The lattice gluon propagator in numerical stochastic perturbation theory

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in collaboration with F. Di Renzo (Parma), C. Torrero (Regensburg)

Outline

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Introduction I

Perturbative results with increasing precision (more loops) are useful and needed:
Relate observables measured in lattice QCD to their physical counterpart via renormalisation

Separate non-perturbative effects in observables assumed to show confinement properties
Example: Extract gluon condensate from plaquette or from Creutz ratios of Wilson loops
Gluon and ghost propagators belong to these observables

Lattice perturbation theory (LPT) in diagrammatic approach much more involved compared to continuum PT of QCD
Only a limited number of two-loop results in LPT
Remember: Standard PT from path integral approach to quantisation

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Alternative:

Use Langevin equation as basis of stochastic quantisation
(Parisi and Wu, 1981)
Non-perturbative application:
Langevin simulations of lattice QCD (Batrouni et al., 1985)

Apply Langevin dynamics for weak coupling expansion of lattice QCD
Powerful numerical approach for higher order calculations:
Numerical stochastic perturbation theory (NSPT)
(Di Renzo et al.,1994)

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New application:

Higher-loop contributions to gluon propagator in Landau gauge (Ilgenfritz, Perlt, Schiller, PoS (LATTICE2007))

Combine efforts with Di Renzo and Torrero

Work supported by DFG under contract FOR 365 (Forschergruppe Gitter-Hadronen-Phänomenologie).

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Langevin equation for lattice QCD

Use Euclidean lattice Langevin equation with "time" t

$$\frac{\partial}{\partial t} U_{x,\mu}(t;\eta) = \mathrm{i}\,(\nabla_{x,\mu} S_G[U] - \eta_{x,\mu}(t))\;\; U_{x,\mu}(t;\eta)$$

$\eta = \eta^a T^a$ random field with Gaussian distribution
$\nabla_{x,\mu}$ left Lie derivative on the group

For $t \to \infty$ link gauge fields U are distributed according to measure $\exp(-S_G[U])$

Discretise $t = n\epsilon$
Get solution at next time step $n+1$ in the Euler scheme

$$U_{x,\mu}(n+1;\eta) = \exp(F_{x,\mu}[U,\eta])\; U_{x,\mu}(n;\eta)$$

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We use the Wilson plaquette gauge action S_G

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Perturbative Langevin equations I

Use that solution for perturbative expansion:
Rescale $\varepsilon = \beta\epsilon$ and expand gauge fields U (and "force" F)

$$U_{x,\mu}(n;\eta) \to 1 + \sum_{l>0} \beta^{-l/2} U_{x,\mu}^{(l)}(n;\eta)$$

Solution transforms to system of equations

$$\begin{aligned} U^{(1)}(n+1) &= U^{(1)}(n) - F^{(1)}(n) \\ U^{(2)}(n+1) &= U^{(2)}(n) - F^{(2)}(n) \\ &\quad + \frac{1}{2}(F^{(1)}(n))^2 - F^{(1)}(n)U^{(1)}(n) \\ &\cdots \end{aligned}$$

Random noise field η enters only in $F^{(1)}$
Higher orders are stochastic via dependence on lower orders

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Perturbative Langevin equations II

Work also with field variables living in the algebra $A = \log U$

Expand

$$A_{x+\hat{\mu}/2,\mu}(t;\eta) \to \sum_{l>0} \beta^{-l/2} A^{(l)}_{x+\hat{\mu}/2,\mu}(t;\eta)$$

$$\begin{aligned} A^{(1)} &= U^{(1)} \\ A^{(2)} &= U^{(2)} - \frac{1}{2}(U^{(1)})^2 \\ \dots \end{aligned}$$

Contributions of some order to an observable

$$\langle \mathcal{O} \rangle \to \sum_{l \geq 0} \beta^{-l/2} \langle \mathcal{O}^{(l)} \rangle$$

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Lattice gluon propagator

Continuum gluon propagator $D^{ab}_{\mu\nu} = \delta^{ab} D_{\mu\nu}$

$$D_{\mu\nu}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu \, q_\nu}{q^2} \right) D(q^2) + \frac{q_\mu \, q_\nu}{q^2} \frac{F(q^2)}{q^2}$$

$F(q^2) = 0$ in Landau gauge $\partial_\mu A_\mu(x) = 0$

Lattice gluon propagator

$$D^{ab}_{\mu\nu}(\hat{q}) = \left\langle \widetilde{A}^a_\mu(k) \widetilde{A}^b_\nu(-k) \right\rangle = \delta^{ab} D_{\mu\nu}(\hat{q})$$

$\widetilde{A}^a_\mu(k)$ – Fourier transform of $A^a_{x+\hat{\mu}/2,\mu}$

$$\hat{q}_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{L_\mu}\right) = \frac{2}{a} \sin\left(\frac{a q_\mu}{2}\right), \; k_\mu \in \left(-\frac{L_\mu}{2}, \frac{L_\mu}{2}\right]$$

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Perturbative gluon propagator

Lattice gluon propagator in NSPT of loop order n (even powers in l)

$$\delta^{ab} D^{(n)}_{\mu\nu}(\hat{q}) = \left\langle \sum_{i=1}^{2n+1} \left[\widetilde{A}^{a,(i)}_{\mu}(k)\, \widetilde{A}^{b,(2n+2-i)}_{\nu}(-k) \right] \right\rangle$$

Tree level $D^{(0)}_{\mu\nu}$ arises from quantum fluctuations of gauge fields with $i = 1$

Inspired by continuum form we consider

$$\sum_{\mu=1}^{4} D^{(n)}_{\mu\mu}(\hat{q}) \equiv 3D^{(n)}(\hat{q})\,, \quad \sum_{\mu,\nu=1}^{4} \hat{q}_{\mu} D^{(n)}_{\mu\nu}(\hat{q})\, \hat{q}_{\nu} \xrightarrow{\text{L.gauge}} 0$$

We present dressing functions $Z^{(n)}$:

$$\hat{Z}^{(n)}(\hat{q}) = \hat{q}^2 D^{(n)}(\hat{q})\,, \quad Z^{(n)}(aq) = (aq)^2 D^{(n)}(\hat{q})$$

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Implementation of NSPT

• Limit $\varepsilon \to 0$

Solve coupled system of equation for $U^{(l)}$'s
Time sequence of gauge fields to all chosen orders
Measure perturbatively constructed observables
Different step sizes: $\varepsilon =$ 0.07,..., 0.01
up to 60000 Langevin steps for smallest ε

• Limit $V \to \infty$

Extract infinite volume loop results
Periodic lattices: $L = 6, 8, 10, 12(16)$ with orders of propagator:
$n_{\max} = 4\,(1)$ (gauge field orders 10 (4))

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Compare with analytic results of standard LPT
Predict new precise numerical results in higher loops

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Landau gauge fixing

Perform Landau gauge fixing and measure gluon propagator (after each 20th Langevin step)

Condition for perturbative Landau gauge

$$\sum_\mu \partial^L_\mu A^{(l)}_{x,\mu} = 0\,, \quad \partial^L_\mu A^{(l)}_{x,\mu} \equiv A^{(l)}_{x+\hat{\mu}/2,\mu} - A^{(l)}_{x-\hat{\mu}/2,\mu}$$

The Landau gauge reached by iterative gauge transformations chosen as perturbative variant of Fourier acceleration (Davies et al, 1987)

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Raw data examples for $\hat{Z}^{(n)}(\hat{q})$

Measured gluon dressing function averaged over equivalent 4-tuples of lattice momenta (k_1, k_2, k_3, k_4)
Contributions for odd l of $\beta^{-l/2}$ have to vanish

Raw data examples for $\hat{Z}^{(n)}(\hat{q})$

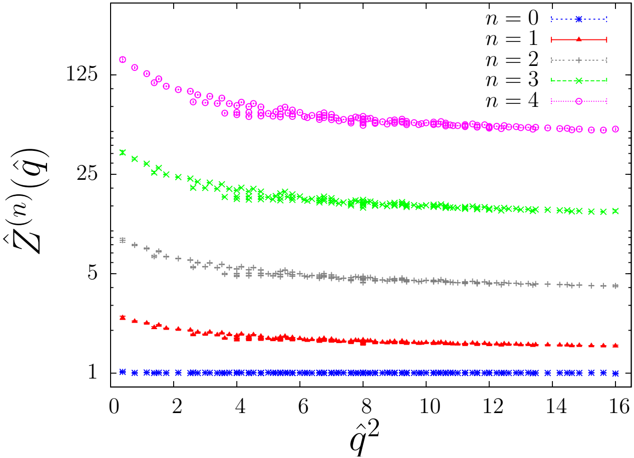

Figure: $\hat{Z}^{(n)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 10$ and $\varepsilon = 0.01$. Loop contributions.

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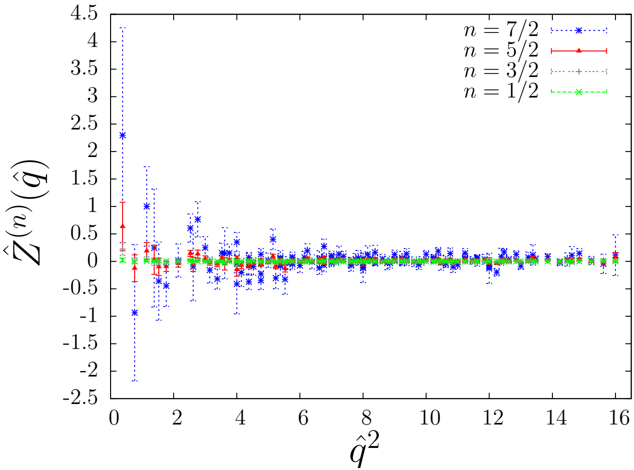

Figure: $\hat{Z}^{(n)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 10$ and $\varepsilon = 0.01$. Vanishing contributions.

Limit $\varepsilon \to 0$ for $\hat{Z}^{(n)}(\hat{q})$ – linear extrapolation

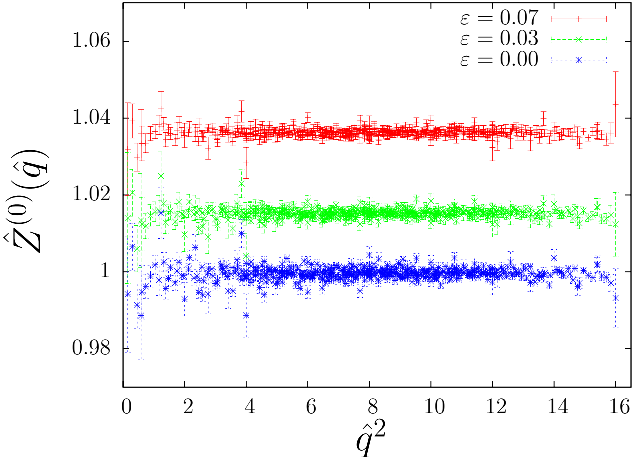

Figure: Tree level dressing function $\hat{Z}^{(0)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 16$.

Limit $\varepsilon \to 0$ for $\hat{Z}^{(n)}(\hat{q})$ – linear extrapolation

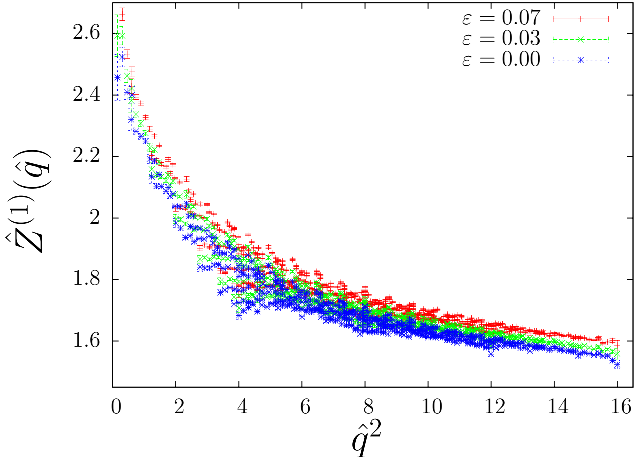

Figure: One-loop dressing function $\hat{Z}^{(1)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 16$.

Limit $V \to \infty$ and momentum cuts for $Z^{(n)}(aq)$

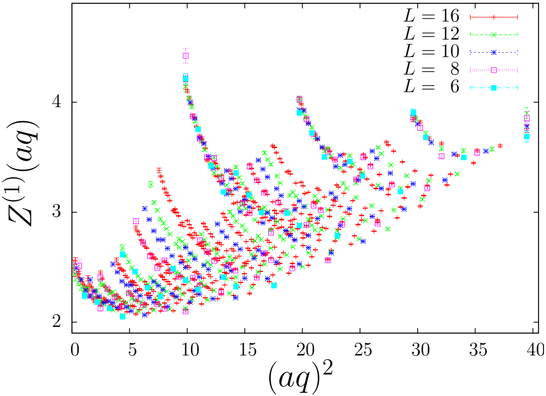

Figure: One-loop dressing function $Z^{(1)}(aq)$ vs. $(aq)^2$ at all volumes for all inequivalent 4-tuples.

Different branches for off-diagonal tuples due to hypercubic group

Limit $V \to \infty$ and momentum cuts for $Z^{(n)}(aq)$

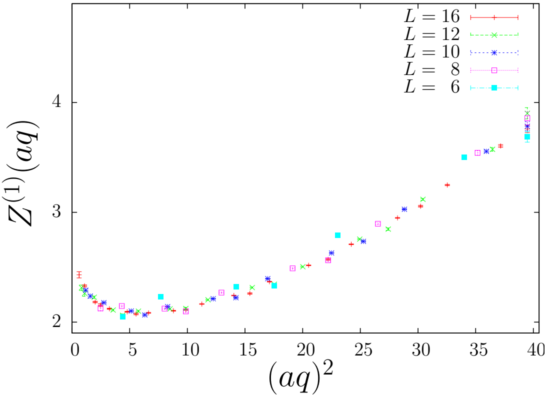

Figure: One-loop dressing function $Z^{(1)}(aq)$ vs. $(aq)^2$ at all volumes for 4-tuples $(k,k,k,k),(k\pm1,k,k,k), k>0$.

Universal $(aq)^2$ (or $\hat{q}^2$) dependence for larger V near diagonal
Behaviour similar in all loop contributions

$\hat{Z}(\hat{q}, n_{\max}) = \sum_{n=0}^{n_{\max}} \hat{Z}^{(n)}(\hat{q})/\beta^n$

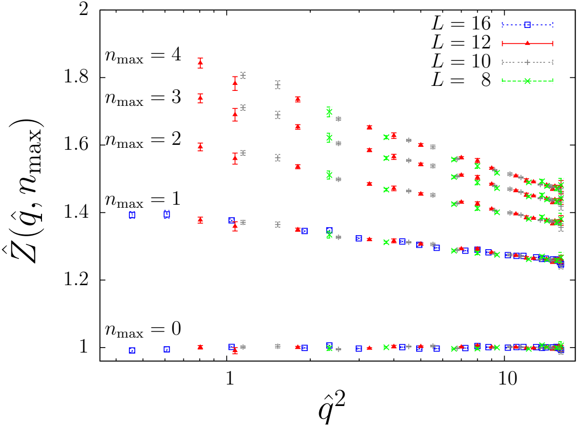

Figure: Summed dressing function near diagonal $\hat{Z}(\hat{q}, n_{\max})$ up to four loops (one loop) vs. $\hat{q}^2$ using $\beta = 6$ at $L = 8, 10, 12$ (16).

Extracting expansion coefficients I

Aim: find gluon wave function renormalisation constant Z_A in LPT
Use regularisation independent scheme (RI')

$$Z_A^{RI'}(a,\mu,\alpha^{RI'})\,\Pi_T(a,q,\alpha^{RI'})\Big|_{q^2=\mu^2} = 1$$

Dressing function $Z(a,q,\alpha)$=1/$\Pi_T(a,q,\alpha)$ – transverse part of the gluon polarisation tensor

$\longrightarrow$ at $\mu^2 = q^2$, in bare α_0

$$Z(a,q,\alpha_0) = Z_A(a,q,\alpha_0)$$

Expected form using $\beta = N_c/(8\pi^2\alpha_0)$:

$$Z(a,q,\beta) = 1 + \frac{1}{\beta}\left[D_{1,1}\,\log(aq)^2 + D_{1,0}\right] + \frac{1}{\beta^2}\left[D_{2,2}\,\log^2(aq)^2 + D_{2,1}\,\log(aq)^2 + D_{2,0}\right] + \cdots$$

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Extracting expansion coefficients II

Determine $D_{i,j}$ for $a \to 0$ and $V \to \infty$ from measured Z (Landau gauge, quenched approximation)
Minimize the coefficient number using results from renormalisation group in continuum QCD PT:

Outcome for lowest orders:
Leading log coefficients $D_{n,n}$ known
Non-leading log coefficients more complicated, e.g. $D_{2,1}(D_{1,0})$
$D_{1,0}$ calculated in lattice PT (Kawai et al.,1981)

$$Z^{2-\mathrm{loop}}(a,q,\beta) = 1 + \frac{1}{\beta}\left(-0.24697\,\log(aq)^2 + 2.29368\right) + \frac{1}{\beta^2}\left(0.0821078\,\log^2(aq)^2 - 1.48445\,\log(aq)^2 + D_{2,0}\right)$$

Check $D_{1,0} = 2.29368$ and predict $D_{2,0}$

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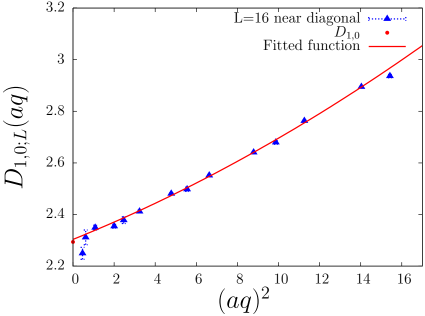
One-loop $D_{1,0}$

Fit non-log contribution of $Z^{(1)}(aq)$ near diagonal

$$D_{1,0;L}(aq) = D_{1,0;L} + c_1(aq)^2 + c_2(aq)^4$$



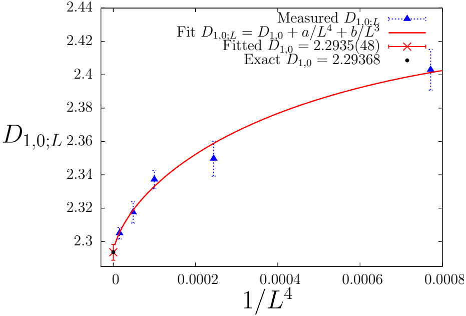


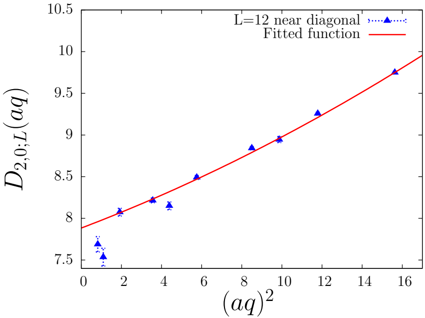
Figure: Left: Limit $a \to 0$: $D_{1,0;16} = 2.3050(35)$. Right: Limit $V \to \infty$:

$$D_{1,0;\infty} = 2.2935(48)$$

Nice agreement !

Two-loop $D_{2,0}$ (preliminary)

Same fit ansatz for the non-log contribution



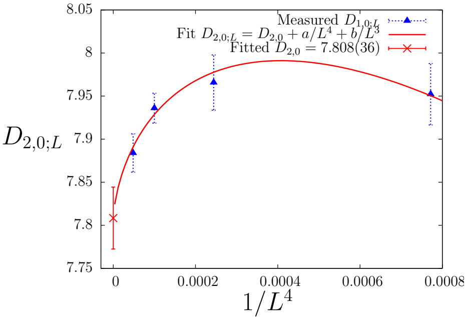


Figure: Left: Limit $a \to 0$: $D_{2,0;12} = 7.884(23)$. Right: Limit $V \to \infty$:

$$D_{2,0;\infty} = 7.808(36)$$

Measurements needed at larger volumes (Parma, in preparation) to confirm that prediction

NSPT works where standard lattice PT is extremely difficult

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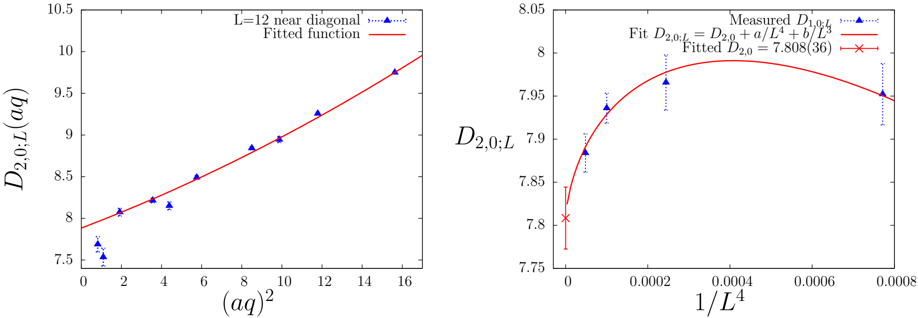

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Summary and Outlook

- Independent code for NSPT – cross-check of available results
- NSPT applied to calculate Landau gauge gluon propagator in higher-loop perturbation theory
- Very good agreement with one-loop standard LPT
- First quantitative prediction for two-loop contribution (prelim.) Large constants as result of lattice artefacts (tadpoles) Tadpole improvement needed
- Results have to be confronted against non-perturbative Monto Carlo results and interpreted
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