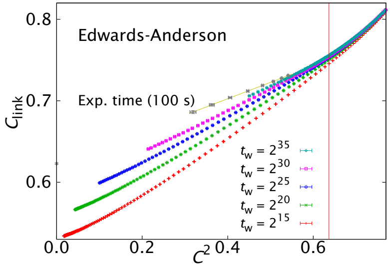

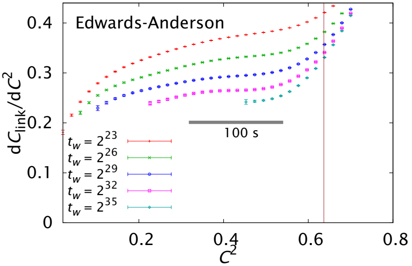

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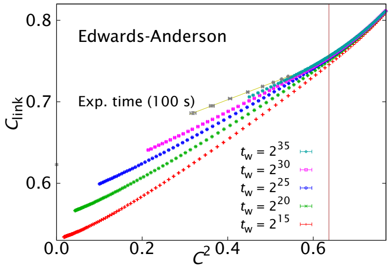

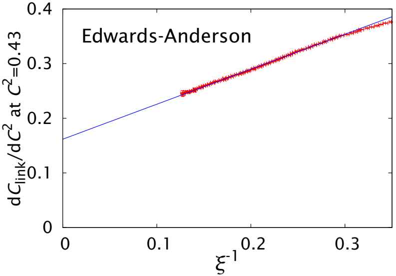

- We find non-coarsening dynamics even for experimental times.

Conclusions and outlook

Summary of our results

- Janus halves the logarithmic time gap between simulations and experiment.
- Our simulations indicate non-coarsening dynamics.
- Further results (PRL **101**, 157201 (2008); arXiv:0811.2864)
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Next steps

- 'Soft quench' $\longleftrightarrow$ memory/rejuvenation experiments.
- Large scale equilibrium simulation with parallel tempering.