

Percolation of negative-weighted loops on $2d$ lattices

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Outline

- Introduction
- Percolation problem
- Results
- Summary

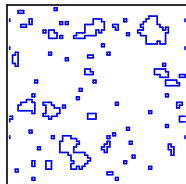
Model

- $L \times L$ lattice, fully periodic boundary conditions
- Undirected edges, weight distribution:

$$P(\omega) = \rho (2\pi)^{-1/2} \exp(-\omega^2/2) + (1-\rho) \delta(\omega - 1)$$

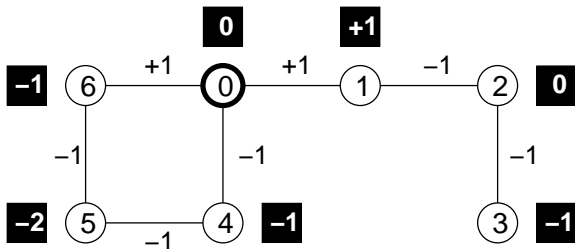
- Allows for loops \mathcal{L} with **negative weight** $\omega_{\mathcal{L}}$
- Configuration \mathcal{C} of loops, with

$$E \equiv \sum_{\mathcal{L} \in \mathcal{C}} \omega_{\mathcal{L}} \stackrel{!}{=} \min$$



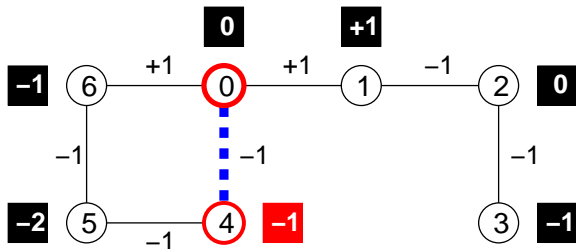
- Obtain \mathcal{C} through mapping to minimum weight perfect matching problem

Minimal distances



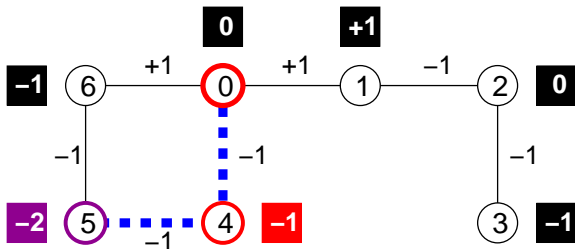
- $d(i) = \min_{j \in N(i)} (d(j) + \omega(i, j))$ not fulfilled

Minimal distances



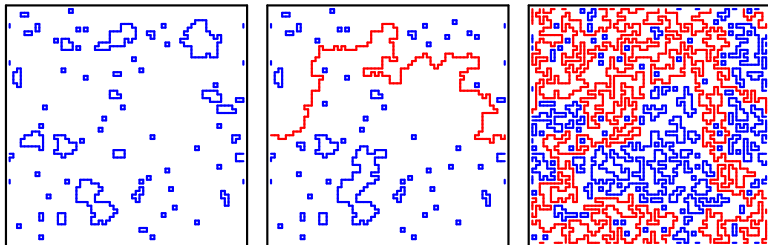
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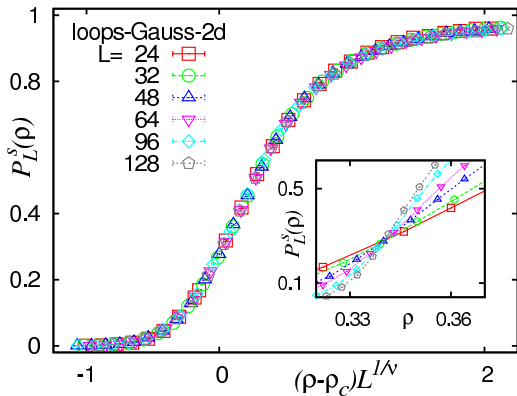
Loop percolation



($L = 64$ at $\rho = 0.335, 0.340, 0.750$)

- Observe system spanning loops above critical ρ
- Disorder induced, geometric transition
- Characterize loops using observables from percolation theory (finite-size scaling (FSS) analysis)

Percolation probability



Percolation probability exhibits FSS:

$$P_L^s \sim f[(\rho - \rho_c)L^{1/\nu}]$$

$$\rho_c = 0.340(1)$$

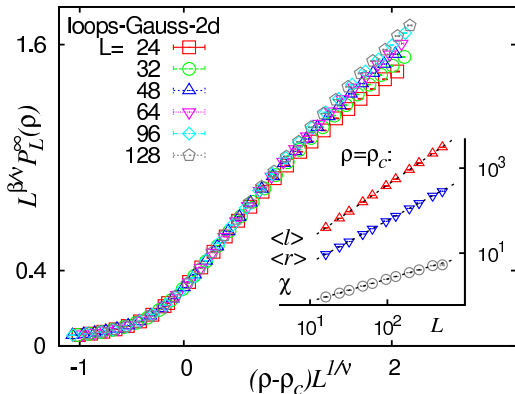
$$\nu = 1.49(7)$$

(rand. perc.: $\nu = 1.33$)

$$S = 0.91$$

- $S =$ “quality” of the scaling assumption
- Similar scaling for mean number of spanning loops

Percolation strength



Exhibits FSS:

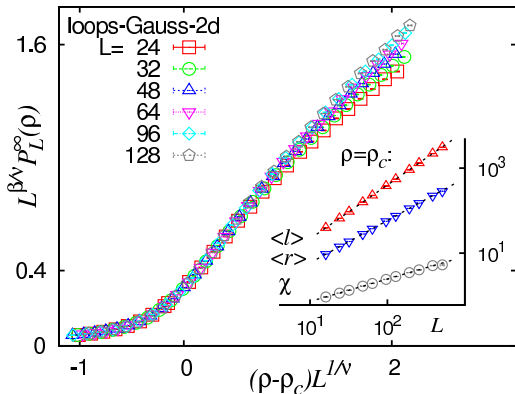
$$P_L^\infty \sim L^{-\beta/\nu} f[(\rho - \rho_c)L^{1/\nu}]$$

$$\beta = 1.07(6)$$

$$S = 1.16$$

- Probability $P_L^\infty \equiv \langle \ell \rangle / L^d$ that edge belongs to percolating loop, finite-size susceptibility $\chi \equiv L^{-d}(\langle \ell^2 \rangle - \langle \ell \rangle^2)$

Percolation strength



At ρ_c ($L_{max} = 512$):

loop length $\langle \ell \rangle \sim L^{d_f}$,

roughness $\langle r \rangle \sim L^{d_r}$,

suszept. $\chi \sim L^{\gamma/\nu}$

$d_f = 1.266(2)$

$d_r = 1.001(4)$

$\gamma = 0.77(7)$

- Probability $P_L^\infty \equiv \langle \ell \rangle / L^d$ that edge belongs to percolating loop, finite-size susceptibility $\chi \equiv L^{-d}(\langle \ell^2 \rangle - \langle \ell \rangle^2)$
- Scaling relations $d_f = d - \beta/\nu$ and $\gamma + 2\beta = d\nu$ are fulfilled

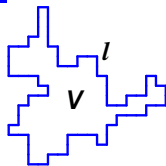
Non-percolating loops

- Scaling properties of the small loops:

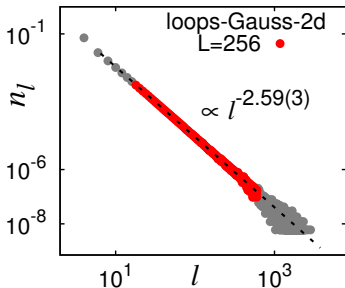
Consistent with percolating loops

$$\langle v \rangle \sim R^2 \text{ (loop spanning length } R)$$

$$\langle \omega \rangle \sim \ell$$



- Distribution n_ℓ of the loop lengths ℓ at ρ_c for $L = 256$



Expected FSS:

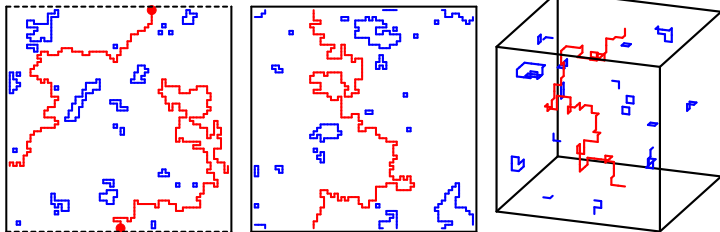
$$n_\ell \sim \ell^{-\tau}$$

$$\tau = 2.59(3)$$

Consistent with

$$\tau = 1 + d/d_f$$

More results



Type	d	ρ_c	ν	β	γ	τ	d_f
$P\pm J$	2	0.1032(5)	1.43(6)	1.03(3)	0.76(5)	2.51(4)	1.268(1)
$L\pm J$	2	0.1028(3)	1.49(9)	1.09(8)	0.75(8)	2.58(6)	1.260(2)
L-GI	2	0.340(1)	1.49(7)	1.07(6)	0.77(7)	2.59(3)	1.266(2)
$L\pm J$	3	0.0286(1)	1.02(3)	1.80(8)	—	3.5(3)	1.30(1)

- Exponents seem to be universal in $2d$
- Random bond Ising model at $T=0$ [Amoruso, PRB 2004]:
 $\rho_c = 0.103(1)$, $\nu = 1.55(1)$, $\beta = 0.9(1)$

Summary

- Negative-weight percolation of loops
- Distinct from random bond/site percolation
- $2d$: critical exponents close to RBIM
- More details: [arXiv:0711.4069](https://arxiv.org/abs/0711.4069)

Summary

- Negative-weight percolation of loops
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- Thank you!