Strong coupling QCD as a dimer model

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arXiv:0811.1931

CompPhys08 Leipzig, Nov. 2008

Motivation

Strong coupling QCD (SCQCD) under investigation for > 25 years...

Analytical (Mean Field $1/d$)

- Mass spectrum: Kawamoto and Smit '81, Kluberg-Stern, Morel, Petersson '82
- Phase diagram, Damgaard, Kawamoto, Shigemoto '84
- Phase diagram with $1/g^2$ corrections: Faldt and Petersson '86, Bilić et al.'92
- Latest: Nishida '04, Kawamoto et al. '05, Miura and Ohnishi '08 (next talk)

Numerical

- Karsch and Mütter '89, MDP-approach ($T \approx 0$, $\mu \approx \mu_c$)
- Boyd et al. '92, MDP at $T \approx T_C, \mu = 0$
- Azcoiti et al. '99 (MDP under scrutiny)
- de Forcrand and Kim '06, HMC, mass spectrum

Strong coupling QCD

Some Definitions:

$$Z = Z(m, \mu, \beta) = \int \mathcal{D}U \mathcal{D}\bar{\chi} \mathcal{D}\chi \, \mathrm{e}^{S_\mathrm{F} + \beta S_\mathrm{G}},$$

μ chemical potential, m staggered quark mass, $\beta = \frac{6}{g_0^2}$ inverse gauge coupling,χ grassmann-valued quark field, links $U_{x,\hat{\nu}} \in SU(3)$

$$\begin{aligned} S_\mathrm{G} &= \sum_P \left(1 - \frac{1}{3} \mathrm{Re}\,\mathrm{tr}[U_P]\right) \\ S_\mathrm{F} &= \sum_{x,\nu} \bar{\chi}_x \left[\eta_{x,\hat{\nu}} U_{x,\hat{\nu}} \chi_{x+\hat{\nu}} - \eta^{-1}_{x,\hat{\nu}} U^\dagger_{x-\hat{\nu},\hat{\nu}} \chi_{x-\hat{\nu}}\right] + 2m \sum_x \bar{\chi}_x \chi_x \end{aligned}$$

$\eta_{x,\hat{\nu}} = \mathrm{e}^{\mu}$ $(\nu = 0)$ and $(-1)^{\sum_{\rho<\nu} x_\rho}$ otherwise.

Strong coupling QCD (SCQCD)

In Strong (infinite) coupling limit $g \to \infty$, $\beta = 0$ - can do integral in links $U_{x,\hat{\nu}}$ first [Rossi & Wolff]:

$$Z(m,\mu) = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \mathrm{e}^{2m\sum_x \bar{\chi}_x \chi_x} \prod_{x,\nu} F_{x,x+\hat{\nu}}$$

where $F_{x,x+\hat{\nu}} = \sum_{k=0}^{3} (-1)^k \alpha_k (M_x M_{x+\hat{\nu}})^k + \left[\bar{B}_x B_{x+\hat{\nu}} \eta_{x,\hat{\nu}}^3 - \bar{B}_{x+\hat{\nu}} B_x \eta_{x,\hat{\nu}}^{-3} \right].$

New degrees of freedom are color singlet

Monomers $M_x = \sum_{a,x} \bar{\chi}_{ax}\chi_{ax}$, $(\bullet)$, monomers per site $n_x = 0, \ldots, 3$

Dimers $D_{k,xy} = \frac{1}{k!}(M_x M_y)^k$ $(-, =, \equiv)$, bond number $n_b = 0, \ldots, 3$

(Anti-)Baryons $B_x = \chi_{1x}\chi_{2x}\chi_{3x}$, $\bar{B}_x = \bar{\chi}_{3x}\bar{\chi}_{2x}\bar{\chi}_{1x}$, $- - -$

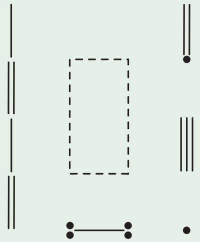
SCQCD loop gas

Self-avoiding loops C of $\bar{B}B_x$ pairs are formed, with *signed* weights $\rho(C)$,

$$Z(m,\mu) = \sum_{\{n_x, n_b, \square\}} \prod_b \frac{(3-n_b)!}{3!\,n_b!} \prod_x \frac{3!}{n_x!}(2m)^{n_x} \prod_{\text{loops } C} \rho(C) \ ,$$

constraint $n_x + \sum_{b_x} n_{b_x} = 3$.

Example



- MDP description [Karsch & Mütter, 1989]: Signed baryonic loops are associated with polymer loops. Mapping the weight

$$\begin{aligned} \rho_B(C) &\rightarrow w_{polymer}(C) \\ \pm\cosh 3\mu/T &\rightarrow 1 \pm \cosh 3\mu/T \end{aligned}$$

softens the sign problem.

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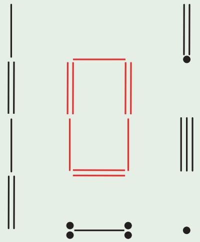

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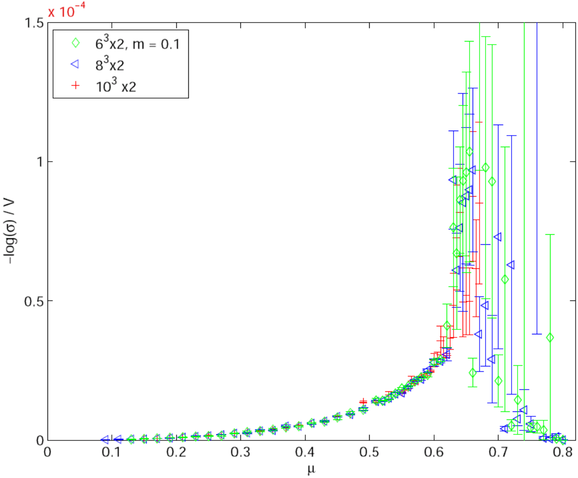
Average sign for varying V, μ



- $\langle \mathrm{sign} \rangle = \frac{Z}{Z_+} \sim \exp\left(-Vf(\mu)/T\right)$

The worm algorithm in strong coupling QCD

Starting over from initial formulation of $Z = \int \ldots$ introduce two additional meson fields M_{bx_1}, M_{cx_2}, i.e.

$$\langle M_{bx_1} M_{cx_2} \rangle = \int \mathcal{D}\bar{\chi}\mathcal{D}\chi \, M_{bx_1} M_{cx_2} \mathrm{e}^{2m \sum_x \bar{\chi}_x \chi_x} \prod_{\langle xy \rangle} F_{xy}.$$

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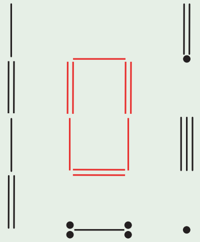

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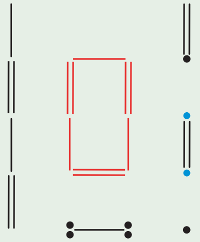

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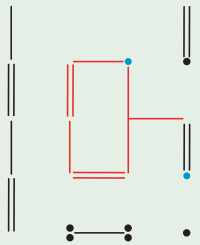

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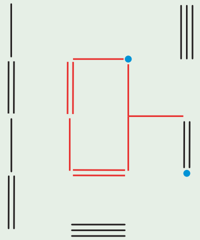

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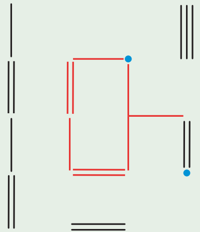

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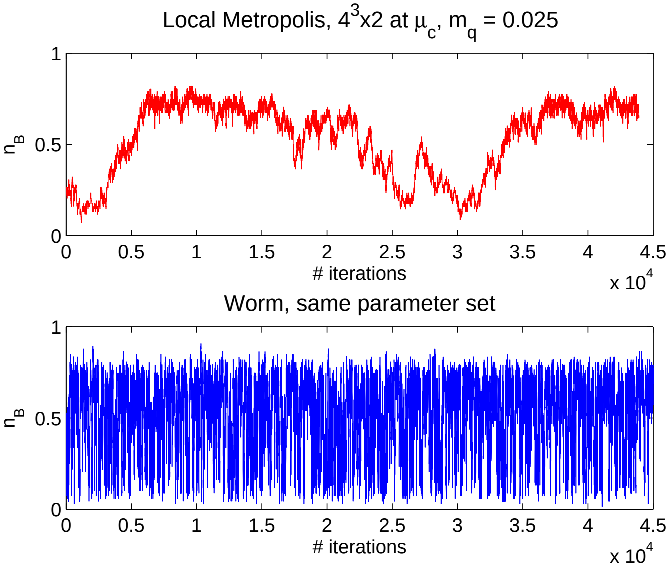
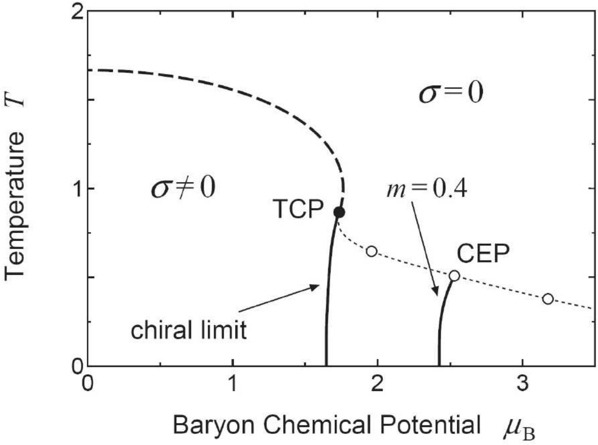
Simple efficiency test



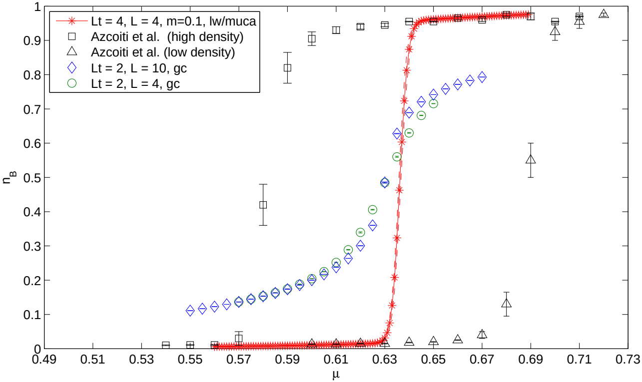
SCQCD chiral restoration transition (1)

Phase diagram as obtained by *mean-field* calculations [Nishida '04]



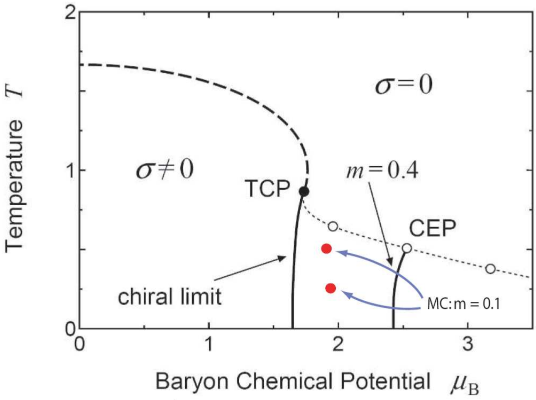
$m = 0$: $U(1)_A$:$\chi \to \chi \exp(i\epsilon(x)\theta)$, $\bar{\chi} \to \bar{\chi} \exp(i\epsilon(x)\theta)$, $\epsilon(x) = (-)^{\sum_\rho x_\rho}$

Consistency check with Karsch & Mütter



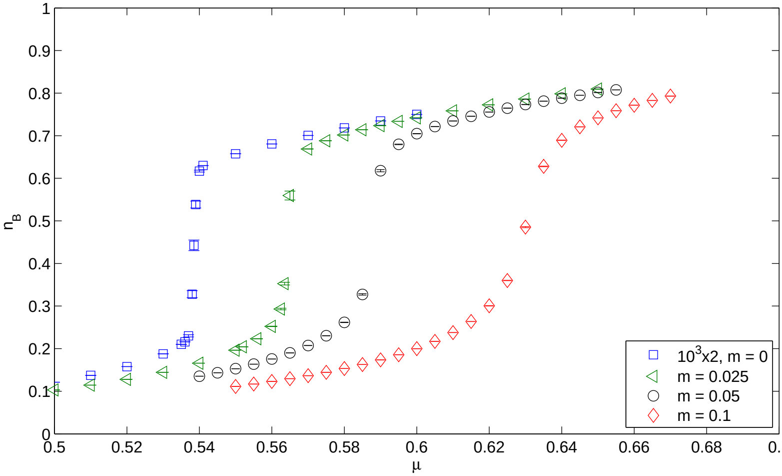
- At low T, transition strong 1st order; multicanonical sampling, wang-landau need for ergodic sampling
- Reveals important corrections to $\mu_c(T = 1/4, m = 0.1)$ value

Compare to mean-field results (1)



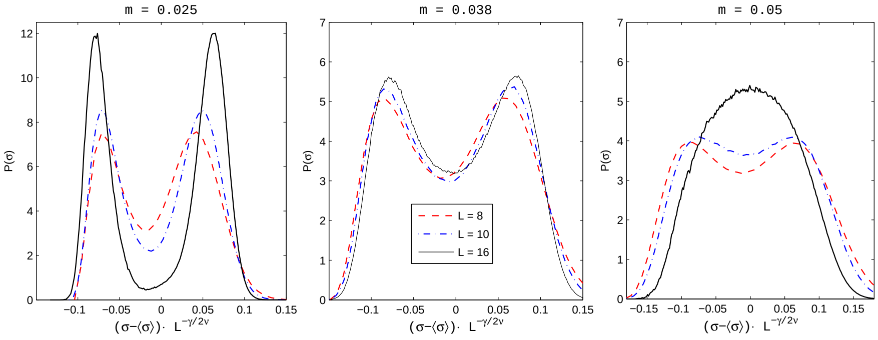
- Good agreement as for location of transition for m = 0.1, T = 1/2, 1/4
- Order? → Mean-field: m = 0.1, T = 1/2 is still first order
- Or: First order line ends in CEP - critical mass for e.g. T = 1/2 ?

Compare to mean-field results (2)



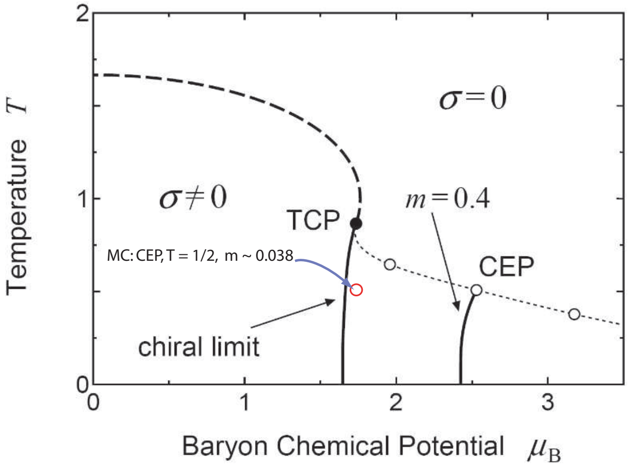
- As quark mass m increases critical μ is shifted and transition becomes smoother

Compare to mean-field results (3)



- FSS for $P(\sigma)$, taking $3d$ Ising critical exponents
- $m_c(T = 1/2) \approx 0.038$

Compare to mean-field results (4)



- Mean-field $m_c(T = 1/2) = 0.4$ deviates $\mathcal{O}(10)$!

SCQCD chiral restoration transition (2)

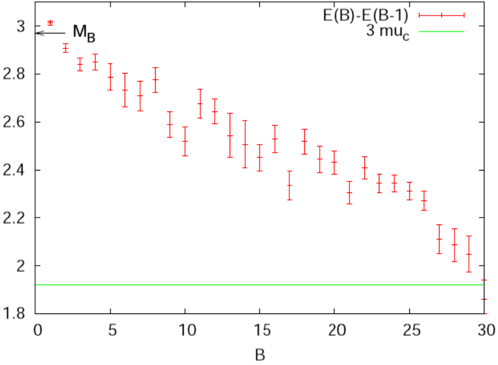
Puzzle

- $T = 1/4, m = 0.1$ MC supports mean-field: $\mu_c \approx 0.64$ (MC).
- However: Expect ($T = 0$)-phase transition when

$$3\mu \geq F_B \approx M_{\mathrm{Nucleon}} \approx 3\,, \mathrm{i.e.} \quad \mu_c \approx 1$$

- Nuclear attraction strong, $\mathcal{O}(300\,\mathrm{MeV})$?
- Or: Finite T effects (MC) and mean field approach inaccurate ?
  $\rightarrow$ Check with canonical simulation.

Internal energy vs B



- $E(B=1) \approx M_B$ ($T \sim 75$ MeV)
- $E(B=2) - 2E(B=1) \approx -0.1$, i.e. "deuteron" binding enhanced from 2 MeV to 30 MeV
- Further binding of one baryon to ca.20 nearest neighbours to give $\mu_c \approx 600$ MeV

Summary & Outlook

- Sizeable corrections to early MC-study
- Quantitative mean-field results may deviate by $\mathcal{O}(10)$
- "Nuclear" matter

"Assignment"

- Extrapolation $T \to 0$ remains open,
  - Include asymmetry γ to vary T continuously
  - Compare real and imaginary μ
- SCQCD Phase diagram
  - In the chiral limit: Locate TCP
  - CEP for varying quark mass
  - Flavor dependence of phase diagram