Out-of-equilibrium bosons on a 1-d optical random Lattice

T. Platini Blacksburg

R.J. Harris London

D. Karevski Nancy

I Model

II Nonequilibrium setup

III Dynamics

IV Steady-state

I Model: Hard-core bosons

Hubbard model $\quad \mathcal{H} = -\sum_{i,j} t\, a_i^+ a_j + \frac{U}{2} \sum_i n_i (n_i - 1) + \varepsilon \sum_i n_i$

$U \gg 1 \quad \longrightarrow$ Hard-core constraint

1-d model

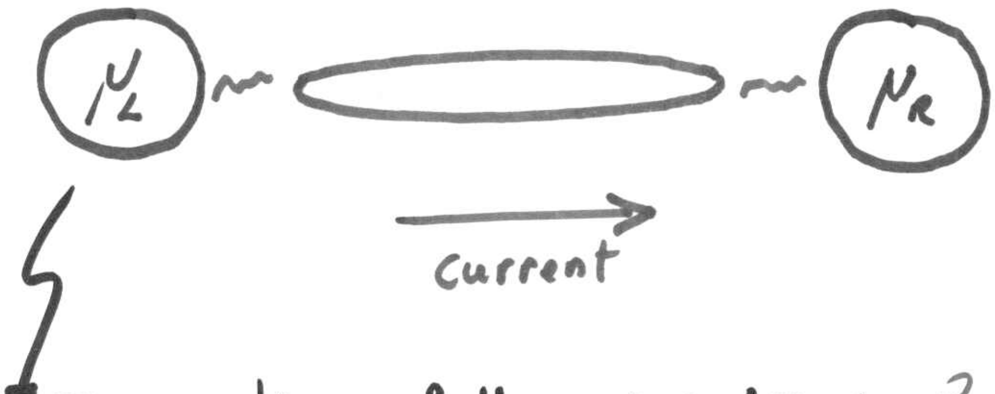
$$\mathcal{H} = -\sum_k t \left[a_k^+ a_{k+1} + a_{k+1}^+ a_k \right] + \varepsilon \sum_k n_k$$

+ hard-core $\quad a^2 = a^{+2} = 0$ and $\{a, a^+\} = \mathbb{1}$

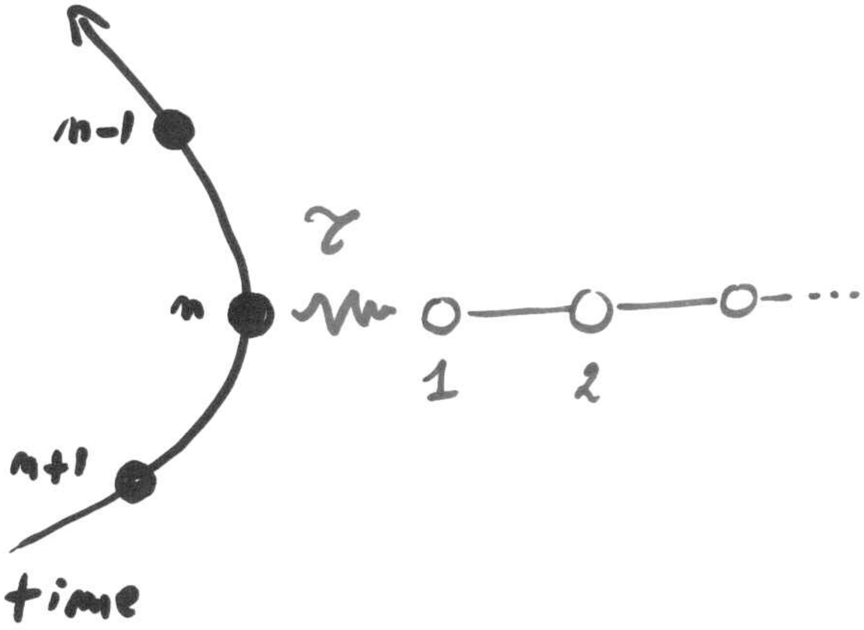
Local Fermi algebra

$\sim \mathcal{H}_{xx}$

II Non-equilibrium setup

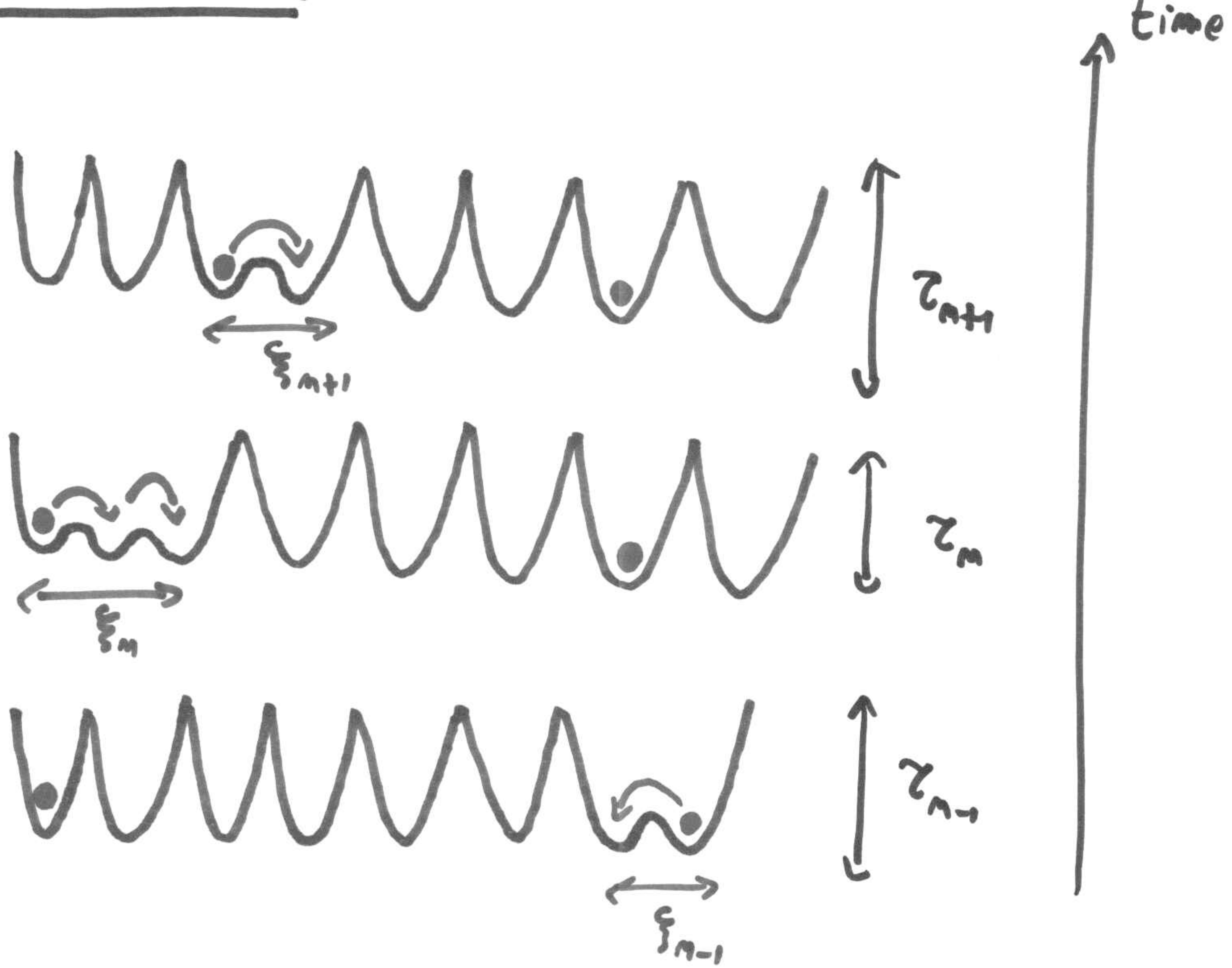


Description of the reservoirs : Repeated interactions.



$$\mathcal{H}_T(n) = \mathcal{H}_{sys} + \mathcal{H}^n + \mathcal{H}_{int}^{n,S}$$

Lattice Fluctuations



III Dynamics

- Reduced density matrix

$$\rho_s(n) = \mathcal{K}_n\left(\rho_s(n-1)\right)$$

where $\mathcal{K}_n$ is a completely positive map

$$\mathcal{K}_n(X) = \sum_{\ell} V_\ell X V_\ell^{+}$$

- Correlation matrix

$$\mathcal{H}_{syst} = \frac{1}{4}\, \Gamma^{\dagger}\, T\, \Gamma \qquad \{\Gamma_k , \Gamma_{k'}\} = 2\, \delta_{kk'}$$

Interaction matrix

$$G \propto -i \langle \Gamma \Gamma \rangle \qquad G_{kk'}(t) = \mathrm{Tr}\{-i\, \Gamma_k \Gamma_{k'}\, \rho_s(t)\}$$

Dynamical equation

$$G(n) = \underbrace{R_s\, G(n-1)}_{\text{syst. part}}\, R_s^{\dagger} + \underbrace{R_{SE}\, G_E\, R_{SE}^{\dagger}}_{\text{Env. part}}$$

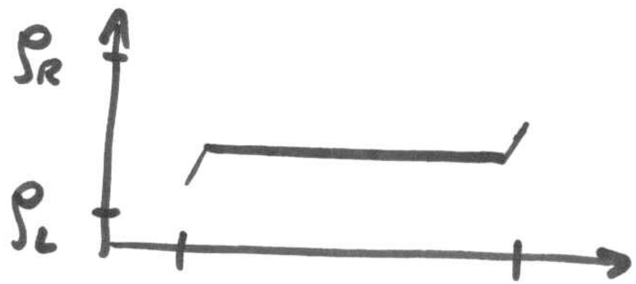
$$G(t) = \begin{pmatrix} \ddots & \ddots & & & \\ & n_{i-1} & j_{i-1} & & \\ & & n_i & j_i & \\ & & & n_{i+1} & \ddots \\ & & & & \ddots \end{pmatrix}$$

$n_i = \langle \hat{n}_i \rangle$ particle density

$j_i = \langle \hat{j}_i \rangle$ particle current : $\dot{n}_k = i[H, n_k] \equiv \hat{j}_{k-1} - \hat{j}_k$

IV Steady-state

* No Fluctuations of the lattice



$$m_k^* = \bar{\rho} = \frac{\rho_L + \rho_R}{2} \Rightarrow \delta^* \equiv m_k^* - m_{k-1}^* = 0$$

density gradient

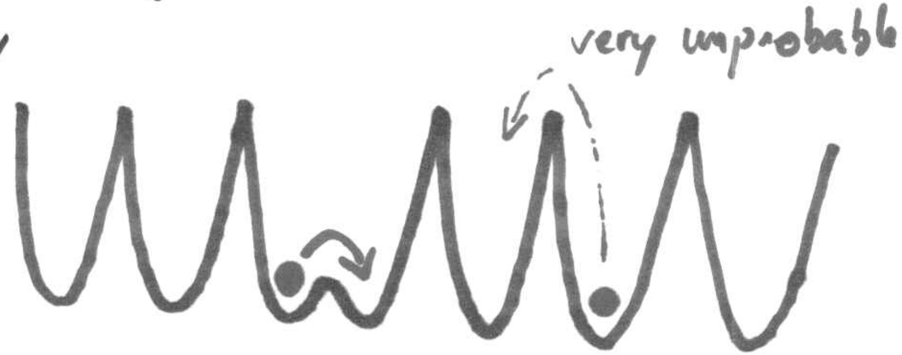
$$j^* \propto -(\rho_R - \rho_L)$$

$$\hookrightarrow \quad \kappa = -\frac{j^*}{\delta^*} \nearrow \infty$$

No Fourier Law: infinite conductivity κ.

* Lattice fluctuations

- Strong localization $Nt_s \ll 1$



→ Closed set of dynamical equations

$$\begin{pmatrix}\delta\\ j\end{pmatrix}_{n+1} = \mathcal{R}_\tau \begin{pmatrix}\delta\\ j\end{pmatrix}_n$$

Steady-state solution

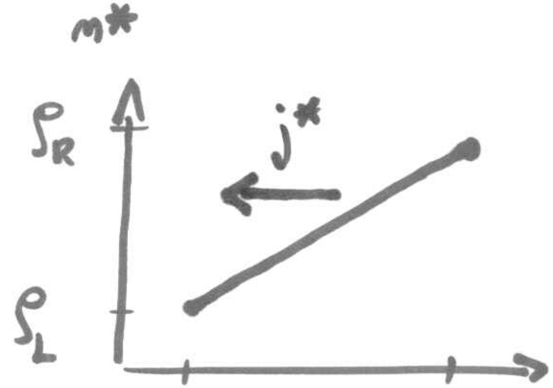
$$\delta^* = \frac{\Delta\rho}{\underbrace{N+1+\gamma(\tau)}_{}} \longrightarrow \frac{\Delta\rho}{N} = \text{const.}$$

extrapolation length

$$\gamma(\tau) \propto \kappa(\tau)$$

$$j^* = -\kappa(\tau)\,\delta^*$$

we recover Fourier Law



Exponential distribution of τ:

$$\kappa(\tau) = \frac{\tau^2+4}{3\tau(\tau^2+2)}$$

For $\tau \ll 1$

$$\left.\begin{array}{l} \kappa \sim 1/\tau \\ \delta^* \sim \tau^2 \end{array}\right\} j^* \sim \tau .$$

* Correction to the strong localized case : $Nt_s \gg 1$

$$\delta^* \sim \frac{\Delta\rho}{N} \cdot \frac{1}{Nt_s}$$

strong attenuation of the gradient

$$j^* \sim \frac{1}{N}$$

$$\kappa \sim \frac{j^*}{\delta^*} \sim N$$

linear divergence of the conductivity.

* Summary

- Exact treatment of the reservoirs
- Analytical solution for $Nt_s \ll 1$
- Fourier law for $Nt_s \ll 1$.