Improving lattice calculations: One-loop determination of c_{sw} for Symanzik gauge action and stout smeared links

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CompPhys07, Leipzig

Outline

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Improvement

Lattice calculations gives access to a number of fundamental physical observables which are typical low energy quantities, e.g.

- masses
- decays
- hadronic structure functions (PDF, GPD, dipole moments, ...)

Basic requirements:

- **powerful computers**
  - IBM Blue Gene/P: $O(220 \times 10^{12})$ floating point operations/second peak performance (Jülich, 2008)
  - special purpose QCD computers
- **effective algorithms**: inverters, solvers, ...
- **clever formulations**: lattice results: (hypercubic, finite volume, finite lattice spacing) $\longleftrightarrow$ measured results: (continuum, infinite volume)

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Starting point of lattice calculations: action of underlying fermionic (ψ) and gauge fields ($U(A) = exp(iagA)$) on a lattice with lattice spacing a

$$S_{lattice}(\psi, U, a) = S_{fermion}(\psi, U, a) + S_{gauge}(U, a)$$

$S_{lattice}(\psi, U, a)$ is not unique - lot of different realizations

Essential constraints:

- underlying symmetries
- $S_{lattice}(\psi, U, a) \overset{(a \to 0)}{=} S_{continuum}(\psi, A)$

Benefit: computational efficiency, diminishing lattice artefacts, acceleration of convergence to continuum

Potential items for improvement:

- $S_{fermion}(\psi, U, a)$ - formulation of fermionc action
- $S_{gauge}(U, a)$ - formulation of gauge action
- $U(A)$ - representation of the gauge field itself

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Symanzik improved gauge action

Symanzik improvement scheme (reducing discretization errors order by order) → developed by *Lüscher/Weisz [1985]* for on-shell quantities

One special class to be used in the future simulations: tree level improved Symanzik action

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\begin{aligned}
S_{gauge}^{Symanzik}(U,a) &= \frac{6}{g^2}\sum_x \Bigg[c_0 \sum_{\mathrm{plaquette}} \frac{1}{3}\,\mathrm{Re\,Tr}\,(1-U_{\mathrm{plaquette}}) \\
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&\overset{(a\to 0)}{=} -\frac{1}{4}\,a^4 \sum_x \Big[\mathrm{Tr}\,F_{\mu\nu}(A)F_{\mu\nu}(A)\Big] + \mathcal{O}(a^6)
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c_1 = -\frac{1}{12},\; c_0 = 1 - 8\,c_1
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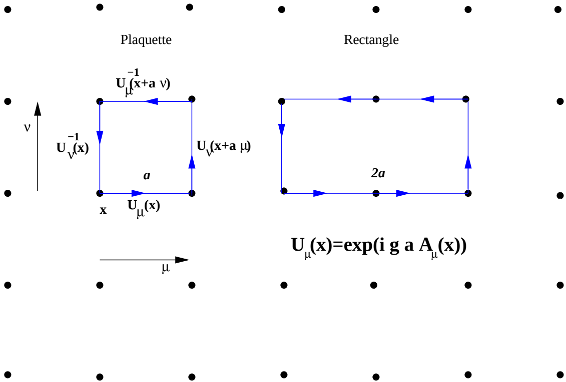

Stout smearing

Reducing the effect of chiral symmetry breaking of light flavors $\rightarrow$ UV-filtering

We use stout smearing(*Mornigstar and Peardon [2004]*) :

$$U \rightarrow U^{(1)} \rightarrow U^{(2)} \ldots \rightarrow U^{(n)} = \tilde{U}$$

$$U_\mu^{(n+1)}(x) = e^{iQ_\mu^{(n)}(U,\omega)}\, U_\mu^{(n)}(x)$$

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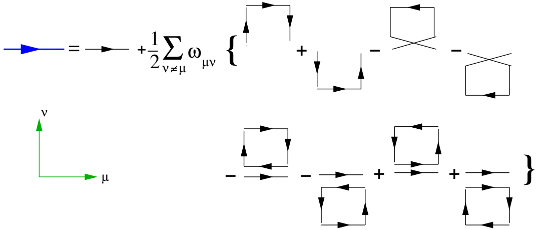

Fermionic action

Many fermions on the market

- Wilson fermions
- domain wall fermions
- staggered fermions
- overlap fermions
- clover fermions

Symanzik improvement scheme (reducing discretization errors order by order) $\rightarrow$ applied to fermions by *Sheikholeslami and Wohlert [1985]* $\rightarrow$ clover fermions

$$S^{clover}_{fermion}(\psi, U, a) = S^{Wilson}_{fermion}(\psi, U, a) + S^{SW}_{fermion}(\psi, U, a)$$

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with (massless case)

$$S_{fermion}^{Wilson}(\psi, U, a) = a^4 \sum_x \bar{\psi}(x) D_W(U,a)\,\psi(x)$$

$$\stackrel{(a\to 0)}{=} a^4 \sum_x \bar{\psi}(x)\gamma_\mu D_\mu(A)\,\psi(x) + \mathcal{O}(a^5)$$

$$D_W(U,a) = \frac{1}{2}\Big(\gamma_\mu(\nabla_\mu^\star + \nabla_\mu) - a\,r\,\nabla_\mu^\star\nabla_\mu\Big)$$

and clover term

$$S_{fermion}^{SW}(\psi, U, a) = -\frac{c_{SW}\,g a^5}{4}\sum_x\sum_{\mu\nu}\bar{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}^{clover}(x)\psi(x)$$

($\sigma_{\mu\nu} = i/2(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, $F_{\mu\nu}^{clover}(x)$: field strength in clover form)

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c_{SW} should be tuned to cancel $\mathcal{O}(a)$ lattice errors; determination in non-perturbative way preferred - but technically difficult

first step: calculation in lattice perturbation theory (LPT):

$$c_{SW} = 1 + g^2\, c_{SW}^{(1)} + \mathcal{O}(g^4)$$

First determinations of $c_{SW}^{(1)}$ in the on-shell regime have been published by:

Wohlert[1987] (twisted antiperiodic b.c., plaquette action)
Lüscher and Weisz[1996] (Schrödinger functional, plaquette action)
Aoki and Kuramashi[2003] (Conventional LPT, improved gauge actions)

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O(a) improvement - off-shell

This talk:

Symanzik gauge action +
one-step stout link smearing +
clover fermions +
off-shell $\rightarrow$
quark field improvement *Martinelli et al. [2001]*:

$$\psi_\star = (1 + a\, c_D\, \overrightarrow{\not{D}} + a\, i\, g\, c_{NGI} \not{A})\psi\,,$$

c_D has been determined to one-loop order (e.g., *QCDSF collaboration [2001]*)

$$c_{NGI} = g^2\, c_{NGI}^{(1)} + \mathcal{O}(g^4)$$

It needs either a two-loop calculation of quark propagator or an one-loop calculation of the quark-quark-gluon vertex.

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c_D has been determined to one-loop order (e.g., *QCDSF collaboration [2001]*)

$$c_{NGI} = g^2\, c_{NGI}^{(1)} + \mathcal{O}(g^4)$$

It needs either a two-loop calculation of quark propagator or an one-loop calculation of the quark-quark-gluon vertex.

qqg-Vertex

Looking for quantity → one-loop information for c_{SW}

quark-quark-gluon-vertex (V^{μ}_{qqg}): it contains to lowest order the improvement parameter c_{SW} → one-loop calculation sufficient

$$
\begin{aligned}
V^{\mu,a}_{qqg}(p_1,p_2) = & -igt^a\gamma_\mu - gt^a\tfrac{1}{2}ar\mathbf{1}(p_1+p_2)_\mu \\
& +(1+g^2\,c^{(1)}_{SW})igt^a\tfrac{1}{2}ar\sigma_{\mu\alpha}(p_1-p_2)_\alpha \\
& +\mathcal{O}(a^2)
\end{aligned}
$$

Strategy: Calculate the related non-amputated three-point function $\mathcal{G}_\mu$ to one-loop and demand that all $\mathcal{O}(a)$ terms cancel → $c^{(1)}_{SW}$, $c^{(1)}_{NGI}$

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Strategy: Calculate the related non-amputated three-point function $\mathcal{G}_\mu$ to one-loop and demand that all $\mathcal{O}(a)$ terms cancel $\rightarrow$ $c^{(1)}_{SW}$, $c^{(1)}_{NGI}$

Improvement relation

$$\mathcal{G}_\mu(p_1, p_2, c_{SW}^{(1)}, c_{NGI}^{(1)}) = S(p_2)\Lambda_\mu(p_1, p_2, c_{SW}^{(1)}, c_{NGI}^{(1)})S(p_1)$$

with quark propagator $S(p)$

$$S(p) = \frac{1}{i\not{p}\Sigma_p(p^2) + \frac{1}{2}arp^2\Sigma_W(p^2)} \approx \frac{1}{i\not{p}\Sigma_p(p^2)} + \frac{1}{2}ar\frac{\Sigma_W(p^2)}{[\Sigma_p(p^2)]^2}$$

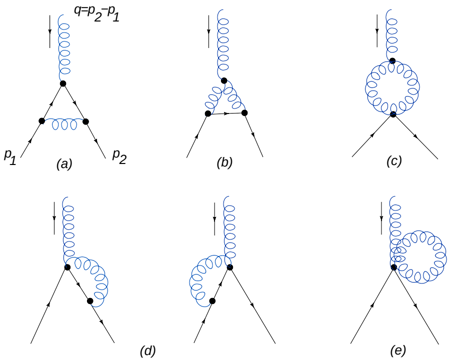
and amputated three-point function Λ_μ

$$\begin{aligned}\Lambda_\mu(p_2, p_1, c_{SW}^{(1)}) = {} & \Lambda_{\star,\mu}(p_2, p_1) + a\, g^3\, c_{NGI}^{(1)}(\not{p}_2\gamma_\mu + \gamma_\mu\not{p}_1) \\ & - \frac{1}{2}\, a\, i\not{p}_2\, \frac{\Sigma_W(p_2)}{\Sigma_p(p_2)}\Lambda_{\star,\mu}(p_2, p_1) - \frac{1}{2}\, a\, i\, \Lambda_{\star,\mu}(p_2, p_1)\, \not{p}_1\, \frac{\Sigma_W(p_1)}{\Sigma_p(p_1)}\,,\end{aligned}$$

$\rightarrow$ conditions on $c_{SW}^{(1)}$ **and** $c_{NGI}^{(1)}$ to get the improved three-point function $\Lambda_{\star,\mu}(p_2, p_1)$

Feynman rules

The diagrams needed for the one-loop calculation of V^{μ}_{qqg} are



Stout smearing makes the Feynman rules very complicated (for local operators see *Capitani, Dürr and Hoelbling [2006]*)

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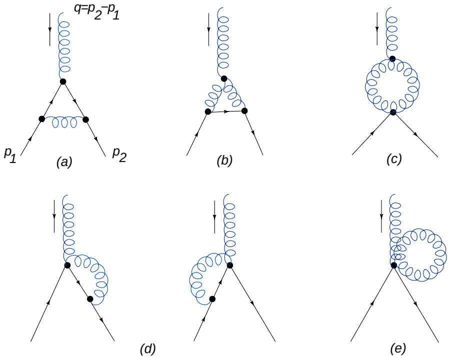

Stout smearing makes the Feynman rules very complicated (for local operators see *Capitani, Dürr and Hoelbling [2006]*)

Example: qqggg-Vertex and stout smearing

$$
\begin{aligned}
V^{abc}_{\alpha\beta\gamma}(p_2, p_1, k_1, k_2, k_3, \omega) &= \frac{1}{6}\, a^2 g^3 \sum_\mu \Big\{ W_{1\mu}(p_2, p_1) \left[F^{abc}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) + \text{cyclic perm.}\right] - \\
&\quad 6\,\omega\, W_{2\mu}(p_2, p_1) \left[T^{abc}_{sa}\, V_{\alpha\mu}(k_1)\, g_{\beta\gamma\mu}(k_2, k_3) + \text{cyclic perm.}\right] \Big\}\,.
\end{aligned}
$$

$$
\begin{aligned}
F^{abc}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) &= T^{abc}_{ss} f^{(1)}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) + T^{abc}_{aa} \left(f^{(2)}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) - f^{(2)}_{\alpha\gamma\beta\mu}(k_1, k_3, k_2)\right) + \\
&\quad \left(T^{abc}_{ss} - \frac{1}{N_c} d^{abc}\right) f^{(3)}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3)\,, \\
f^{(1)}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) &= \frac{1}{2}\, V_{\alpha\mu}(k_1, \omega)\, V_{\beta\mu}(k_2, \omega)\, V_{\gamma\mu}(k_3, \omega)\,, \\
f^{(2)}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) &= \frac{1}{2}\, V_{\alpha\mu}(k_1, \omega)\, V_{\beta\mu}(k_2, \omega)\, \delta_{\gamma\mu} - \frac{1}{2}\, \delta_{\alpha\mu}\delta_{\beta\mu}\, V_{\gamma\mu}(k_3, \omega) + \\
&\quad 6\,\omega\, \delta_{\alpha\beta} \left[\mathrm{c}_\mu(k_1 - k_2)\, \mathrm{c}_\beta(2k_3 + k_1 + k_2)\, \delta_{\gamma\mu} + \mathrm{s}_\mu(k_3)\, \mathrm{s}_\gamma(k_3 + 2k_1)\, \delta_{\beta\mu}\right] \\
f^{(3)}_{\alpha\beta\gamma\mu}(k_1, k_2, k_3) &= 2\,\omega\, \delta_{\beta\gamma} \Big[\left(3\, w_{\alpha\mu}(k_1, k_2 + k_3) + v_{\alpha\mu}(k_1 + k_2 + k_3)\right) \delta_{\alpha\beta} + \\
&\quad 12\, \mathrm{s}_\beta(k_1)\, \mathrm{s}_\alpha(k_2)\, \mathrm{s}_\alpha(k_3) \left(\mathrm{s}_\beta(k_1 + k_2 + k_3)\, \delta_{\alpha\mu} - \mathrm{s}_\alpha(k_1 + k_2 + k_3)\, \delta_{\beta\mu}\right) \Big]
\end{aligned}
$$

Example: qqggg-Vertex and stout smearing

Notation:

$$T^{abc}_{ss} = \{T^a, \{T^b, T^c\}\}\,, \quad T^{abc}_{aa} = [T^a, [T^b, T^c]]\,, \quad T^{abc}_{sa} = \{T^a, [T^b, T^c]\}$$

$$\mathrm{s}_\mu(k) = \sin\left(\frac{a}{2}k_\mu\right), \quad \mathrm{c}_\mu(k) = \cos\left(\frac{a}{2}k_\mu\right), \quad s^2(k) = \sum_\mu \mathrm{s}^2_\mu(k)\,,$$

$$s^2(k_1, k_2) = \sum_\mu \mathrm{s}_\mu(k_1 + k_2)\,\mathrm{s}_\mu(k_1 - k_2) \equiv s^2(k_1) - s^2(k_2)$$

$$\begin{aligned} W_{1\mu}(p_2, p_1) &= \mathrm{i}\,\mathrm{c}_\mu(p_2 + p_1)\,\gamma_\mu + r\,\mathrm{s}_\mu(p_2 + p_1) \\ W_{2\mu}(p_2, p_1) &= \mathrm{i}\,\mathrm{s}_\mu(p_2 + p_1)\,\gamma_\mu - r\,\mathrm{c}_\mu(p_2 + p_1) \end{aligned}$$

$$\begin{aligned} V_{\alpha\mu}(k, \omega) &= \delta_{\alpha\mu} + 4\,\omega\, v_{\alpha\mu}(k) \\ v_{\alpha\mu}(k) &= \mathrm{s}_\alpha(k)\,\mathrm{s}_\mu(k) - \delta_{\alpha\mu}\, s^2(k) \\ g_{\alpha\beta\mu}(k_1, k_2) &= \delta_{\alpha\beta}\,\mathrm{c}_\alpha(k_1 + k_2)\,\mathrm{s}_\mu(k_1 - k_2) - \\ &\quad \delta_{\alpha\mu}\,\mathrm{c}_\alpha(k_2)\,\mathrm{s}_\beta(2k_1 + k_2) + \delta_{\beta\mu}\,\mathrm{c}_\beta(k_1)\,\mathrm{s}_\alpha(2k_2 + k_1) \\ w_{\alpha\mu}(k_1, k_2) &= \mathrm{s}_\alpha(k_1 + k_2)\,\mathrm{s}_\mu(k_1 - k_2) - \delta_{\alpha\mu}\, s^2(k_1, k_2)\,, \quad w_{\alpha\mu}(k, 0) = v_{\alpha\mu}(k) \end{aligned}$$

$c_{NGI}^{(1)}$

Ward identity demands $c_{NGI}^{(1)}$ to be independent on color factor C_F
We get

$$
\begin{aligned}
c_{NGI}^{(1,plaq)} &= 0.0014260\, N_c - 0.0116643\, N_c\, \omega \\
c_{NGI}^{(1,Sym)} &= 0.0011781\, N_c - 0.0096247\, N_c\, \omega
\end{aligned}
$$

$c_{SW}^{(1)}$

Results for $c_{sw}^{(1)}$ have been published for Wilson fermions and various gauge actions.
For stout smearing and plaquette action we get for $N_c = 3$

$$c_{sw}^{(1,plaq)} = 0.268588 + 1.46772\,\omega - 5.76993\,\omega^2$$

which coincides for $\omega = 0$ with all previous given results.
For Symanzik action we get

$$c_{sw}^{(1,Sym)} = 0.196244 + 1.137452\,\omega - 4.180291\,\omega^2$$

which should be compared to the $\omega = 0$ value of Aoki/Kuramashi: 0.19624449(1)

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Summary

- Using standard perturbation theory we have calculated one-loop non-amputated Green's function related to the qqg-vertex with plaquette/Symanzik gauge actions and stout smeared links in the fermionic action
- The result is used to determine the improvement coefficient c_{SW} including stout smearing
- We have used symbolic and numerical methods
- $c_{SW}^{(1)}$: we have reproduced earlier results for non-smeared links and plaquette and Symanzik action
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