The collapse dynamics of a classical self-gravitating Brownian system: analogy with the chemotactic aggregation and with the Bose-Einstein condensation

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Outline

The Brownian model: a canonical description of self-gravitating systems

- systems with long-range interactions: *inequivalence of statistical ensembles* at equilibrium [Thirring, Z. Phys. **235**, 339 (1970), Lynden-Bell (70's)]
- ▶ microcanonical (MC) and canonical (C) descriptions $\Rightarrow \neq$ conclusions
- 2 *differents* microscopic dynamical models [Chavanis *et al.*, Phys. Rev. E **66**, 036105 (2002)] $\rightsquigarrow$ *dynamical* inequivalence

HAMILTONIAN (MC)

- isolated systems $\rightsquigarrow E$
- conservative dynamics

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{m}\nabla U_N = \mathbf{F}_{grav}$$

- collapse $\rightsquigarrow$ binary surrounded by a hot halo

BROWNIAN (C)

- systems strongly coupled with a thermal bath $\rightsquigarrow T$
- dissipative dynamics

$$\frac{d\mathbf{v}}{dt} = -\xi\mathbf{v} + \mathbf{F}_{grav} + \sqrt{T}\mathbf{W}$$

- collapse $\rightsquigarrow$ Dirac peak

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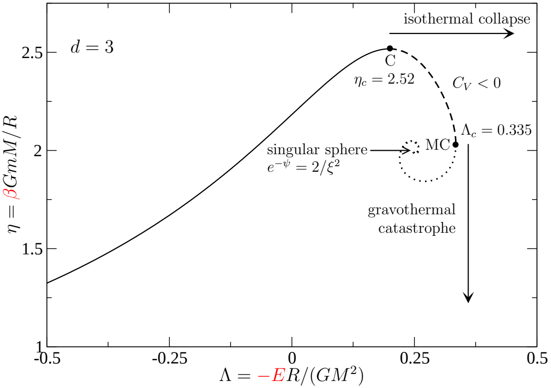
- MC and C equilibrium states: solutions *same* equations *but* stability $\neq$

Statistical equilibrium states and series of equilibrium



- MC and C instabilities $\rightsquigarrow$ no equilibrium state
- gravothermal catastrophe (MC) : $\Lambda > \Lambda_c = 0.335 \ \ (E < E_c)$
- *isothermal collapse* (C) : $\eta > \eta_c = 2.52 \ \ (\beta > \beta_c)$

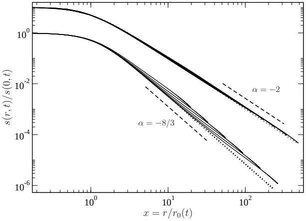
The pre-collapse regime

Smoluchowski-Poisson (SP) (MF, $\xi \to +\infty$)

$$\frac{\partial \rho}{\partial t} = \frac{1}{\xi}\nabla \cdot \left(\frac{1}{m\beta}\nabla\rho + \rho\nabla\Phi \right) \equiv -\nabla \cdot (\mathbf{J}_{diff} + \mathbf{J}_{drift})$$

$$\Delta\Phi = 4\pi G\rho$$

- $\beta \leq \beta_c \rightsquigarrow$ equilibrium state
- $\beta > \beta_c \rightsquigarrow$ isothermal collapse: Dirac peak $[M(0,t) = M]$



[Sire and Chavanis, Banach Center Publ. **66**, 287 (2004)]

- pre-collapse $\rightsquigarrow$ *self-similar* evolution

$$\rho(r,t) = \rho_0(t) S[r/r_0(t)]$$

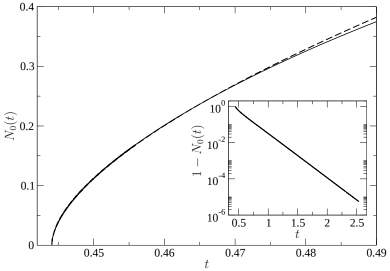
$$r_0(t) \sim 1/\sqrt{\beta\rho_0(t)}$$

$$\rho_0(t) \sim \rho(0,t) \sim (t_{coll} - t)^{-1}$$

$$S(x) = \frac{1}{\pi}\frac{3+x^2}{(1+x^2)^2} \sim x^{-2} \quad (x \gg 1)$$

The post-collapse regime

- $\rho(r, t_{coll}) \sim r^{-2} \Leftrightarrow M(r, t_{coll}) \sim r \rightsquigarrow M(0, t_{coll}) = 0 \neq M$!
- incomplete collapse $\Rightarrow$ continues after t_{coll} (post-collapse)
- $t > t_{coll}$: formation of the Dirac peak $\rightsquigarrow$ central mass $N_0(t) \to M = 1$

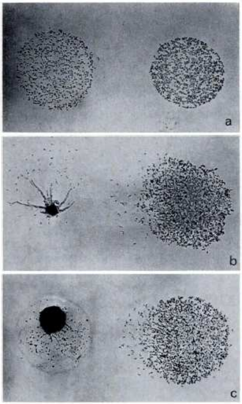


[Sire and Chavanis, Phys. Rev. E **69**, 066109 (2004)]

$$N_0(t) \sim \sqrt{t - t_{coll}} \qquad (t \gtrsim t_{coll})$$
$$N_0(t) = 1 - e^{-\lambda t} \qquad (t \to +\infty)$$

Analogy with the chemotactic aggregation

- chemotaxis: aggregation process of a biological population mediated by a chemical secreted by the population itself



[Konijn, Biol. Bull. **134**, 298 (1968)]

- dynamical model of chemotaxis [Keller and Segel, J. Theor. Bio. **26** 399 (1970)]
- chemical: no degradation, produced with a constant rate, large diffusivity limit

Modified Keller-Segel (KS)

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (D \nabla \rho + \chi \rho \nabla c)$$

$$\Delta c = -\lambda \rho$$

- KS $\Leftrightarrow$ SP $\rightsquigarrow$ reinterpretation [Chavanis and Sire, Phys. Rev. E **70**, 026115 (2004)]

$$\Phi \leftrightarrow -\frac{4\pi G}{\lambda} c \quad , \quad \beta \leftrightarrow \frac{\lambda \chi}{4\pi G D} \quad , \quad \xi \leftrightarrow \frac{4\pi G}{\lambda \chi}$$

The dynamical model: BEC in the canonical ensemble

- (semi-classical) Fokker-Planck equation of free "bosons" strongly coupled with a thermal bath T [Kaniadakis and Quarati, Phys. Rev. E **49**, 5103 (1994)]

bosonic Fokker-Planck equation

(in k-space, homogeneous in r-space)

$$\frac{\partial \rho}{\partial t} = \frac{1}{\xi} \nabla_{\mathbf{k}} \cdot \left[\frac{1}{\beta} \nabla_{\mathbf{k}} \rho + \rho(1+\rho)\mathbf{k} \right]$$

bosonic Fokker-Planck vs. (integrated) **Smoluchowski-Poisson**

[Sopik, Sire and Chavanis, Phys. Rev. E **74**, 011112 (2006)]

$$\begin{aligned} \frac{\partial M}{\partial t} &= \frac{1}{\beta}\left(\frac{\partial^2 M}{\partial k^2} - \frac{2}{k}\frac{\partial M}{\partial k} \right) + k \frac{\partial M}{\partial k}\left(\frac{1}{k^2}\frac{\partial M}{\partial k} + 1 \right) \\ \frac{\partial M}{\partial t} &= \frac{1}{\beta}\left(\frac{\partial^2 M}{\partial r^2} - \frac{2}{r}\frac{\partial M}{\partial r} \right) + \frac{M}{r^2}\frac{\partial M}{\partial r} \end{aligned}$$

- *same* diffusion terms and nonlinear terms have *same* dimension
- ▸ dynamical analogies between self-gravitating systems and bosonic systems

Isothermal collapse vs. BEC

GRAVITATION

- ***canonical*** description
  - $\beta > \beta_c$: isothermal collapse
  - $t \leq t_{coll}$: self-similar pre-collapse
  - $t > t_{coll}$: post-collapse $\rightsquigarrow$ Dirac peak

$$N_0(t) \sim \sqrt{t - t_{coll}} \qquad (t \gtrsim t_{coll})$$
$$M_0(\infty) - N_0(t) \sim e^{-\lambda t} \qquad (t \to +\infty)$$

- $M_0(\infty) = M$

BOSONS

- ***canonical*** description
  - $\beta \geq \beta_c$: condensation $\mathbf{k} = \mathbf{0}$
  - $t \leq t_{coll}$: self-similar pre-condensation
  - $t > t_{coll}$: post-condensation $\rightsquigarrow$ condensate

$$N_0(t) \sim t - t_{coll} \qquad (t \gtrsim t_{coll})$$
$$M_0(\infty) - N_0(t) \sim e^{-ct} \qquad (t \to +\infty)$$

- $M_0(\infty) = M_0(T) \neq M$

Summary and remarks

The Brownian model

- exact results can be derived
- reinterpretation of the parameters $\rightsquigarrow$ chemotactic aggregation of biological populations
- shares many analogies with the Bose-Einstein condensation in k-space of free bosons strongly coupled with a thermal bath

- inequivalence: canonical description differs from the microcanonical one

	C	MC
gravity ($t \leq t_{coll}$)	$\rho_0 \sim (t_{coll} - t)^{-0.5}$	$\rho_0 \sim (t_{coll} - t)^{-0.53}$ [1]
BEC ($t \gtrsim t_{coll}$)	$M_0(t) \sim (t - t_{coll})$	$M_0(t) \sim (t - t_{coll})^{1.234}$ [2]

[1]: Takahashi, Publ. Astron. Soc. Japan **47**, 561 (1995)

[2]: Lacaze *et al.*, Physica D **152**, 779 (2001)