Thermodynamics of SU(3) gauge theory in 2 spatial dimensions

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At very high temperature Nuclear Matter is in a new phase:

The Quark Gluon Plasma

- **Early Universe**
- **High Energy Heavy Ion Collisions**

Properties fundamentally non-perturbative near T_c and at large distances for all T

Use lattice gauge theory simulations and extrapolate to the continuum

Simpler models:

Pure gluon theory in 3 and 2 spatial dimensions

SU(3) Gauge Theory in 2 spatial dimensions

$Dim(g^2) = 1$ — **Superrenormalizable**

$g^2_{eff}(T) = \frac{g^2}{T}$ — **Effective dimensionless coupling constant**

Linear confinement at low T, phase transition at T_c

$g^2_{eff}(T_c) = 1.82(1)$ — **Large**

$\frac{\sqrt{\sigma}}{T_c} = 1.02(1)$ — **As predicted from NG string**

$m_{GB} \geq 4.3 T_c$

Breakdown of P. T. in the Gluon Plasma in d+1 dimensions

$2+1$

$$g^2_{eff}(T) = \frac{g^2}{T} \xrightarrow{T\to\infty} 0$$

$3+1$

$$g^2_4(T) \simeq \frac{c}{\log T/\wedge} \xrightarrow{T\to\infty} 0$$

$$f_N(T) \sim g^{2N} \prod_{i=1}^{N+1} \left(T \sum_{n_i} \int d^d k_i \right) \prod_{j=1}^{2N} \left(\vec{q}_j^{\,2} + (2\pi l_j T)^2 \right)^{-1}$$

A field theory in d dimensions with infinite tower of masses

$$m_n = 2\pi n T$$

But IR div. when all $n_i = 0$ and:

$$4N \geq (N+1)d$$

$$N_{cr} = \frac{d}{4-d}$$

$N_{cr} = 1$

$N_{cr} = 3$

Thermodynamics in 2 spatial dimensions

Large, homogeneous system

$$p(T) = -f(T)$$

$$\frac{p(T)}{T^3} = \frac{1}{T^2 V} log Z$$ **Dimensionless**

$$T\frac{d}{dT}\frac{p(T)}{T^3} = \frac{\Theta_{\mu\mu}}{T^3} = \frac{\epsilon - 2p}{T^3}$$

Equilibrium finite temperature lattice gauge theory

$N_s^2 \times N_\tau$ **lattice:**

$$T = 1/aN_\tau \qquad V = (aN_s)^2$$

Let $N_s \to \infty$ **and then** $N_\tau \to \infty$

$$S(U) = \beta \sum_P (1 - \frac{1}{3} ReTrU_P(x))$$

β = coupling constant **P = plaquette on the lattice**

$$Z = \int \prod dU_\mu(x) exp(-S(U))$$

Vary T at fixed N_τ by varying β $T/T_c = f(\beta, N_\tau)$

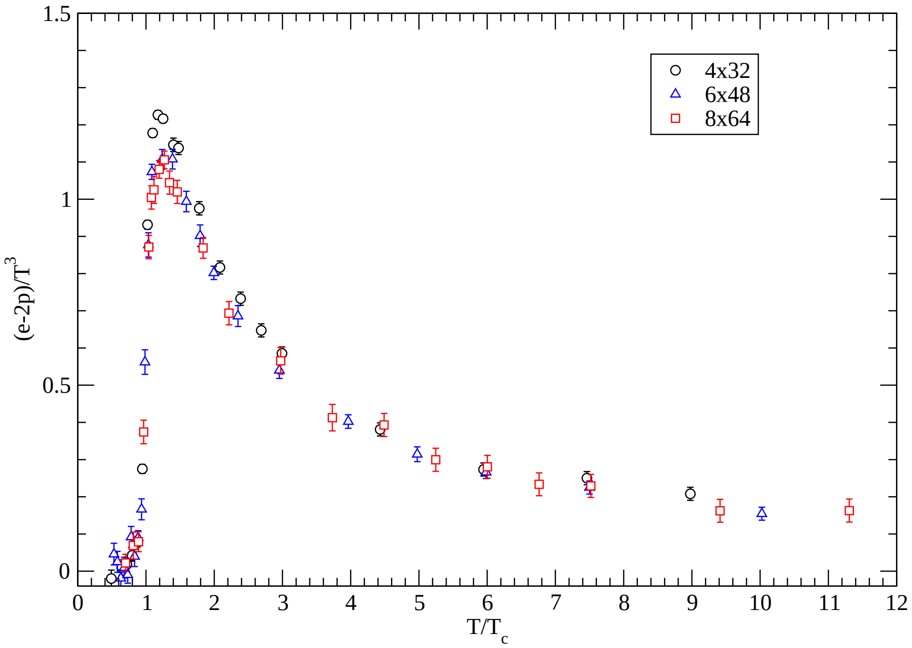
Normalized trace

$$\frac{\epsilon - 2p}{T^3} = T \frac{d\beta}{dT} N_\tau^3 (\langle P_s + 2P_\tau \rangle_{\beta, N_\tau, N_s} - 3 \langle P \rangle_{\beta, N_s, N_s})$$

$$\frac{p(T)}{T^3} - \frac{p(T_0)}{T_0^3} = \int_{T_0}^{T} dT' \frac{\epsilon - 2p}{T'^3}$$

Normalized Trace of the Energy-Momentum Tensor

$2+1$



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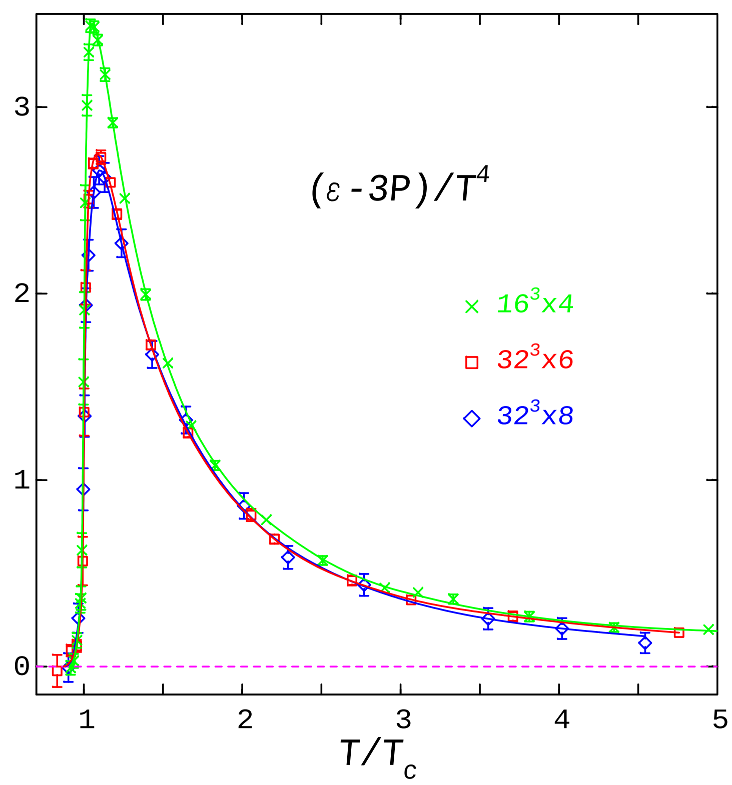
$3+1$

SU(3)



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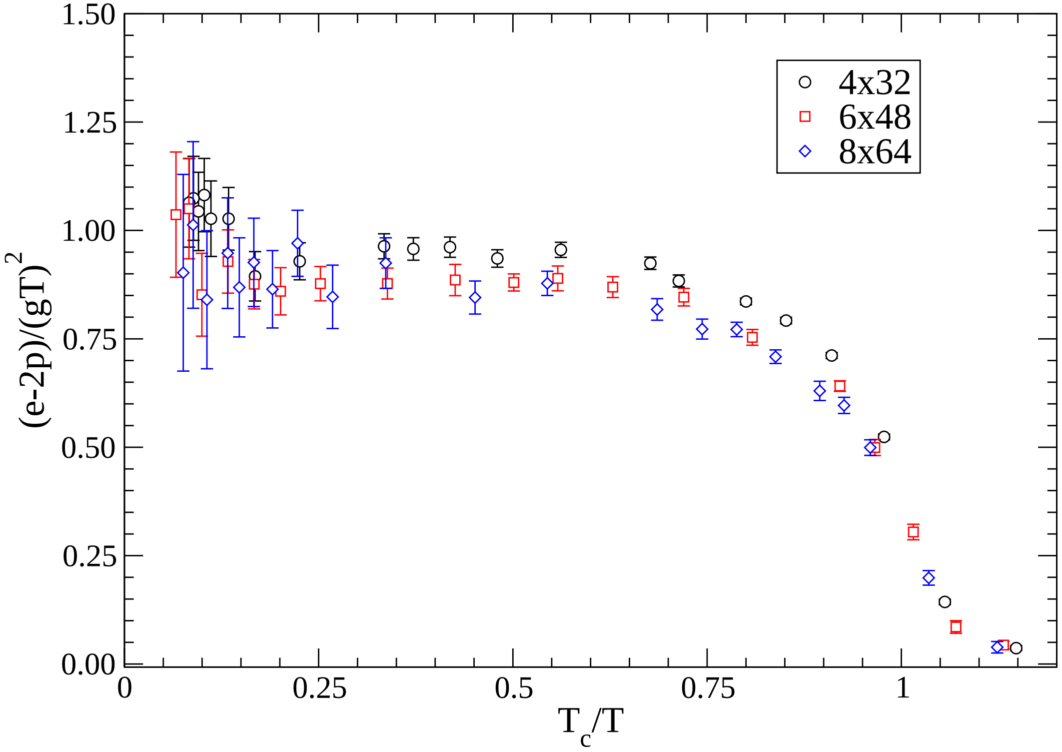
$(\epsilon - 2p)/T^3 = 0$ **for free massless gluons in 2 + 1 dimensions**

Expect

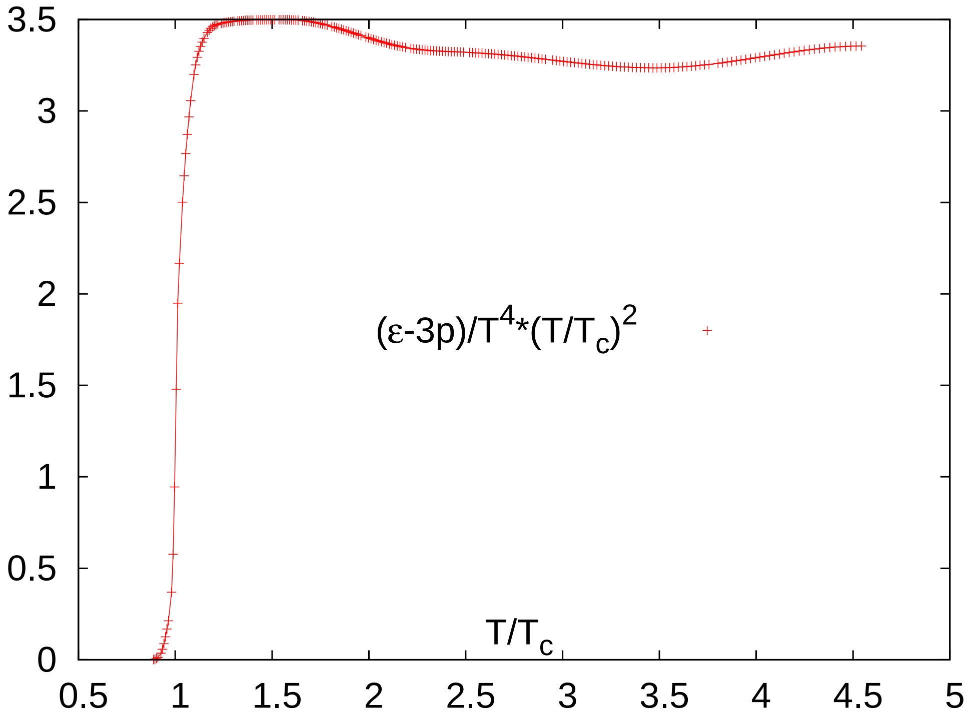
$(\epsilon - 2p)/T^3 \simeq A\,\frac{g^2}{T}\,f(\log(T/g^2))$ $\qquad \frac{\epsilon-2p}{(gT)^2} = Af(log(T/T_c))$



$(\epsilon - 3p)/T^4 = 0$ **for free massless gluons in 3 + 1 dimensions**

Assume

$(\epsilon - 3p)/T^4 \simeq A(\frac{T_c}{T})^2$ **Pisarski**

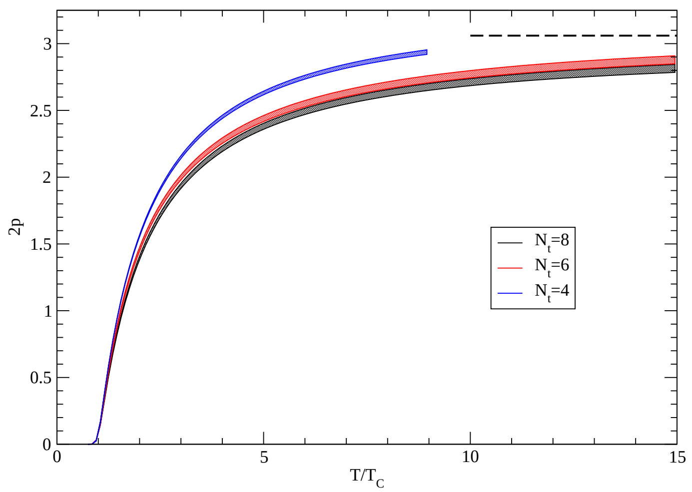


Pressure

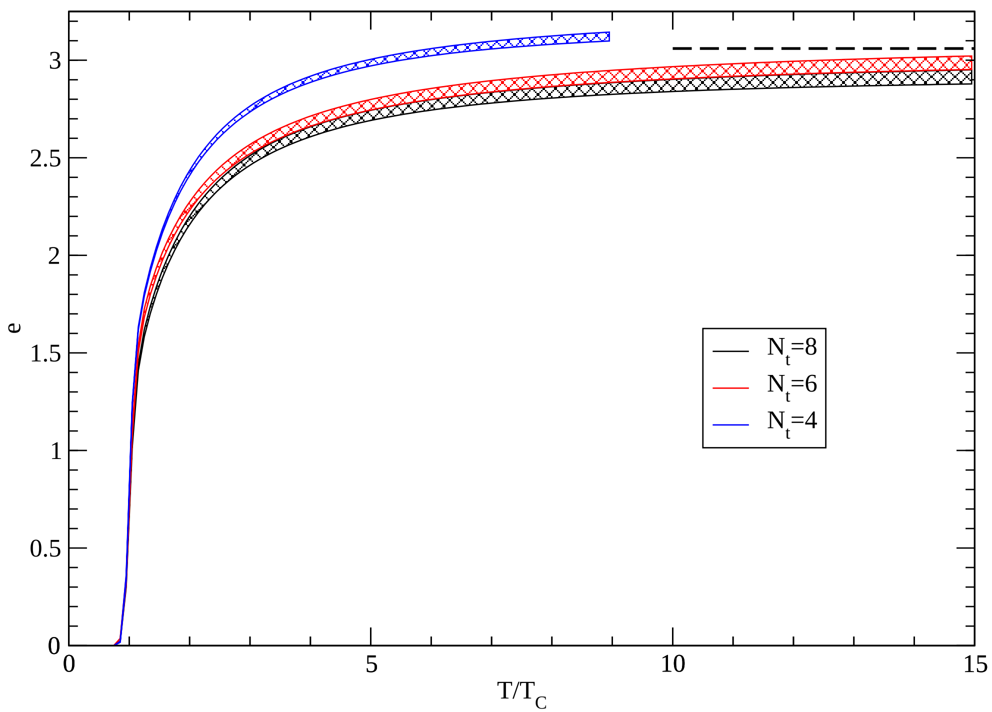
Free massless gluons in 2 + 1 dimensions:

$$\frac{2p}{T^3} = 3.06\left(1 + \frac{1.51}{N_\tau^2}\ldots\right)$$

MC data:

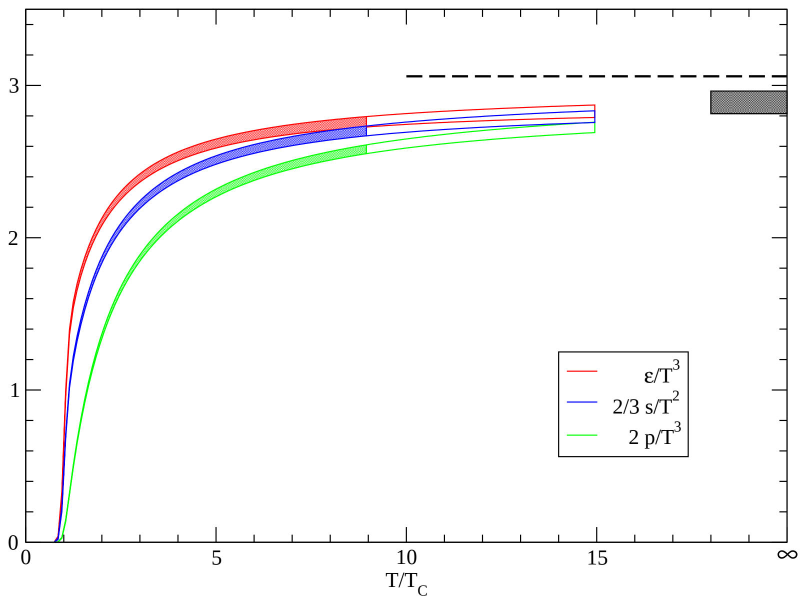


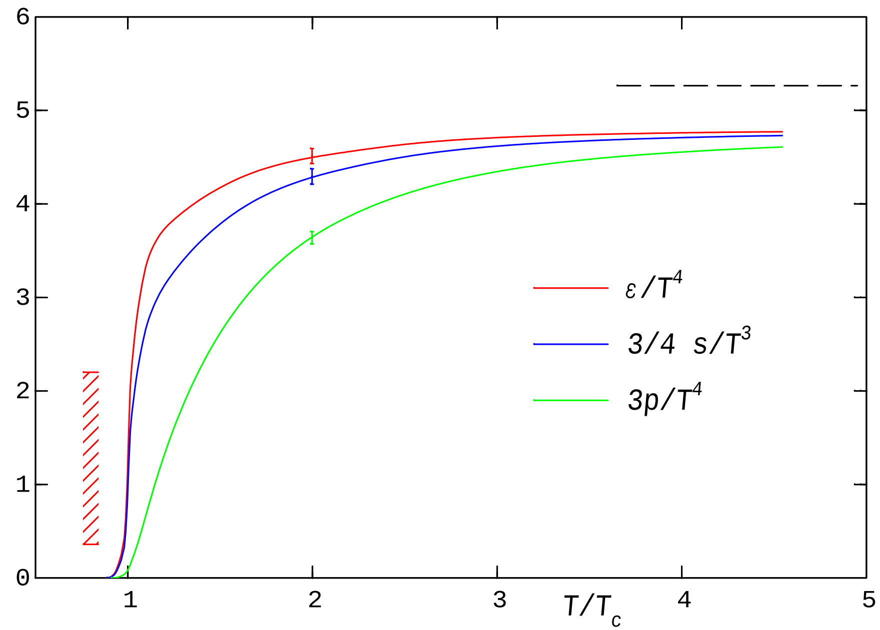
Energy Density



Thermodynamics, continuum extrapolation

2 + 1 preliminary | **3 + 1**





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Screening mass

$$\langle P(\bar{x})P(\bar{0})\rangle - \langle P\rangle^2 \sim e^{-m|\bar{x}|}$$

lowest order perturbation theory $m = 2m_g$ **for** $ReP(\bar{x})$ $m = 3m_g$ **for** $ImP(\bar{x})$

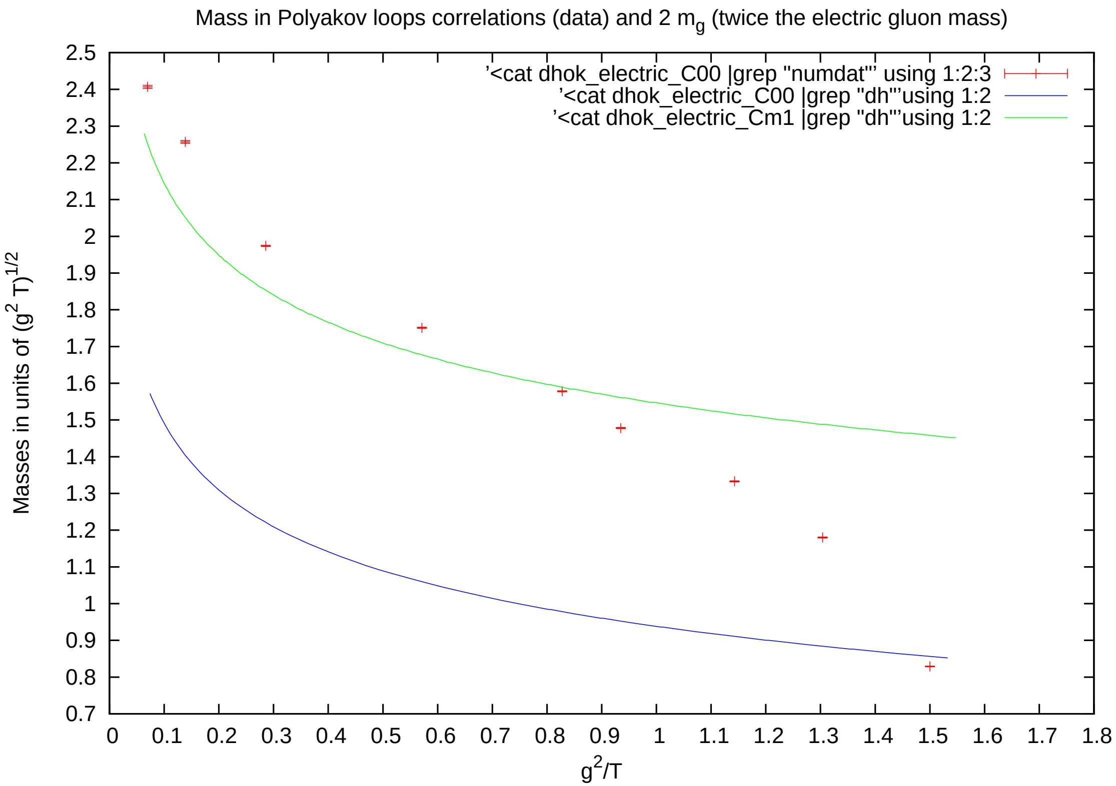
Selfconsistent Perturbation Theory D'Hoker 1981

$$\frac{m_g^2}{T^2} \sim \frac{g^2}{T}\left(a \log \frac{T}{m_g} + b + O\left(1/\log \frac{T}{m_g}\right)\right)$$

scales $\begin{cases} T \\ \sqrt{\frac{g^2}{T}}T \ll T \quad \text{for } T \text{ large} \end{cases}$ **note:** $\frac{g^2}{T_c} = 1.81(2)$ **Legeland, B.P.**

Comparison with the MC data

SCPT: $$\frac{m_g^2}{g^2T} = \frac{3}{2\pi}\left(\log\frac{T}{m_g} + C\right) + O\left(\frac{1}{\log T/m_g}\right); \qquad C = -1$$



$C = +1$ **!**

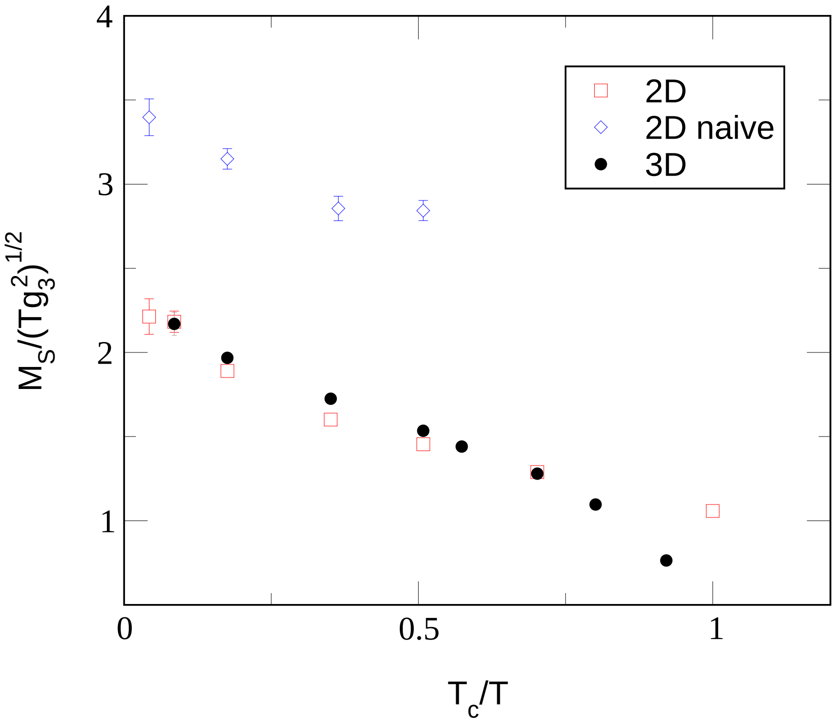
Dimensional reduction

gauge fields $\longrightarrow$ fermions	**QCD at high T** **in** $D = d_s + 1$	Ginsparg Appelquist, Pisarski Kärkkäinen, Lacock, BP, Reisz Kajantie, Laine, Rummukainen Hart, Philipsen Gavai, Gupta Laine, Philipsen

$\Downarrow$ $\longleftarrow$ **perturbative integration over non-static modes (no IR div.)**

scalar $\longrightarrow$ gauge	**Adj. SU(3) Higgs** **model** **in** d_s **dim.**	$\longleftarrow$ **Lattice calculations**

Comparison with MC data



$m/\sqrt{g^2T}$ vs T_c/T

In the reduced model the two exchanged states corresponding to $Re\,P(\bar{x})$ and $Im\,P(\bar{x})$ are simple poles, not 2 gluon and 3 gluon cuts respectively

Conclusions:

I. We have performed lattice simulations with high precision of SU(3) gauge theory in 2 spatial dimensions at finite temperature, extracted thermodynamical quantities, and extrapolated them to the continuum in the range $T_c \leq T \leq 10T_c$

II. SU(3) gauge theories in $2+1$ and $3+1$ dimensions have very similar qualitative behaviour

III. Results can be compared with resummed perturbation theory, dimensional reduction and phenomenological models (in progress)

IV. Similar comparisons for screening masses exist