Scaling behavior of domain walls at the $T = 0$ ferromagnet to spin-glass transition

O. Melchert, A.K. Hartmann

Institut für Physik
Universität Oldenburg

Outline

- Introduction
- Algorithm
- Results
- Summary

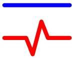
Model

- $N = L \times L$ Ising spins $\sigma_i = \pm 1$ on square lattice
- Periodic boundary conditions in one direction
- Edwards-Anderson Hamiltonian: $\mathcal{H}(\sigma) = -\sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$

interaction strength:

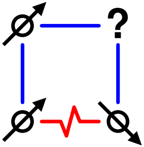
$J_{ij} > 0$:

$J_{ij} < 0$:



quenched disorder

frustration:



- Here: “Gaussian-like” distributed bonds

$$P(J) = (1-\rho)\, e^{-J^2/2}/\sqrt{2\pi} \;+\; \rho\, \delta(J-1)$$

$\rho < \rho_c$: Spin-glass (SG)
$\rho > \rho_c$: Ferromagnet (FM)

Domain Walls (DWs)

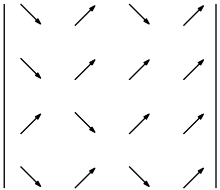
- Exact ground states (GSs) using sophisticated matching algorithms (up to $L = 512$)
- DWs defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$:
  $\sigma^{(2)}$:
- Separates regions of agreeing/disagreeing spin orient.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

DW energy:

$$\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by 1 config.



Domain Walls (DWs)

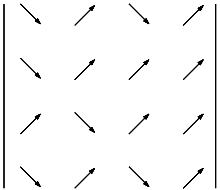
- Exact ground states (GSs) using sophisticated matching algorithms (up to $L = 512$)
- DWs defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$: GS for periodic boundary conditions (BCs)
  $\sigma^{(2)}$:
- Separates regions of agreeing/disagreeing spin orient.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

DW energy:

$$\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by 1 config.



Domain Walls (DWs)

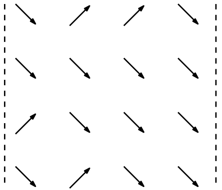
- Exact ground states (GSs) using sophisticated matching algorithms (up to $L = 512$)
- DWs defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$: GS for periodic boundary conditions (BCs)
  $\sigma^{(2)}$: GS for antiperiodic BCs
- Separates regions of agreeing/disagreeing spin orient.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

DW energy:

$$\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by 1 config.



Domain Walls (DWs)

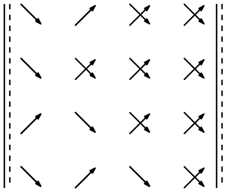
- Exact ground states (GSs) using sophisticated matching algorithms (up to $L = 512$)
- DWs defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$: GS for periodic boundary conditions (BCs)
  $\sigma^{(2)}$: GS for antiperiodic BCs
- Separates regions of agreeing/disagreeing spin orient.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

DW energy:

$$\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by 1 config.



Domain Walls (DWs)

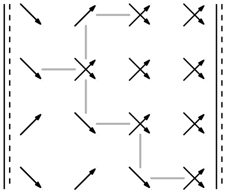
- Exact ground states (GSs) using sophisticated matching algorithms (up to $L = 512$)
- DWs defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$: GS for periodic boundary conditions (BCs)
  $\sigma^{(2)}$: GS for antiperiodic BCs
- Separates regions of agreeing/disagreeing spin orient.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

DW energy:

$$\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by 1 config.



Domain Walls (DWs)

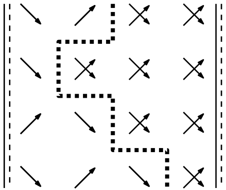
- Exact ground states (GSs) using sophisticated matching algorithms (up to $L = 512$)
- DWs defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$: GS for periodic boundary conditions (BCs)
  $\sigma^{(2)}$: GS for antiperiodic BCs
- Separates regions of agreeing/disagreeing spin orient.

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

DW energy:

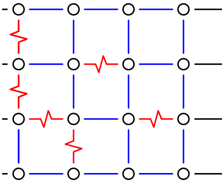
$$\delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by 1 config.



Dual Graph

- Construct weighted graph $G = (V, E, \omega)$
  - $V(G)$ elementary plaquettes (EP)
  - $E(G)$ connect EP with common side
  - ω energy contribution to DW



Consider GS σ for periodic BCs:

(i) Bond satisfied for σ, e.g.

 : $\omega \geq 0$

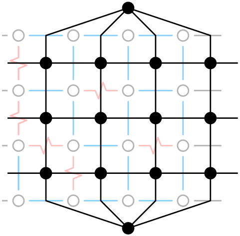
(ii) Bond not satisfied for σ, e.g.

 : $\omega \leq 0$

Dual Graph

- Construct weighted graph $G = (V, E, \omega)$

$V(G)$	elementary plaquettes (EP)
$E(G)$	connect EP with common side
ω	energy contribution to DW



Consider GS σ for periodic BCs:

(i) Bond satisfied for σ, e.g.

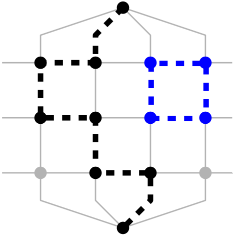

: $\omega \geq 0$

(ii) Bond not satisfied for σ, e.g.


: $\omega \leq 0$

Dual Graph

- Construct weighted graph $G = (V, E, \omega)$
  - $V(G)$ elementary plaquettes (EP)
  - $E(G)$ connect EP with common side
  - ω energy contribution to DW



no loops with negative weight:

$$\omega(\mathcal{C}) = \sum_{\langle ij \rangle \in \mathcal{C}} J_{ij}\sigma_i\sigma_j \geq 0$$

- DW: minimum-weight $(\mathrm{top}, \mathrm{bottom})$ path

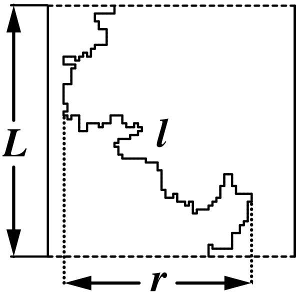
Minimum-Weight Paths

- G: undirected graph, allowing for negative edge weights
- Here: standard minimum-weight path algorithms, e.g. Bellman-Ford, Floyd-Warshall, don't work
- Minimum-weight path problem on dual requires matching techniques
  - i) Dual graph $\rightarrow$ auxiliary graph
  - ii) Find minimum-weighted perfect matching (MWPM)
  - iii) Interpret MWPM as min.-weight path

[R.K. Ahuja, T.L. Magnanti and J.B. Orlin, *Network flows*]

Previous results

- Excitation energy of DWs:
  $\langle|\delta E|\rangle \sim L^{\theta}$, $\theta = -0.287(4)$
  [AKH and A.P. Young, PRB 2001]
- Scaling behavior of DWs:
  $\langle\ell\rangle \sim L^{d_f}$, $d_f = 1.274(2)$
  $\langle r\rangle \sim L^{d_r}$, $d_r = 1.008(11)$
  [OM and AKH, PRB 2007]



DWs can be described by Schramm-Loewner evolutions (SLEs) [Amoruso *et. al.*, PRL 2006], possibility to relate exponents via

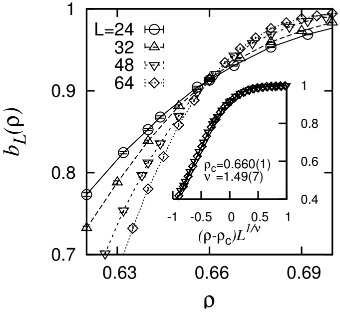
$$d_f = 1 + 3/[4(3+\theta)]$$

Universality: SLE scaling relation also valid for $\rho > 0$?

Location of the critical point

- Magnetization: $m_L = |\sum_i \sigma_i|/L^2$

  Binder parameter: $b_L = (3 - \frac{\langle m_L^4 \rangle}{\langle m_L^2 \rangle^2})/2$



Finite size scaling form:

$b_L(\rho) \sim f_1[(\rho - \rho_c)L^{1/\nu}]$

$\rho_c = 0.660(1)$
$\nu = 1.49(7)$

Quality: $S = 1.3$

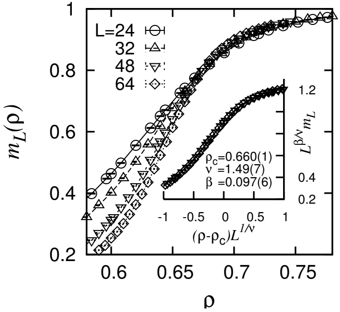
- Critical exponents in agreement with numerical values $\nu = 1.55(1)$ and $\beta = 0.09(1)$ found for $\pm$J model

  [Amoruso and A.K. Hartmann, PRB 2004]

Location of the critical point

- Magnetization: $m_L = |\sum_i \sigma_i|/L^2$

  Binder parameter: $b_L = (3 - \frac{\langle m_L^4 \rangle}{\langle m_L^2 \rangle^2})/2$



Finite size scaling form:

$m_L(\rho) \sim L^{-\beta/\nu} f_2[(\rho-\rho_c)L^{1/\nu}]$

$\rho_c = 0.660(1)$
$\nu = 1.49(7)$, $\beta = 0.097(6)$

Quality: $S = 1.8$

- Critical exponents in agreement with numerical values $\nu = 1.55(1)$ and $\beta = 0.09(1)$ found for $\pm$J model

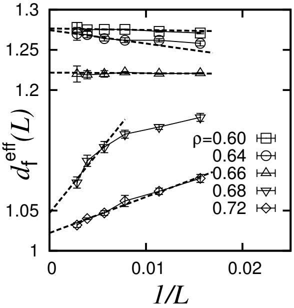
  [Amoruso and A.K. Hartmann, PRB 2004]

Scaling behavior of DWs

- Scaling analysis up to $L = 512$

ρ	d_f	d_r	θ_2
0.00	1.274(2)	1.008(11)	-0.287(4)
0.60	1.275(1)	1.003(3)	-0.28(2)
0.64	1.275(2)	1.012(4)	-0.28(4)
0.66	1.222(1)	1.002(2)	0.16(1)
0.68	1.05(2)	0.74(3)	0.35(3)
0.72	1.022(1)	0.698(6)	0.27(2)

where $\sigma(\delta E) = \sqrt{\langle \delta E^2 \rangle - \langle \delta E \rangle^2} \sim L^{\theta_2}$



- Effective exponents d_f^{eff}: describe the scaling of $\langle \ell \rangle$ for 4 successive values of L according to $\langle \ell \rangle \sim L^{d_f}$
- Spin glass phase up to ρ close to ρ_c: Scaling behavior of DW energy and DW length consistent with scaling relation

$$d_f = 1 + 3/[4(3+\theta)]$$

derived from SLE processes

Summary

- Groundstate study on 2D Ising spin glasses with short ranged interactions
- DWs obtained via minimum-weight path approach
- Scaling behavior of DWs near SG-FM transition at $T = 0$
- $\rho < \rho_c$: SLE scaling relation consistent with exponents found from numerical simulations