Ageing in bosonic contact and pair-contact process with Lévy flights : exacts results

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Article with M. Henkel : in preparation

CompPhys08, Universität Leipzig, 27$^{\text{th}}$ of December 2008

Contents

- Physical ageing phenomena, Bosonic contact processes
- Results : phase diagram, exponents
- Cross-over time towards ageing regime, Fluctuation dissipation ratio

I. Physical ageing phenomena

- Why do materials look old after some time ?
- Which (reversible) microscopic processes lead to such macroscopic effects ?
- One of the first experiments by Struik (1978)

a priori behaviour should depend on entire prehistory

but evidence for reproductible and **universal** behaviour

- Ageing : defining characteristics and symmetry properties :
  1. slow dynamics
  2. breaking of time-translation invariance
  3. scaling invariance

I.2 Scaling behaviour

System studied easier than glassy system : ex. simple magnet
single relevant time-dependent length scale : $L(t) \sim t^{1/z}$

$$
\begin{aligned}
\text{correlator } C(t,s) &= \langle\phi(t)\phi(s)\rangle - \langle\phi(t)\rangle\langle\phi(s)\rangle \approx s^{-b} f_C(t/s) \\
\text{response } R(t,s) &= \left.\frac{\delta\langle\phi(t)\rangle}{\delta h(s)}\right|_{h=0} \approx s^{-1-a} f_R(t/s)
\end{aligned}
$$

In ageing regime : $t, s >> 1$ and $t - s >> 1$
for $y = t/s \to \infty$: $f_C(y) \sim y^{-\lambda_C/z}$, $f_R(y) \sim y^{-\lambda_R/z}$

z : dynamical exponent
a, b : ageing exponents
λ_R : autoresponse's exponent
λ_C : correlator's exponent

II. Bosonic contact process : definition

Bosonic contact process :

- Non-negative arbitrary number of particles on each site
- Reaction-diffusion rate :

$$mA \longrightarrow (m+k)A\,;\ \text{with rate } \mu$$
$$pA \longrightarrow (p-l)A\,;\ \text{with rate } \lambda$$

Usual case : Particles move diffusively with rate D

Studied case : long range hopping with a diffusion rate D(r)

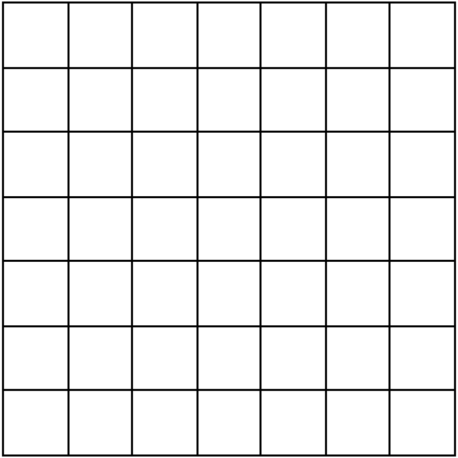
Bosonic contact process (BCPL) : $p = m = 1$
Bosonic pair-contact process (BPCPL) : $p = m = 2$

Houchmandzadeh, 2002

Paessens, Schütz, 2004

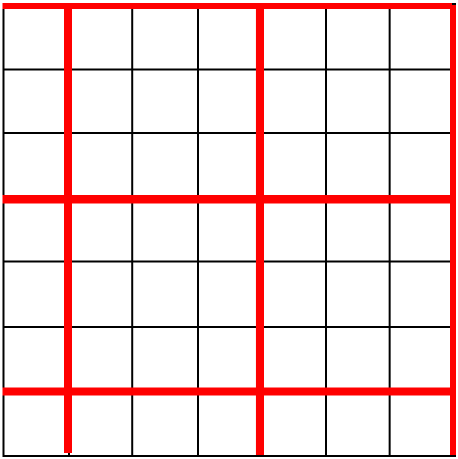
Baumann, Henkel, Pleimling, Richert, 2005

II.1 Why Lévy flights ?



Microscopic real lattice :

$\rightarrow$ particles move randomly at distance x_{black}



Coarsening lattice :

Red particles move such as : $x_{red} = \sum_{\text{red square}} x_{black}$.

Random variables x_i are identically distributed.

Generalized central limit theorem :
A sum of independant random variables x_i from the same distribution belongs to the domain of attraction of a stable distribution.

Lévy, Khintchine, 30s

If the first and second moment exist, this distribution is gaussian $(\eta = 2)$. $\rightarrow$ diffusive motion.

Distribution of the diffusion rate $D(\overrightarrow{r})$

$$D(\overrightarrow{r}) = D\frac{1}{(2\pi)^d}\int_{-\infty}^{\infty} d^d\overrightarrow{q}\, e^{\left(i\overrightarrow{q}.\overrightarrow{r} - \|\overrightarrow{q}\|^{\eta}c\right)}$$

where $0 < \eta \leq 2$: control parameter

II.2 Master equation

Master equation in quantum hamiltonian formulation

$$\partial_t |P(t)\rangle = -H|P(t)\rangle$$

where $|P(t)\rangle = \sum_{\{n\}} P(\{n\};t)|\{n\}\rangle$: state vector

Glauber, 1963 ...

$$\frac{\partial}{\partial t}\langle a(\overrightarrow{x},t)\rangle = \sum_{\overrightarrow{n}\neq\overrightarrow{0}} D(\overrightarrow{n})[\langle a(\overrightarrow{x}+\overrightarrow{n},t)\rangle - \langle a(\overrightarrow{x},t)\rangle]$$
$$-\lambda l\langle a(\overrightarrow{x},t)^p\rangle + \mu k\langle a(\overrightarrow{x},t)^m\rangle + h(\overrightarrow{x},t)$$

Critical line

$$\sigma = \frac{\mu k - \lambda l}{D} = 0$$

Essential control parameter

$$\alpha = \frac{\mu k(k+l)}{2D}$$

In Fourier space

$$\frac{\partial}{\partial t}\langle a(\overrightarrow{q},t)\rangle = -\frac{1}{2}\sum_{\overrightarrow{n}\neq\overrightarrow{0}}\frac{D(\overrightarrow{n})}{D}(1-e^{i\overrightarrow{q}.\overrightarrow{r}})\langle a(\overrightarrow{q},t)\rangle + \frac{\sigma}{2}\langle a(\overrightarrow{q},t)\rangle + h(\overrightarrow{q},t)$$

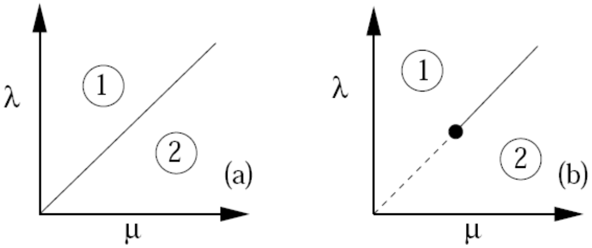
Dispersion relation $\omega(\overrightarrow{q})$

$$\omega(\overrightarrow{q}) = \sum_{\overrightarrow{n}\neq\overrightarrow{0}}\frac{D(\overrightarrow{n})}{D}\left(1-e^{i\overrightarrow{q}.\overrightarrow{r}}\right)$$

$$\omega(\overrightarrow{q}) = 1-e^{-\|\overrightarrow{q}\|^{\eta}c} \underset{\overrightarrow{q}\to\overrightarrow{0}}{\approx} \|\overrightarrow{q}\|^{\eta}c$$

Dynamical exponent

$$z=\eta$$



Schematics phase diagrams for $d \neq 0$ of :

- BCPL in $d \leq \eta$
- BPCPL in $d \leq \eta$

BPCPL in $d > \eta$

The absorbing phase 1 is separated by the critical line ($\sigma = \frac{\mu k - \lambda l}{D} = 0$) from the active region 2.

Full line : clustering ; broken line : homogeneous.

phase 1 : density vanishes.

phase 2 : density diverges.

III.2 Results : Exponents

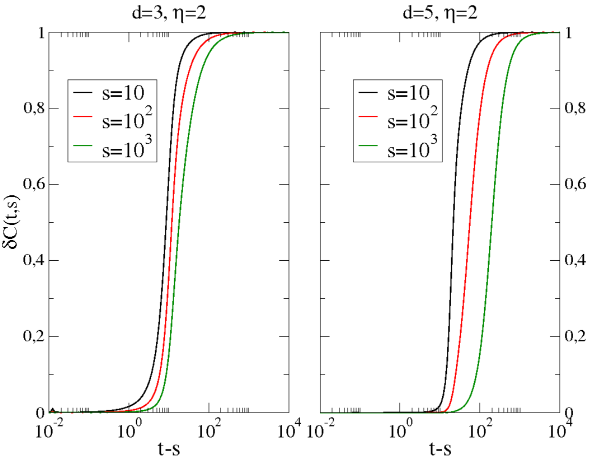
	Bosonic contact process	Bosonic pair-contact process	
		$\alpha < \alpha_c$	$\alpha = \alpha_c$
a	$\frac{d}{\eta} - 1$	$\frac{d}{\eta} - 1$	$\frac{d}{\eta} - 1$
b	$\frac{d}{\eta} - 1$	$\frac{d}{\eta} - 1$	0 si $\eta < d < 2\eta$ $\frac{d}{\eta} - 2$ si $d > 2\eta$
λ_R	d	d	d
λ_C	d	d	d
z	η	η	η

- For $\eta = 2$, recover **diffusive motion** case.
- $a = b$ for BCPL and BPCPL with $\alpha < \alpha_C$: we can define a Fluctuation Dissipation Ratio.
- Study the behaviour of the FDR in the BCPL case.

IV. Cross-over time towards ageing regime

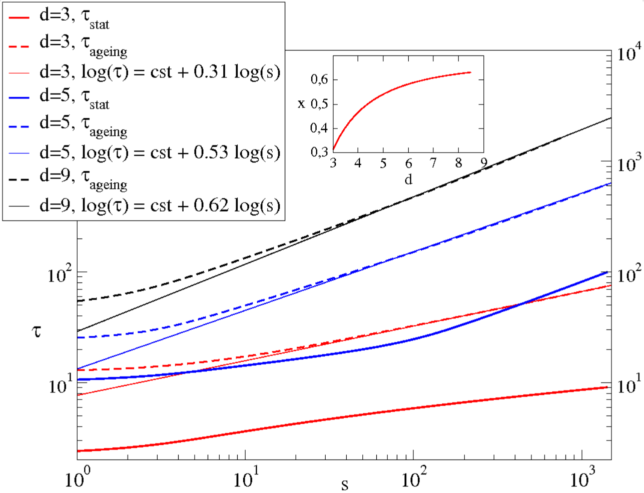
Relative error defined such as

$$\delta C(t,s) = \left| \frac{C(t,s) - C_{stat}(t,s)}{C_{ageing}(t,s) - C_{stat}(t,s)} \right|$$



τ_{stat} such as $\delta C(t,s) = 10\%$; τ_{ageing} such as $\delta C(t,s) = 90\%$

IV.1 Results on critical line



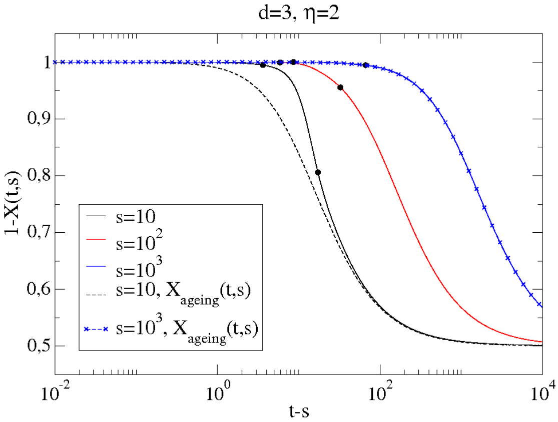
Asymptotic behaviour : $\tau_{ageing} \sim cst.s^x$ with $x(d) < 1$ and $cst(d)$.

Similar to ageing in spherical model for $T < T_c$

W. Zippold, R. Kuehn, H. Horner, 2000

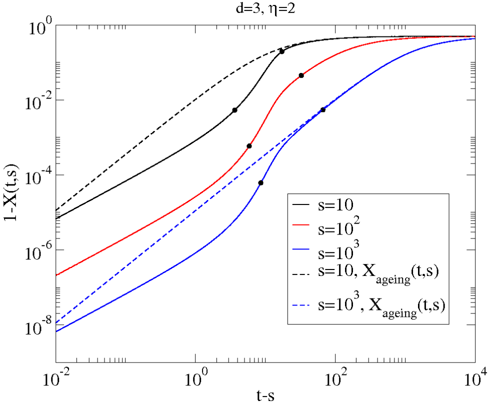
IV.2 Fluctuation-dissipation ratio

$$X(t,s) = \frac{\frac{\partial C}{\partial s}(s,s)}{R(s,s)} \frac{R(t,s)}{\frac{\partial C}{\partial s}(t,s)} \qquad X_{ageing}(t,s) = \frac{1}{1+\left(\frac{t-s}{t+s}\right)^{d/\eta}}$$



Behaviour analogous to ageing in simple magnet with detailed balance

IV.3 Fluctuation dissipation ratio (loglog plot)



Three differents regimes :

- Stationary regime : microscopic relaxation
- Transition regime : non analytic
- Ageing regime : out of equilibrium

Conclusion

- Study dynamics of relaxation and ageing in systems without detailed balance
- BCP and BPCP choosen because exactly solvable
- Analyze effects of Lévy flights transport
- Find exact scaling functions for correlation and response
- Can define fluctuation dissipation ratio
- Analyze cross-over time from stationary regime to ageing regime

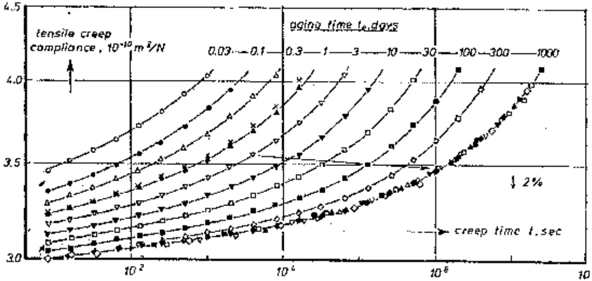
Heisenberg equation of motion :

$$\frac{\partial}{\partial t} g(\overrightarrow{x}, t) = [H; g(\overrightarrow{x}, t)]$$

Hamiltonian :

$$\begin{aligned} H = &- \sum_{\overrightarrow{n} \neq \overrightarrow{0}} \sum_{\overrightarrow{x}} D(\overrightarrow{n})[a(\overrightarrow{x})a^{\dagger}(\overrightarrow{x} + \overrightarrow{n}) - n(\overrightarrow{x})] \\ &- \lambda \sum_{\overrightarrow{x}} [a^{\dagger}(\overrightarrow{x})^{p-l} a(\overrightarrow{x})^{p} - \prod_{i=1}^{p} (n(\overrightarrow{x}) - i + 1)] \\ &- \mu \sum_{\overrightarrow{x}} [a^{\dagger}(\overrightarrow{x})^{m+k} a(\overrightarrow{x})^{m} - \prod_{i=1}^{m} (n(\overrightarrow{x}) - i + 1)] \\ &- \sum_{\overrightarrow{x}} h(\overrightarrow{x}, t) a^{\dagger}(\overrightarrow{x}) \end{aligned}$$

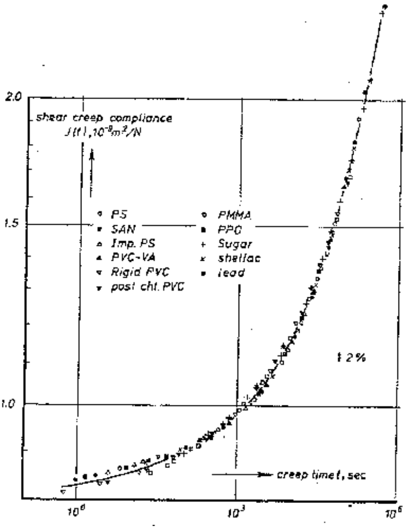
I.1 Struik's experiments



STRUIK 78

1. slow relaxation after quenching PVC from melt to low temperature
2. creep curves depend on **waiting time** t_e and du **creep time** t
3. find master curve for all (t, t_e) $\longrightarrow$ scaling invariance

$\rightarrow$ three defining properties of **physical ageing** $\rightarrow$Universality



les courbes maitresses de différents materiaux sont **identiques**

$\longrightarrow$ on parle d'universalité !

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III.1 Results : Scaling functions ($\sigma = 0$)

correlator $C(t,s) \approx s^{-b} f_C(t/s) \underset{s,t/s\to\infty}{\longrightarrow} s^{-b}(t/s)^{-\lambda_C/z}$

response $R(t,s) \approx s^{-1-a} f_R(t/s) \underset{s,t/s\to\infty}{\longrightarrow} s^{-1-a}(t/s)^{-\lambda_R/z}$

Autoresponse function : $f_R(t/s) = (t/s-1)^{-d/\eta}$

Autocorrelator function :

BCPL

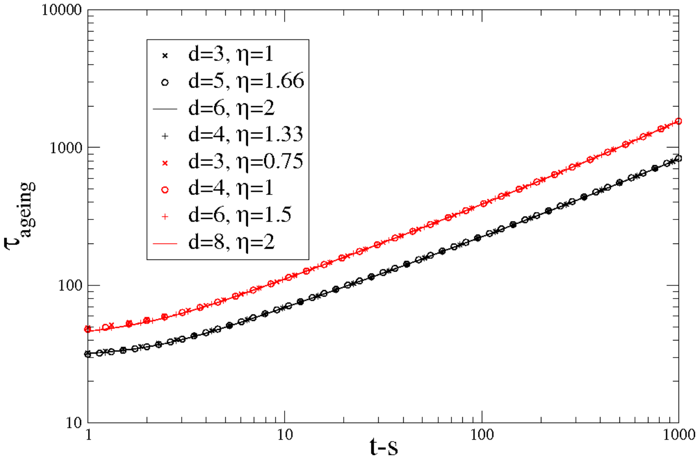
$$f_C(t/s) = (t/s-1)^{-d/\eta+1} - (t/s+1)^{-d/\eta+1}$$

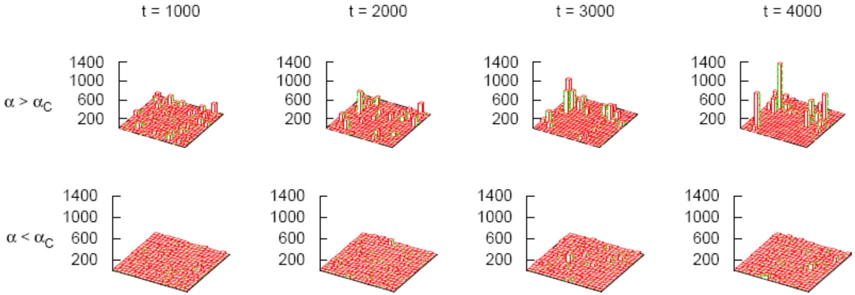
BPCPL

$\alpha > \alpha_c$: no scaling behaviour for ageing.

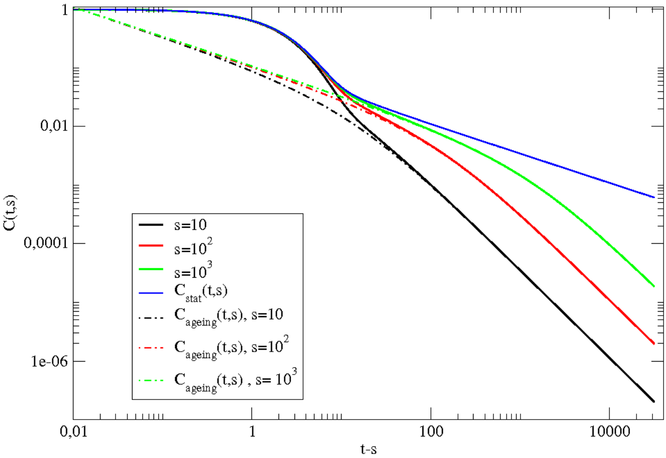
$\alpha \leq \alpha_c$:

		$f_C(t/s)$
$\alpha < \alpha_c$	$d > \eta$	$(t/s-1)^{-d/\eta+1} - (t/s+1)^{-d/\eta+1}$
$\alpha = \alpha_c$	$\eta < d < 2\eta$	$(t/s+1)^{-d/\eta}\, {}_2F_1\left(\frac{d}{\eta}, \frac{d}{\eta}; \frac{d}{\eta}+1; \frac{2}{t/s+1}\right)$
	$d > 2\eta$	$(t/s-1)^{1-\frac{d}{\eta}} - \frac{\eta}{2d-2\eta}\left[(t/s+1)(t/s-1)^{1-\frac{d}{\eta}} - (t/s+1)^{2-\frac{d}{\eta}}\right]$


10000
1000
100
10
1
10
100
1000
τ_{ageing}
t-s
d=3, η=1
d=5, η=1.66
d=6, η=2
d=4, η=1.33
d=3, η=0.75
d=4, η=1
d=6, η=1.5
d=8, η=2



Thèse Baumann 07


1
0,01
0,0001
1e-06
C(t,s)
0,01
1
100
10000
t-s
s=10
s=10^2
s=10^3
C_stat(t,s)
C_ageing(t,s), s=10
C_ageing(t,s), s=10^2
C_ageing(t,s) , s= 10^3