Nonequilibrium critical dynamics of the two-dimensional Ising model quenched from a correlated initial state

Ferenc Iglói (Budapest)

in collaboration with

László Környei (Szeged, Budapest)

Michel Pleimling (Virginia Tech)

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AGENDA

- Nonequilibrium relaxation
  - ordered ($T_i = 0$) initial state
  - disordered ($T_i = \infty$) initial state
  - analogy with static semi-infinite criticality
- Models
  - Ising model
  - Baxter-Wu (BW) model
  - Turban model
- Results
  - Relaxation from a critical initial state
  - Relaxation from a 1^{st}-order initial state
- Conclusions

Nonequilibrium relaxation

- prepare the system in an initial state for $t < 0$
- quench to the critical point (T_c) at $t = 0$
- let evolve with the dynamical rules at T_c for $t > 0$
- measure
  - relaxation of the magnetization:

$$m(t) = \langle \sigma(t) \rangle$$

  - autocorrelation function:

$$G(t,s) = \langle \sigma(s)\sigma(t) \rangle$$

s: waiting time t: observation time
in equilibrium:

$$\boxed{G(t,s) \sim (t-s)^{-2x/z}}$$

time dependence for $t > 0$

- **ordered initial state ($T_i = 0$)**

  relaxation involves only equilibrium critical exponents

  $$\boxed{m(t) \sim t^{-x/z}, \quad T_i = 0}$$

  $x = \beta/\nu$: anomalous dimension of the magnetization

  z: dynamical exponent

  .

  autocorrelation function:

  $$\boxed{G(t,s) \sim (t-s)^{-2x/z} g(t/s), \quad T_i = 0}$$

position dependence for $y > 0$

- magnetization profile in a semi-infinite system $y \geq 0$ with **fixed boundary condition**

  $$\boxed{m(y) \sim y^{-x}, \quad h_s = \infty}$$

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  correlation function:

  $$\boxed{C(y_1,y_2) \sim (y_1 - y_2)^{-2x} g(y_1/y_2), h_s = \infty}$$

time dependence for $t>0$

- **disordered initial state ($T_i=\infty$)**

$$\boxed{m(t)\sim m_i t^{\theta}, \quad t<t_i}$$

θ: initial slip exponent

$m_i\to 0$: initial magnetization

$t_i\sim m_i^{-z/x_i}$: initial time

$x_i=\theta z+x$: anomalous dimension of m_i

$$\boxed{G(t,s)\sim t^{-\lambda/z}, \quad t\gg s, \quad T_i=\infty}$$

$\lambda=d-\theta z=d-x_i+x$

position dependence for $y>0$

- magnetization profile in a semi-infinite system **with small surface field**: $h_s\ll 1$

$$\boxed{m(y)\sim y^{x_s-x}, \quad y<y_s}$$

$h_s\to 0$: surface field

$y_s\sim h_s^{-1/x_s}$: surface region

x_s: anomalous dimension of m_s

.

$$\boxed{C(y_1,y_2)\sim y_1^{-\eta_\perp}, \quad y_1\gg y_2}$$

$\eta_\perp=x_s+x$

Power-law correlations in the initial state

previous studies

- 2d XY model: $T_i < T_c < T_{KT}$ *(Berthier, Holdsworth, Sellitto) (Abriet, Karevski)*

$$x(T_c) \to x(T_c) - x(T_i)$$

- spherical model *(Picone, Henkel)*

Our aim is to study nonequilibrium dynamics after a sudden change of the form of the interaction

Experimental realization: atoms in optical lattices.

Models

- **2d Ising model** $\rightarrow$ dynamics at T_c

$$\boxed{\mathcal{H}_I = -\sum_{i,j} J \left(\sigma_{i,j}\sigma_{i,j+1} + \sigma_{i,j}\sigma_{i+1,j}\right)}$$

critical point: $\boxed{\sinh(2J/kT_c) = 1}$

- **Baxter-Wu model**

$$\mathcal{H}_{BW} = -\sum_{i,j} J_{BW} \left(\sigma_{i,j}\sigma_{i-1,j}\sigma_{i,j-1} + \sigma_{i,j}\sigma_{i+1,j}\sigma_{i,j+1}\right)$$

critical point:

$$\boxed{\sinh(2J_{BW}/kT_c) = 1}$$

4-state Potts universality

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- **Turban model**

$$\boxed{\mathcal{H}_n = -\sum_{i,j} \left(J_2\sigma_{i,j}\sigma_{i,j+1} + J_n \prod_{k=0}^{n-1} \sigma_{i+k,j}\right)}$$

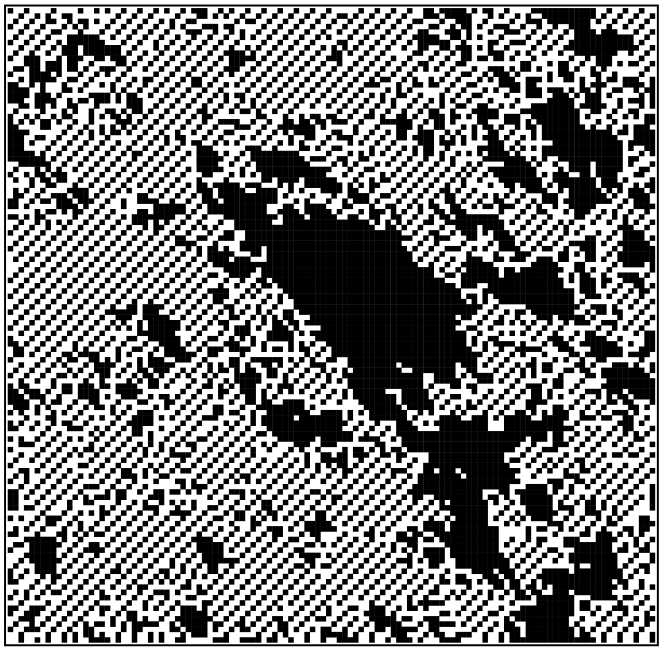
critical point:

$$\boxed{\sinh(2J_2/kT_c)\sinh(2J_n/kT_c) = 1}$$

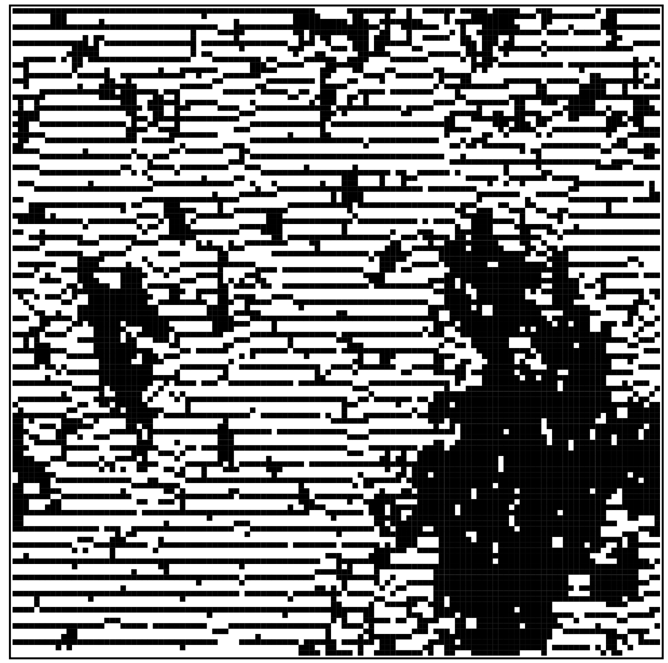
  - $n = 3$, 2^{nd}-order 4-state Potts universality
  - $n = 4$, 1^{st}-order

Relaxation from second-order transition points

[L. Környei, M. Pleimling, F. I.,Phys. Rev. E ***77****, 011127 (2008)]*

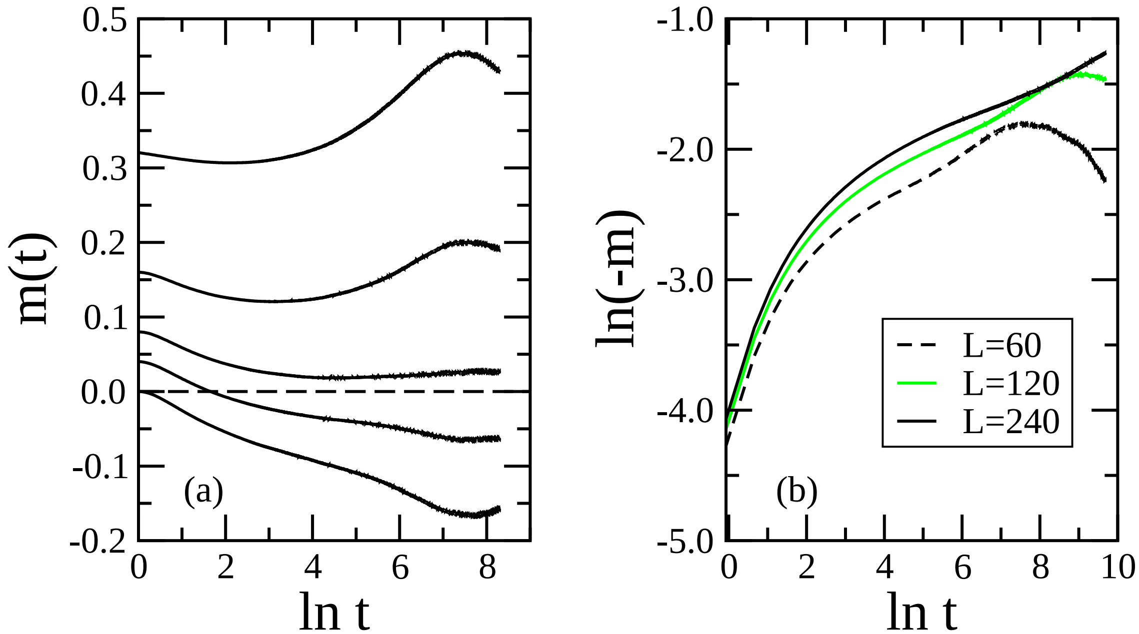


BW model



$n = 3$ Turban model

Relaxation from a BW critical state

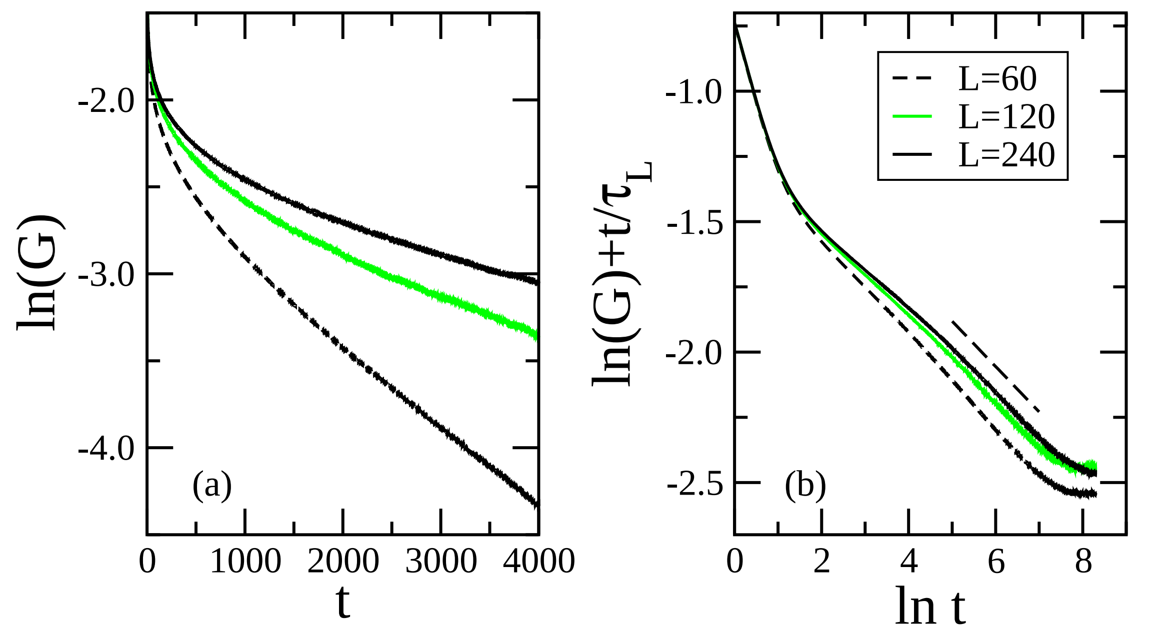


$m_i = 0.33, 0.16, 0.08, 0.04, 0.$

(from top to bottom)

$m_i = 0.$

$\theta = 0.18(1)$

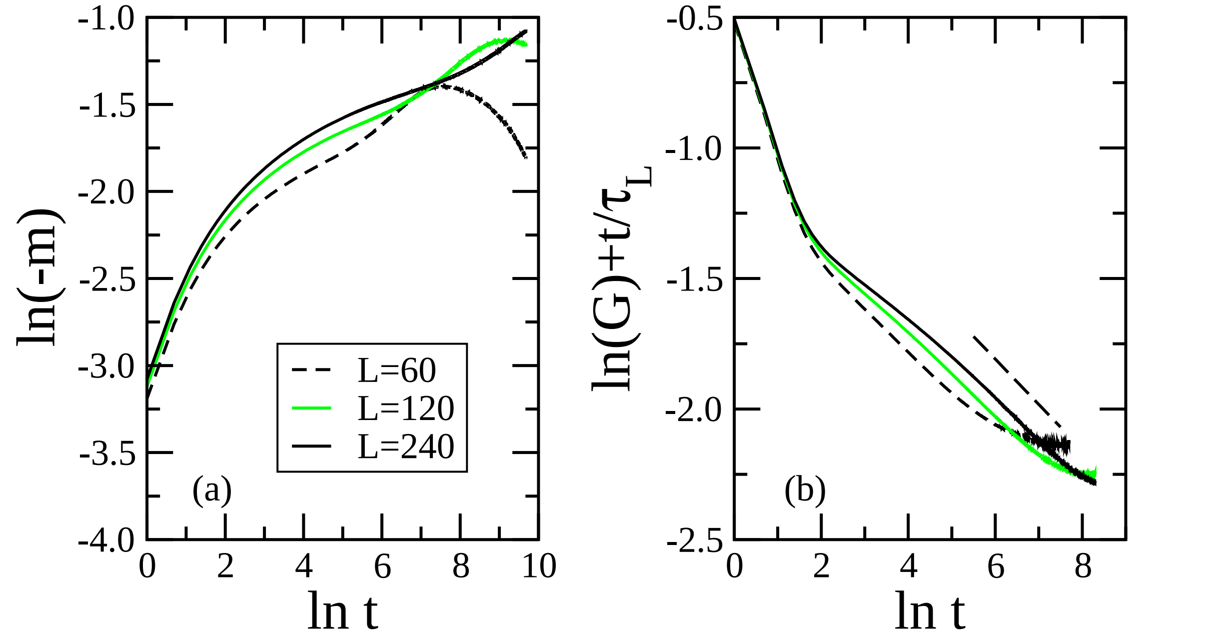


$G(t) = At^{-\lambda/z}\exp(-t/\tau_L)$

$\tilde{G}(t) = G(t)\exp(t/\tau_L)$

$\lambda/z = 0.18(1)$

Relaxation from an $n=3$ critical state



magnetization

$\theta = 0.18(1)$

$\tilde{G}(t) = G(t)\exp(t/\tau_L)$

$\lambda/z = 0.165(10)$

Relaxation from a first-order transition point

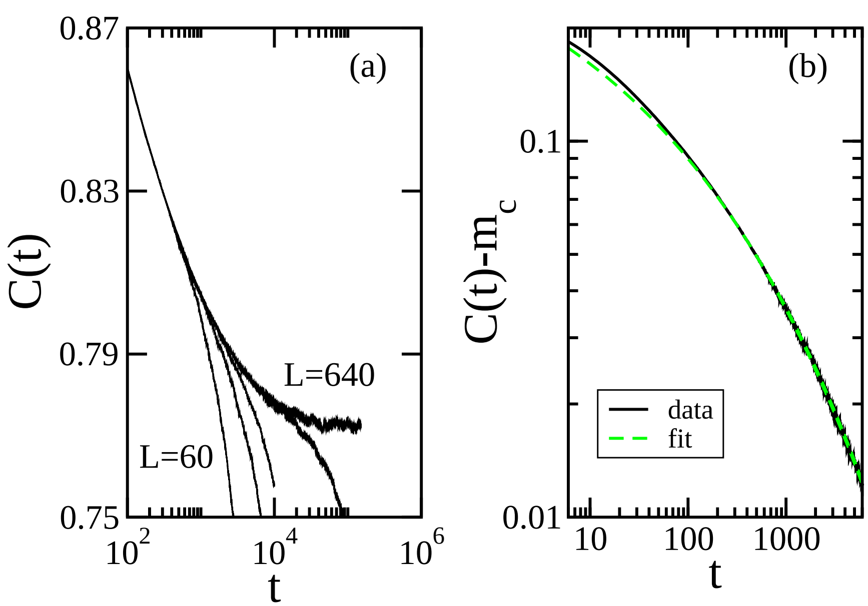
$n = 4$ Turban model $T < T_c$ $\quad \rightarrow$ 8-fold degenerate state

- latent heat: $\Delta / kT_c = 0.146(3)$
- jump in the magnetization: $m_c = 0.769(6)$

Reminder: nonequilibrium relaxation from $T_i = \infty$

[M. Pleimling, F. I.,Europhys. Lett. **79**, *56002 (2007)]*

Autocorrelation

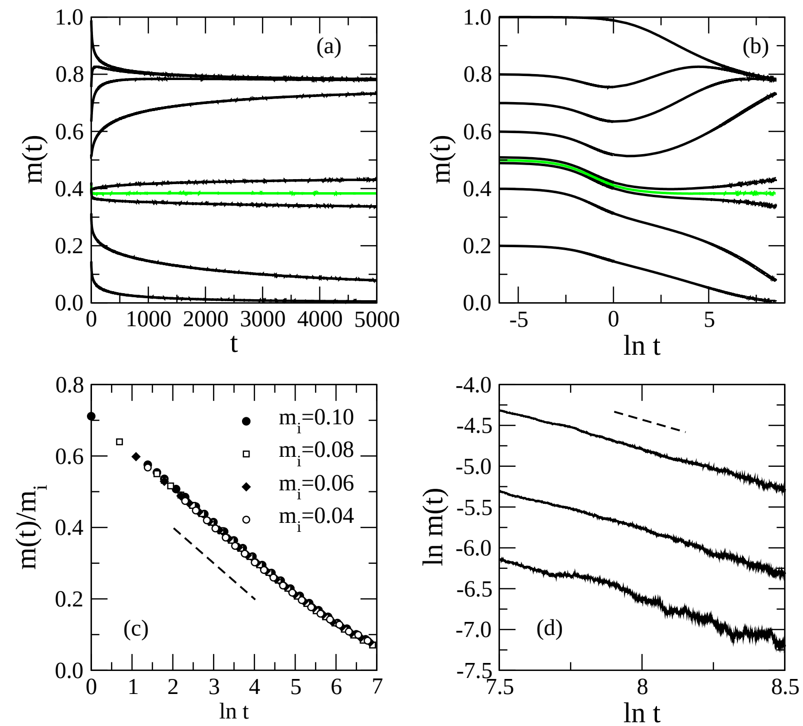


stretched exponential approach of m_c

$$C(t) - m_c \sim \exp(-bt^a)$$

$$a = 0.198(3)$$

Relaxation from $m_i > 0$



$$m_i > m^* = 0.5 \quad m(t) \to m_c$$

$$m_i < m^* = 0.5 \quad m(t) \to 0 \quad m(t) \sim t^{-d/z}$$

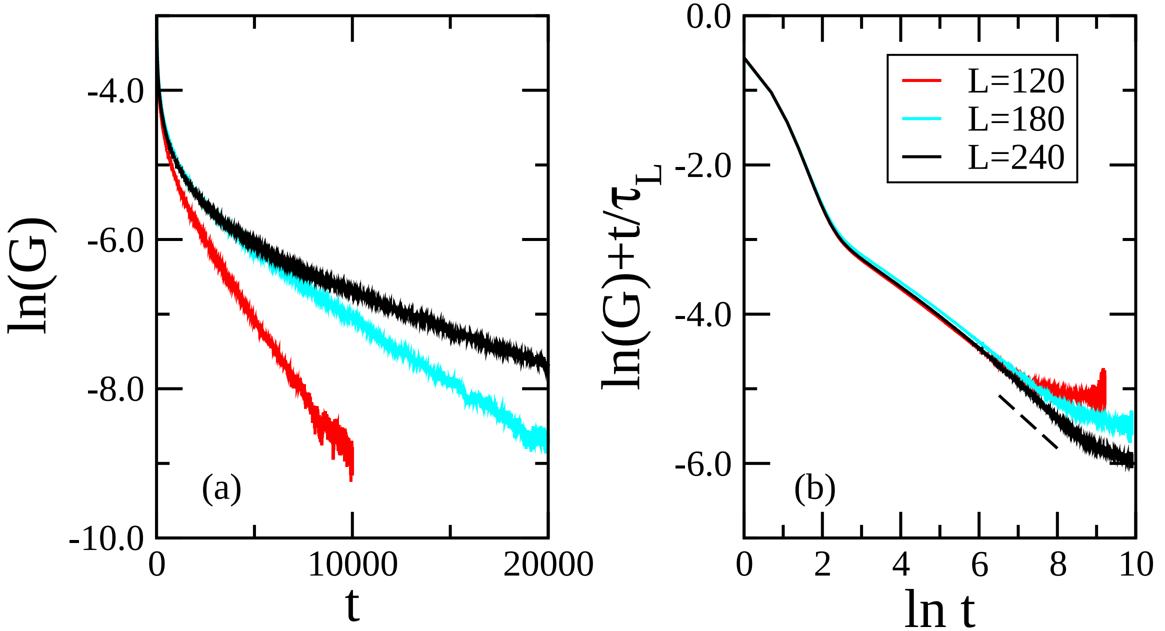
$$m_i < m^* = 0.5 \quad m(t) \to m_c/2$$

discontinuous equilibrium transition

continuous nonequilibrium transition

Wetting in time

Autocorrelation of the 2d Ising model starting with $n=4$ initial state



$$G(t) = At^{-\lambda/z}\exp(-t/\tau_L)$$

$$\lambda/z = 0.475(10) \quad \rightarrow \quad \boxed{\lambda = d/2}$$

Conclusions

	x	z	λ/z	θ
Ising	$1/8$	2.17	$0.74(2)$	0.187
BW	$1/8$	$2.29(1)$	$1.13(6)$	$-0.186(2)$
$n=3$	$1/8$	$2.3(1)$	$0.98(2)$	$-0.03(1)$
$n=4$	$0.^{(a)}$	$2.05(10)$	$\infty^{(b)}$	$-1.00(5)$
BW			$0.17(1)$	$0.18(1)$
$n=3$			$0.165(10)$	$0.18(1)$
$n=4$			$0.475(10)$	

$^{(a)}$: discontinuity fixed-point value $^{(b)}$: stretched exponential decay

- a critical initial state has an influence on the nonequilibrium exponents
- initial states in the same universality class have the same effect
- analogy with static critical behavior at an interface separating two critical systems
- if the initial state corresponds to a 1^{st} order transition then $\lambda = d/2$

Acknowledgment

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• 2d random-field Ising model

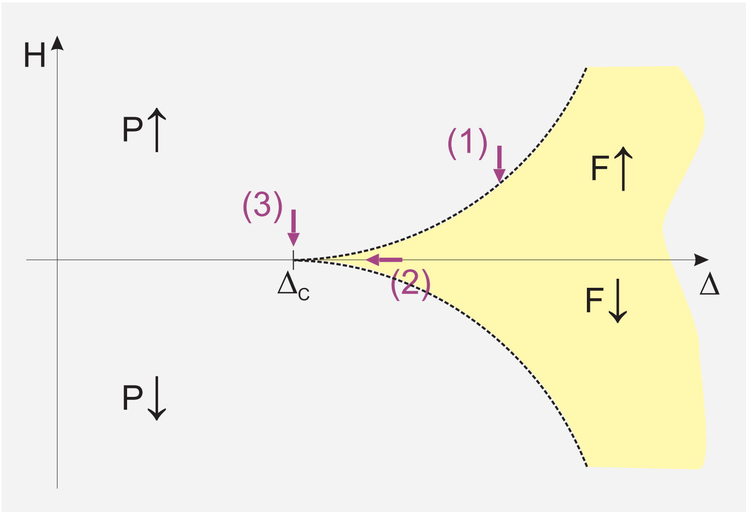
$$\mathscr{H}_{RF} = \mathscr{H}_I - \sum_i (H + h_{i,j})\sigma_{i,j}$$

$h_{i,j}$ random field with Gaussian distribution:

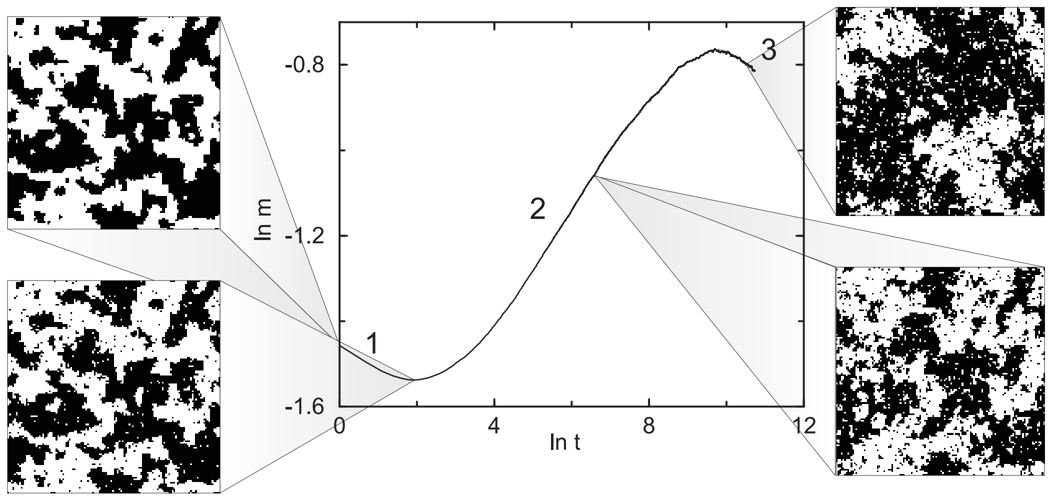
$$P(h_{i,j}) = \frac{1}{\sqrt{2\pi h^2}} \exp\left[-\frac{h_{i,j}^2}{2h^2}\right]$$

for $H = 0$:

- no magnetic long-range order
- geometrical clusters (with parallel spins)
  - percolating for $\Delta = J/h < \Delta_c \approx 1.65$
  - homogeneous domains of size: $l_b \sim \exp(A/\Delta^2)$: breaking-up length

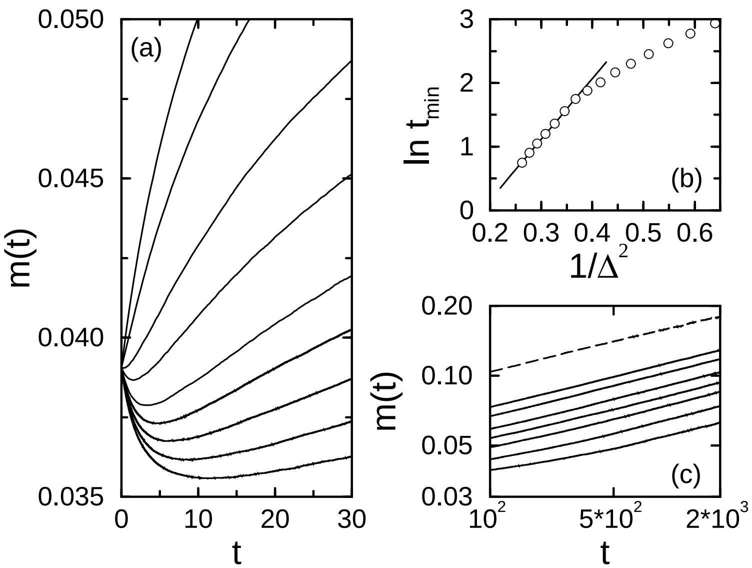


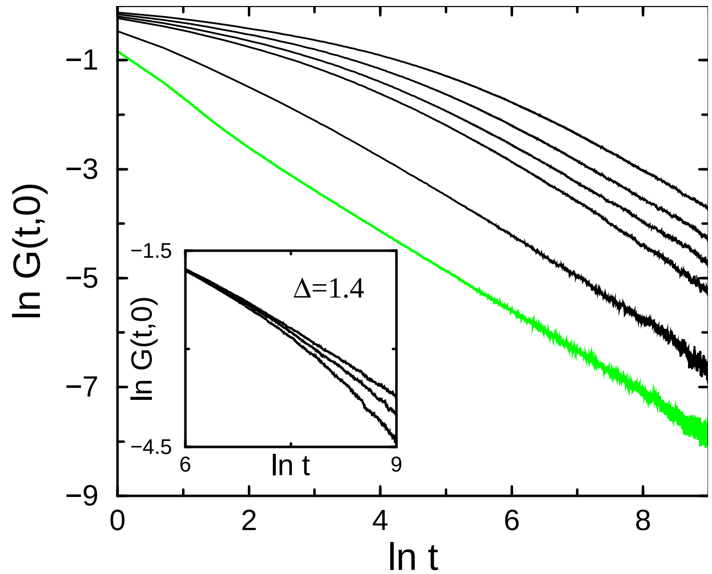
Relaxation from the RFIM initial state



$$m_i = 0.25, \quad \Delta = 1.7, \quad \text{lattice} : 200 \times 200$$

- 1) cluster dissolution $\rightarrow \ln t_{min} \sim \ln l_b \sim 1/\Delta^2$
- 2) domain growth $\rightarrow t_i \sim m_i^{-z/x_i}$
- 3) equilibrium relaxation





a) $\Delta = 1.4, 1.5, 1.6, 1.7, 1.8, 2.0, 2.2, 2.6, 3.0$ (from bottom to top)

c) $\Delta = 1.4, 1.6, 1.8, 2.0, 2.2, 2.6, 3.0$ (from bottom to top), dashed: $T_i = \infty$

$\Delta = 1.4, 1.6, 1.8, 2.0, 4.0$ (from top to bottom), green: $T_i = \infty$

inset: $L = 128, 200, 500$ (from bottom to top)

same nonequilibrium exponents as for $T_i = \infty$