Nonequilibrium critical dynamics of the two-dimensional Ising model quenched from a correlated initial state

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in collaboration with

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AGENDA

• Nonequilibrium relaxation
  – ordered \((T_i = 0)\) initial state
  – disordered \((T_i = \infty)\) initial state
  – analogy with static semi-infinite criticality

• Models
  – Ising model
  – Baxter-Wu (BW) model

  – Turban model

• Results
  – Relaxation from a critical initial state
  – Relaxation from a 1\textsuperscript{st}-order initial state

• Conclusions
**Nonequilibrium relaxation**

- prepare the system in an initial state for \( t < 0 \)
- quench to the critical point \( (T_c) \) at \( t = 0 \)
- let evolve with the dynamical rules at \( T_c \) for \( t > 0 \)
- measure
  - relaxation of the magnetization:
    \[
    m(t) = \langle \sigma(t) \rangle
    \]
  - autocorrelation function:
    \[
    G(t,s) = \langle \sigma(s)\sigma(t) \rangle
    \]

\( s \): waiting time \( t \): observation time

**in equilibrium:**

\[
G(t,s) \sim (t-s)^{-2x/z}
\]
time dependence for \( t > 0 \)

- **ordered initial state** (\( T_i = 0 \))
  relaxation involves only equilibrium critical exponents

\[
m(t) \sim t^{-x/z}, \quad T_i = 0
\]

\( x = \beta/\nu \): anomalous dimension of the magnetization
\( z \): dynamical exponent

autocorrelation function:

\[
G(t,s) \sim (t-s)^{-2x/z} g(t/s), \quad T_i = 0
\]

position dependence for \( y > 0 \)

- magnetization profile in a semi-infinite system \( y \geq 0 \) with **fixed boundary condition**

\[
m(y) \sim y^{-x}, \quad h_s = \infty
\]

correlation function:

\[
C(y_1,y_2) \sim (y_1 - y_2)^{-2x} g(y_1/y_2), h_s = \infty
\]
time dependence for \( t > 0 \)

- **disordered initial state** \((T_i = \infty)\)

  \[
  m(t) \sim m_i t^\theta, \quad t < t_i
  \]

  \( \theta \): initial slip exponent

  \( m_i \to 0 \): initial magnetization

  \( t_i \sim m_i^{-z/x_i} \): initial time

  \( x_i = \theta z + x \): anomalous dimension of \( m_i \)

  \[
  G(t,s) \sim t^{-\lambda/z}, \quad t \gg s, \quad T_i = \infty
  \]

  \( \lambda = d - \theta z = d - x_i + x \)

position dependence for \( y > 0 \)

- magnetization profile in a semi-infinite system **with small surface field** \( h_s \ll 1 \)

  \[
  m(y) \sim y^{x_s-x}, \quad y < y_s
  \]

  \( h_s \to 0 \): surface field

  \( y_s \sim h_s^{-1/x_s} \): surface region

  \( x_s \): anomalous dimension of \( m_s \)

  \[
  C(y_1, y_2) \sim y_1^{-\eta_\perp}, \quad y_1 \gg y_2
  \]

  \( \eta_\perp = x_s + x \)
Power-law correlations in the initial state

previous studies

- 2d XY model: $T_i < T_c < T_{KT}$ (Berthier, Holdsworth, Sellitto) (Abriet, Karevski)

\[ x(T_c) \rightarrow x(T_c) - x(T_i) \]

- spherical model (Picone, Henkel)

**Our aim is to study nonequilibrium dynamics after a sudden change of the form of the interaction**

Experimental realization: atoms in optical lattices.
Models

- **2d Ising model** → dynamics at $T_c$

  \[
  \mathcal{H}_I = -\sum_{i,j} J \left( \sigma_{i,j} \sigma_{i,j+1} + \sigma_{i,j} \sigma_{i+1,j} \right)
  \]

  critical point: $\sinh\left(\frac{2J}{kT_c}\right) = 1$

- **Baxter-Wu model**

  \[
  \mathcal{H}_{BW} = -\sum_{i,j} J_{BW} \left( \sigma_{i,j} \sigma_{i-1,j} \sigma_{i,j-1} + \sigma_{i,j} \sigma_{i+1,j} \sigma_{i,j+1} \right)
  \]

  critical point: $\sinh\left(\frac{2J_{BW}}{kT_c}\right) = 1$

  4-state Potts universality.

- **Turban model**

  \[
  \mathcal{H}_n = -\sum_{i,j} \left( J_2 \sigma_{i,j} \sigma_{i,j+1} + J_n \prod_{k=0}^{n-1} \sigma_{i+k,j} \right)
  \]

  critical point:

  $\sinh\left(\frac{2J_2}{kT_c}\right) \sinh\left(\frac{2J_n}{kT_c}\right) = 1$

  - $n = 3$, $2^{nd}$-order
  - $n = 4$, $1^{st}$-order

4-state Potts universality
Relaxation from second-order transition points


BW model

$n = 3$ Turban model
Relaxation from a BW critical state

\[ m_i = 0.33, 0.16, 0.08, 0.04, 0. \]

(from top to bottom)

\[ m_i = 0. \]

\[ \theta = 0.18(1) \]
\begin{align*}
G(t) &= A t^{-\lambda/z} \exp(-t/\tau_L) \\
\tilde{G}(t) &= G(t) \exp(t/\tau_L)
\end{align*}

\[\frac{\lambda}{z} = 0.18(1)\]
Relaxation from an $n = 3$ critical state

magnetization

$\theta = 0.18(1)$

$\tilde{G}(t) = G(t) \exp(t/\tau_L)$

$\frac{\lambda}{z} = 0.165(10)$
Relaxation from a first-order transition point

\( n = 4 \) Turban model \( T < T_c \rightarrow 8\)-fold degenerate state

- latent heat: \( \Delta/kT_c = 0.146(3) \)

- jump in the magnetization: \( m_c = 0.769(6) \)

**Reminder:** nonequilibrium relaxation from \( T_i = \infty \)

[M. Pleimling, F. I., Europhys. Lett. 79, 56002 (2007)]

**Autocorrelation**

\[
C(t) - m_c \sim \exp(-bt^a)
\]

\[ a = 0.198(3) \]
Relaxation from $m_i > 0$


discontinuous equilibrium transition
continuous nonequilibrium transition

Wetting in time
Autocorrelation of the 2d Ising model starting with $n = 4$ initial state

\[ G(t) = A t^{-\lambda/z} \exp(-t/\tau_L) \]

\[ \lambda/z = 0.475(10) \quad \rightarrow \quad \lambda = d/2 \]
Conclusions

<table>
<thead>
<tr>
<th></th>
<th>$x$</th>
<th>$z$</th>
<th>$\lambda/z$</th>
<th>$\theta$</th>
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<tr>
<td>Ising</td>
<td>1/8</td>
<td>2.17</td>
<td>0.74(2)</td>
<td>0.187</td>
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<tr>
<td>BW</td>
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<td>2.29(1)</td>
<td>1.13(6)</td>
<td>−0.186(2)</td>
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<td>$n = 3$</td>
<td>1/8</td>
<td>2.3(1)</td>
<td>0.98(2)</td>
<td>−0.03(1)</td>
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<td>0.0(0)</td>
<td>2.05(10)</td>
<td>$\infty(b)$</td>
<td>−1.00(5)</td>
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<tr>
<td>BW</td>
<td>$n = 3$</td>
<td>0.17(1)</td>
<td>0.165(10)</td>
<td>0.18(1)</td>
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<td></td>
<td>$n = 4$</td>
<td>0.475(10)</td>
<td></td>
<td>0.18(1)</td>
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</tbody>
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(a): discontinuity fixed-point value  (b): stretched exponential decay

• a critical initial state has an influence on the nonequilibrium exponents

• initial states in the same universality class have the same effect

• analogy with static critical behavior at an interface separating two critical systems

• if the initial state corresponds to a 1$^{st}$ order transition then $\lambda = d/2$
Acknowledgment

László Környei (Szeged, Budapest)

Michel Pleimling (Virginia Tech)
• 2d random-field Ising model

\[ \mathcal{H}_{RF} = \mathcal{H}_I - \sum_i (H + h_{i,j}) \sigma_{i,j} \]

- homogeneous domains of size:

\[ l_b \sim \exp(A/\Delta^2) : \quad \text{breaking-up length} \]

- no magnetic long-range order

- geometrical clusters (with parallel spins)

- percolating for

\[ \Delta = J/h < \Delta_c \approx 1.65 \]
Relaxation from the RFIM initial state

\[ m_i = 0.25, \quad \Delta = 1.7, \quad \text{lattice: } 200 \times 200 \]

- 1) cluster dissolution \( \rightarrow \ln t_{\text{min}} \sim \ln l_b \sim 1/\Delta^2 \)

- 2) domain growth \( \rightarrow t_i \sim m_i^{-z/x_i} \)

- 3) equilibrium relaxation
a) $\Delta = 1.4, 1.5, 1.6, 1.7, 1.8, 2.0, 2.2, 2.6, 3.0$ (from bottom to top)

b) $\Delta = 1.4, 1.6, 1.8, 2.0, 2.2, 2.6, 3.0$ (from bottom to top), dashed: $T_i = \infty$

c) $\Delta = 1.4, 1.6, 1.8, 2.0, 2.2, 2.6, 3.0$ (from top to bottom), green: $T_i = \infty$

inset: $L = 128, 200, 500$ (from bottom to top)

**same nonequilibrium exponents as for $T_i = \infty$**