Work fluctuations in small quantum spin chains

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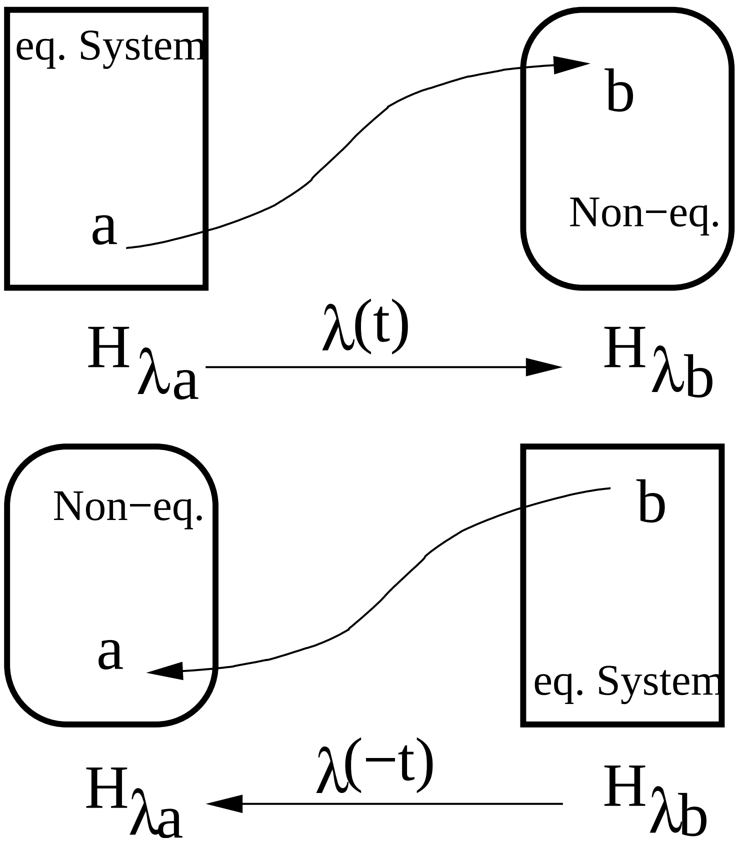
- $\Diamond$ Classical Crooks and Jarzynski relations
- $\Diamond$ Time reversal operator
- $\Diamond$ Quantum Crooks and Jarzynski relations
- $\Diamond$ Ising chain - integrable model
- $\Diamond$ XX chain with h_x - non-integrable model

Classical relations



$$\frac{\mathcal{P}_F(W)}{\mathcal{P}_B(-W)} = e^{-\beta(\Delta F - W)}$$

$$\langle e^{-\beta W} \rangle = e^{-\beta \Delta F}$$

Time reversal

The transition probability between two state ϕ and ψ in the forward process is given by

$$\mathcal{P}_F(\phi \to \psi) = |(\psi, U\phi)|^2 w_F^\phi, \qquad w_F^\phi = (\phi, \rho_F \phi)$$

In quantum mechanics an antiunitary operator K is associated to the time reversal, such as $KK^\dagger = 1$.

$$\phi \to \bar{\phi} = K\phi \qquad U \to \bar{U} = KU^\dagger K^\dagger$$

In the backward protocol one has

$$\mathcal{P}_B(\bar{\psi} \to \bar{\phi}) = |(\bar{\phi}, \bar{U}\bar{\psi})|^2 w_B^{\bar{\psi}}$$

Quantum relation

$$\frac{\mathcal{P}_F(\phi \to \psi)}{\mathcal{P}_B(\bar{\psi} \to \bar{\phi})} = \frac{w_F^{\phi}}{w_B^{\bar{\psi}}} = e^{-\beta(\Delta F - \Delta E_{\phi,\psi})}$$

$$P_F(W) = \sum_{\phi,\psi} \mathcal{P}_F(\phi \to \psi)\delta(W - (\epsilon_\psi - \epsilon_\phi))$$

Then one obtains the quantum relation

$$\mathcal{P}_F(W)e^{-\beta W} = \mathcal{P}_B(-W)e^{-\beta \Delta F}$$

Two quantum spin chains

We study two quantum spin chains described by

$$H^{Ising} = -\frac{1}{2}\sum_j \sigma_j^x \sigma_{j+1}^x - \frac{h(t)}{2}\sum_j \sigma_j^z$$

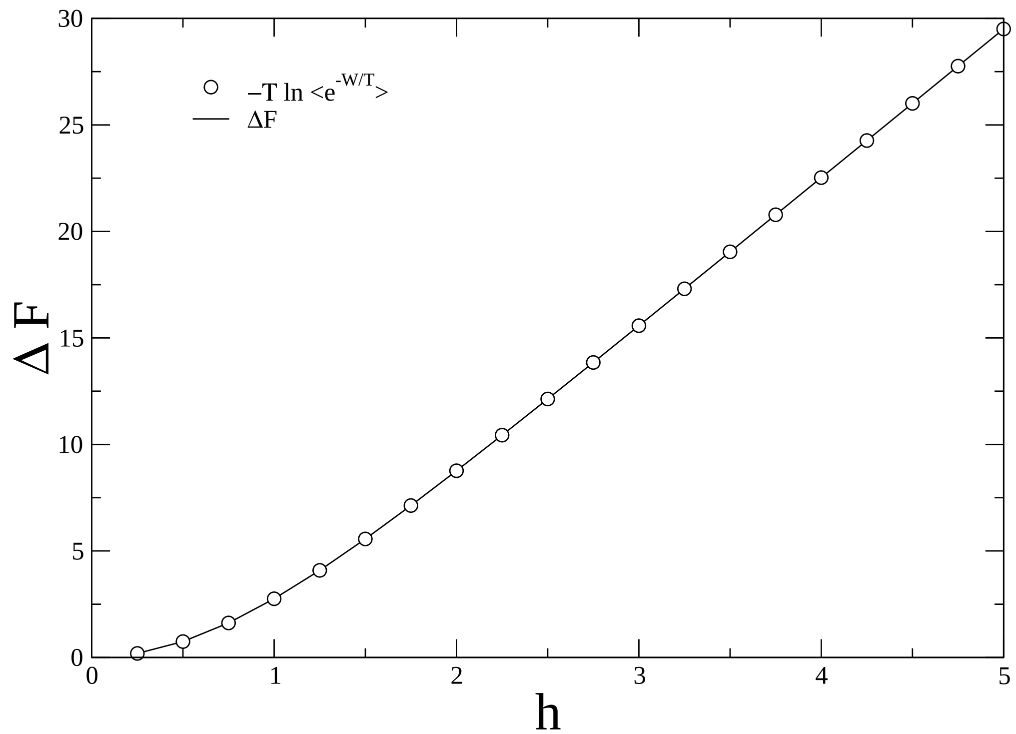
$$H^{XX^+} = -\frac{1}{2}\sum_j \left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^z \sigma_{j+1}^z\right) - \frac{h(t)}{2}\sum_j \sigma_j^z$$

in a periodic magnetic field $h(t) \propto |\sin(\omega t)|$

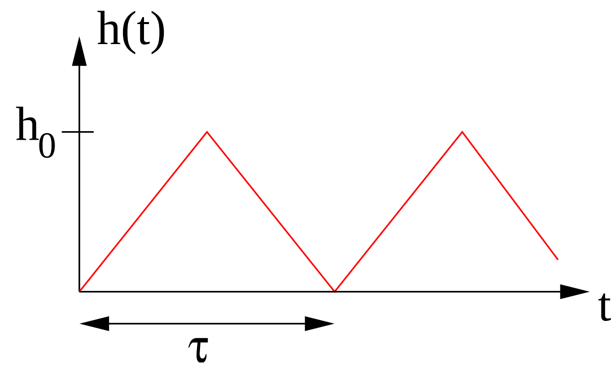
Test of the Jarzynski relation

We compare the free energy differences obtained from the Jarzynski equality with the equilibrium free energy differences calculated from the partition function

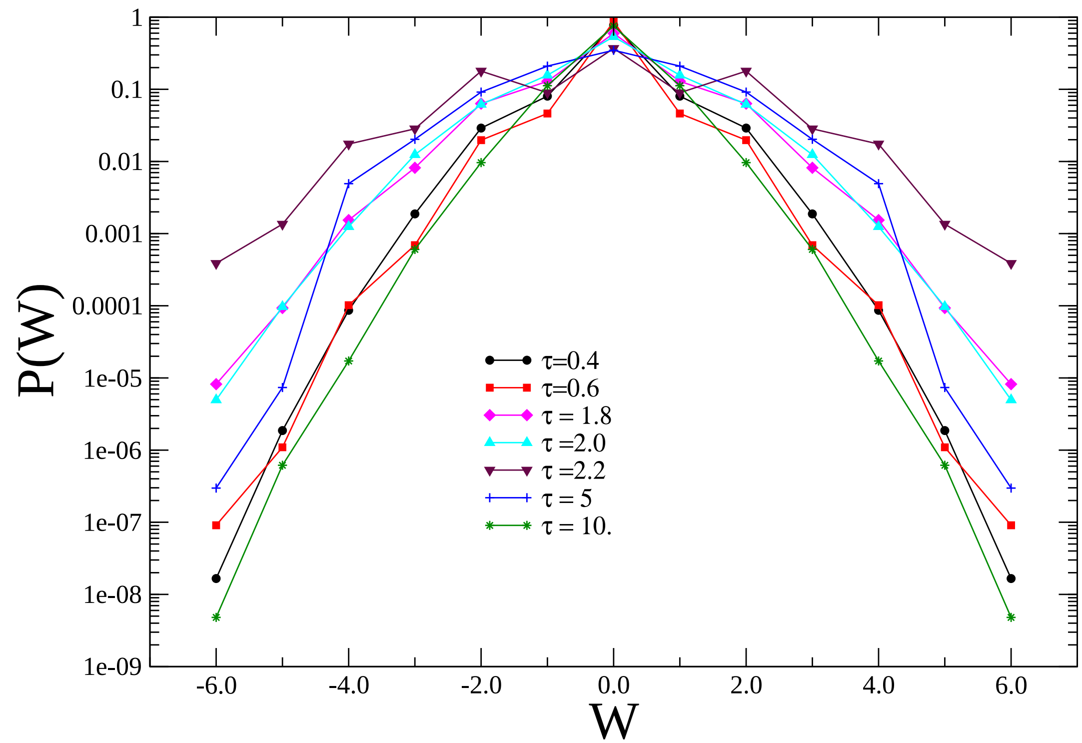
$$F(h(t)) = -\frac{1}{\beta} Tr\{e^{-\beta H(h(t))}\}$$



Numerical study of the Ising Model



Fluctuation of the work after one periode, $\beta = 0$



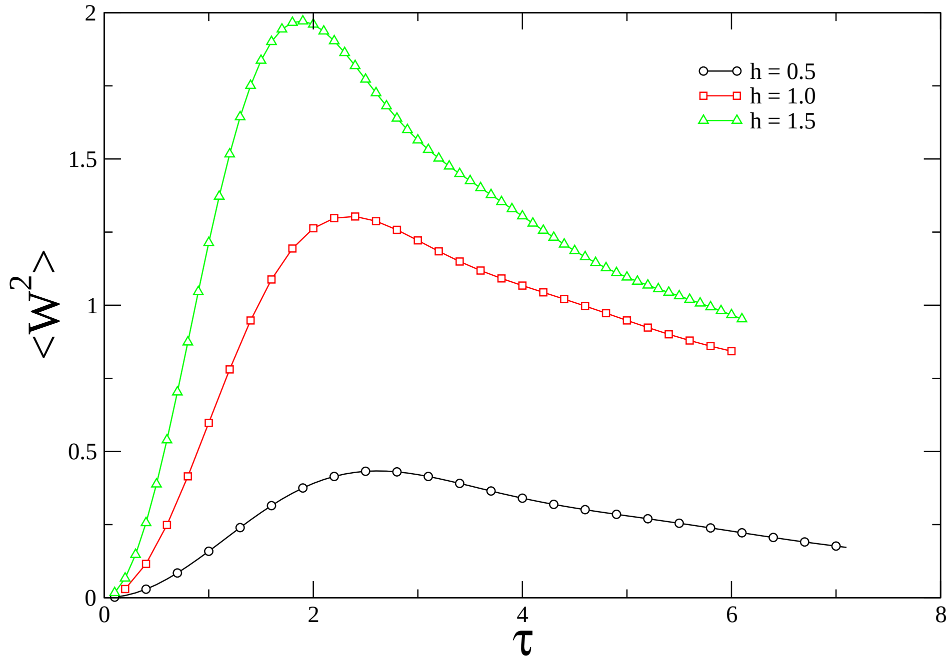
$$\langle W \rangle \propto \int ds \dot{h} \langle M^z \rangle (s) = 0$$

$$\begin{aligned} \mathcal{P}_F(W) e^{-\beta W} &= \mathcal{P}_B(-W) e^{-\beta \Delta F} \\ \mathcal{P}_F(W) &= \mathcal{P}_B(-W) \\ \mathcal{P}(W) &= \mathcal{P}(-W) \end{aligned}$$

Numerical study of the Ising Model

The width of the distribution increases with h_0 since the perturbation is more effectively coupled with the chain.

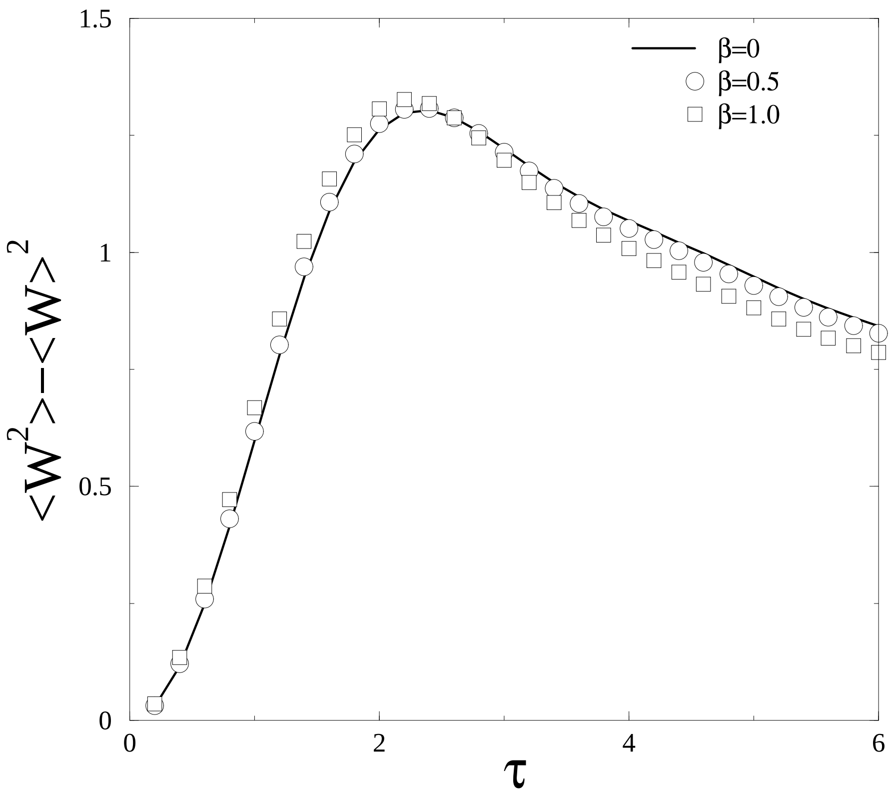
$$\tau_{max} \propto 1/h_0$$



Numerical study of the Ising Model

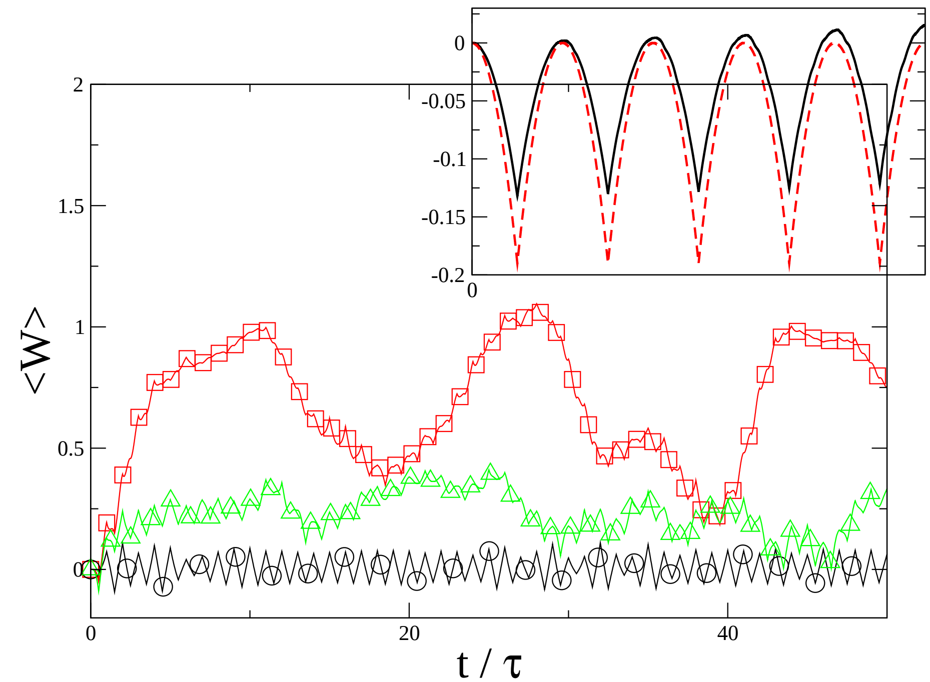
The temperature effect is simply expressed as

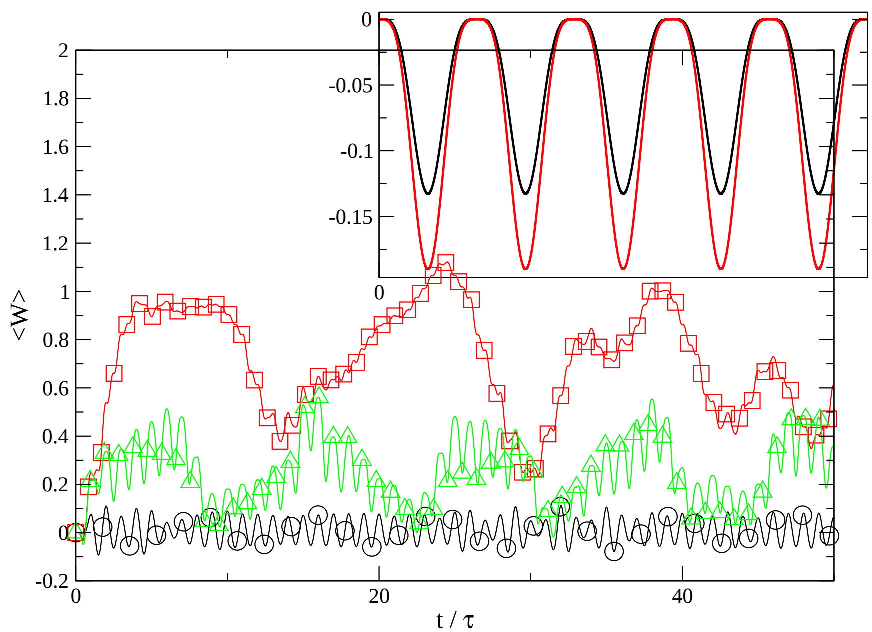
$$\langle W \rangle_\beta = \tanh\left(\frac{\beta}{2}\right) \langle W \rangle_\infty \qquad \langle W^2 \rangle_0 = \langle W^2 \rangle_\beta - \langle W \rangle_\beta^2$$



The driven situation at finite temperature $T = 1$

For large periods $\tau >> 1$ the evolution is almost adiabatic.





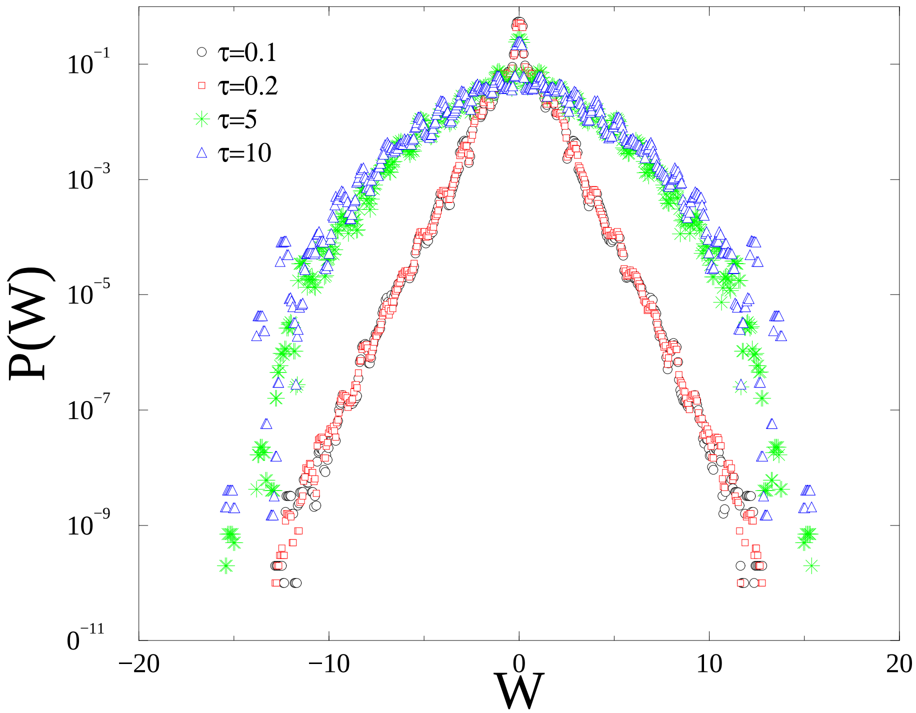
An exact solution is obtained in the case of $h(t) = |\sin(\omega t)|$ and leads to $\langle W \rangle_\beta = \tanh\left(\frac{\beta}{2}\right) \langle W \rangle_\infty$.

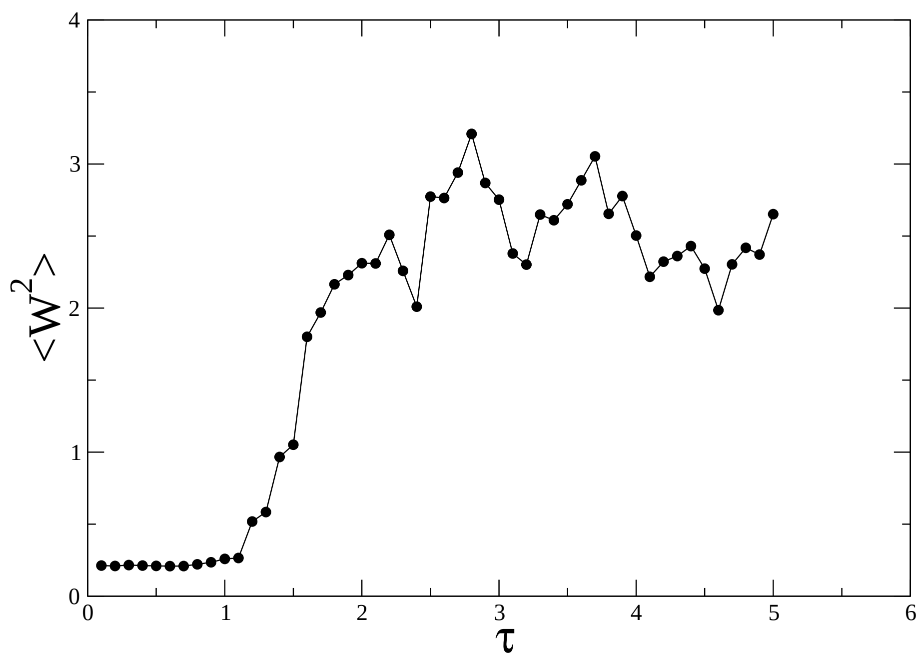
XX Chain with longitudinal magnetic field

Two different regimes appears, depending on the time scale τ.

$$P(W) \propto e^{-|W|/\alpha} \qquad \tau << 1$$

$$P(W) \propto e^{-W^2/\sigma^2} \qquad \tau >> 1$$





Is the change of behaviour from exponential to Gaussian distribtution can be linked to the integrability of the model ?

Summary and Questions

◊ The existence of an optimal frequency τ_{max} for the variance of work fluctuations

◊ Adiabatic behaviour for slow perturbation

◊ No steady work distribution in the Ising chain

◊ Two different regimes depending on τ, $\tau_c \sim 1$

◊ What is the temperature effect ? Is the transition still present at finite temperature?