Fractal Dimension of Loop Gases
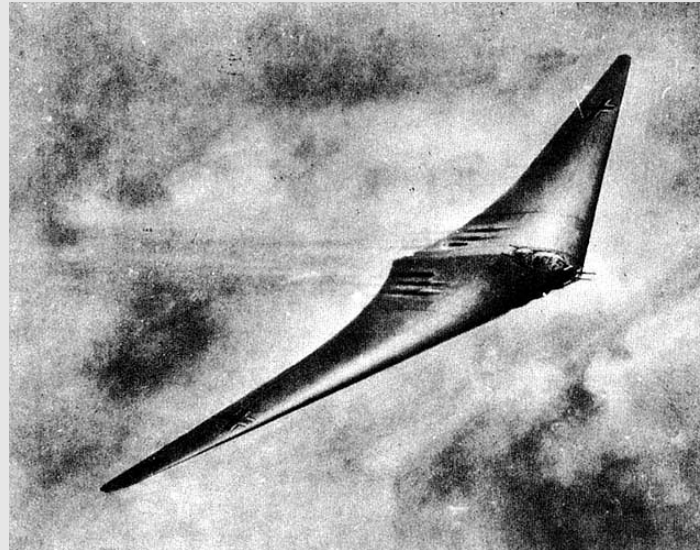on
Fluctuating Planar Lattices

ADRIAAN M.J. SCHAKEL
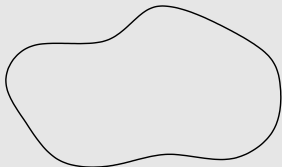w/
WOLFHARD JANKE

▶ Critical O(N), $N = 2\cos(\pi/m)$
($1 \le m \le \infty$)
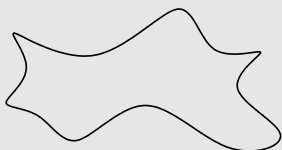Dilute loop gas:

Fractal dimensions:

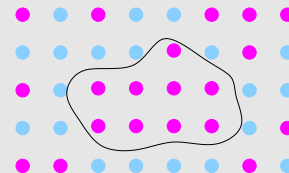- $D_{\text{HT}}(m)$ (2-leg operator)
- $D_{\text{Interior}}(m)$

▶ Tricritical O(N^{T}), $N^{\text{T}} = \sqrt{Q} - 1/\sqrt{Q}$
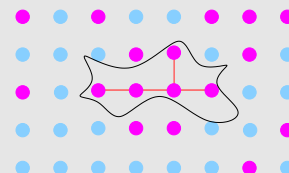Collapsing loops:

Conjecture [JANKE & A.S., 2005]:

- $D^{\text{T}}_{\text{HT}}(m) = D^{\text{FK}}_{\text{Hull}}(m)$
- $D^{\text{T}}_{\text{Interior}}(m) = D^{\text{FK}}(m)$

▶ Q-state Potts, $\sqrt{Q} = 2\cos[\pi/(m+1)]$
Potts spin clusters:

- $D^{\text{Potts}}_{\text{Hull}}(m) = D_{\text{HT}}(m)$
- $D^{\text{Potts}}(m) = D_{\text{Interior}}(m)$
- $D^{\text{Potts}}_{\text{RB}}(m) < 0$ (4-leg operator)

▶ Fortuin Kasteleyn spin clusters:

- $D^{\text{FK}}_{\text{Hull}}(m) = D^{\text{Potts}}_{\text{Hull}}(m \to -m-1)$
- $D^{\text{FK}}(m) = D^{\text{Potts}}(m \to -m-1)$
- $D^{\text{FK}}_{\text{RB}}(m) = D^{\text{Potts}}_{\text{RB}}(m \to -m-1) > 0$
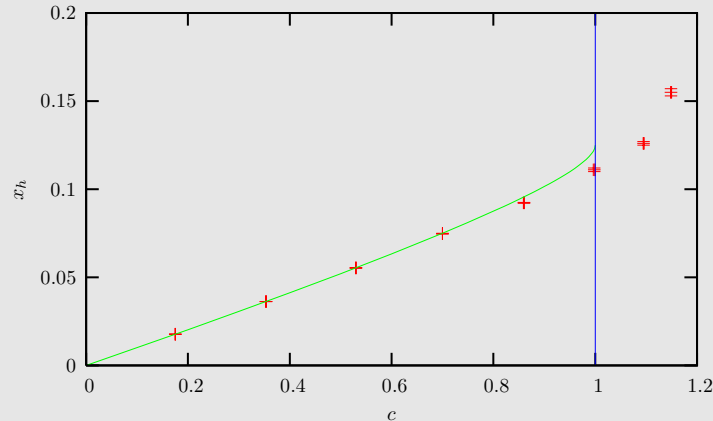
2nd conjecture for tricritical $O(N^{\text{T}})$ [JANKE & A.S., 2005]:
Magnetic scaling dimension:
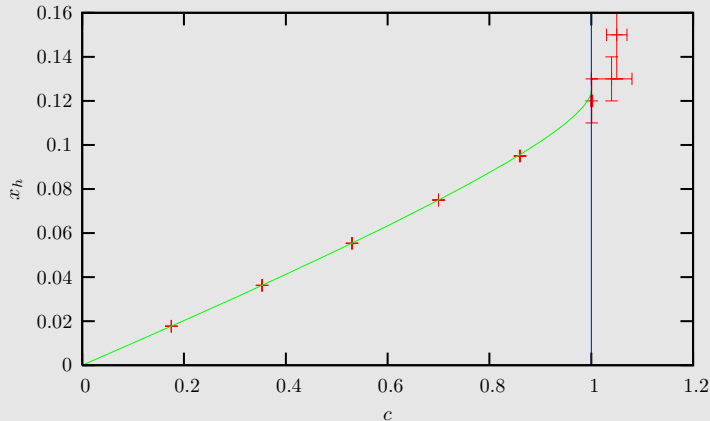
$$x_h^{\text{T}}(m) = x_h(m \to -m-1) = 2h_{m/2,m/2}$$

w/ (Kac table)

$$h_{p,q} = \frac{[(m+1)p - mq]^2 - 1}{4m(m+1)}$$

(probably the only tricritical index which appears in the Kac table)

[GUO, BLÖTE & LIU, 2004]   [GUO, NIENHUIS & BLÖTE, 2006]

w/ $c = 1 - 6/m(m+1)$ central charge

KPZ map [KNIZHNIK, POLYAKOV & ZAMOLODCHIKOV, 1988]:

[JANKE & WEIGEL, 2006]:
KPZ works for Potts and FK clusters of $Q(=2)$-state Potts model on fluctuating
planar lattices (which has intrinsic dimension $d_i \approx 4$)

Transcribed to loop gases:

$$\widetilde{D}_{\mathrm{HT}}/d_{\mathrm{i}} = 1/2, \quad \widetilde{D}_{\mathrm{HT}}^{\mathrm{T}}/d_{\mathrm{i}} = (1+m)/2m$$

for dilute and collapsing loops, respectively.

▶ $\widetilde{D}_{\mathrm{HT}}$ independent of m—as if in upper critical dimension

▶ $m = 1$: $\widetilde{D}_{\mathrm{HT}}^{\mathrm{T}} = d_{\mathrm{i}}$ collapsing loops fill entire available space

For magnetic scaling dimensions KPZ gives:

$$\widetilde{x}_h/d_{\mathrm{i}} = (m-1)/4m, \quad \widetilde{x}_h^{\mathrm{T}}/d_{\mathrm{i}} = (m-2)/4m$$

Scaling relations yield as cricical exponents of tricritical $\mathrm{O}(N^{\mathrm{T}})$ model:

$$\widetilde{\beta}^{\mathrm{T}}/\widetilde{v}^{\mathrm{T}}d_{\mathrm{i}} = (m-2)/4m, \quad \widetilde{\gamma}^{\mathrm{T}}/\widetilde{v}^{\mathrm{T}}d_{\mathrm{i}} = (m+2)/2m$$