Entanglement evoution after connecting finite to infinite quantum chains

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Introduction

- entanglement properties of *equilibrium* spin chains are widely investigated and understood
- simplest way leading to time evolution is a *quench*
- first studies were carried out on *global* qunches, leading to a drastic change of the state
- what happens when the system is perturbed *locally*?

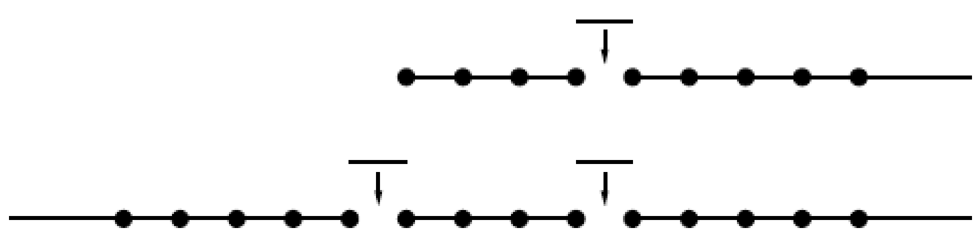
Model and geometry

$$H = -\frac{1}{2}\sum_{n=1}^{N-1}\left[\frac{1+\kappa}{2}\sigma_n^x\sigma_{n+1}^x + \frac{1-\kappa}{2}\sigma_n^y\sigma_{n+1}^y\right] - \frac{h}{2}\sum_{n=1}^{N}\sigma_n^z$$

Consider only the special cases:

- $\kappa = 0$ and h=0 (XX chain, critical)
- $\kappa = 1$ (Ising chain, critical for h = 1)

Time evolution after connecting segments:



Calculation of entropy

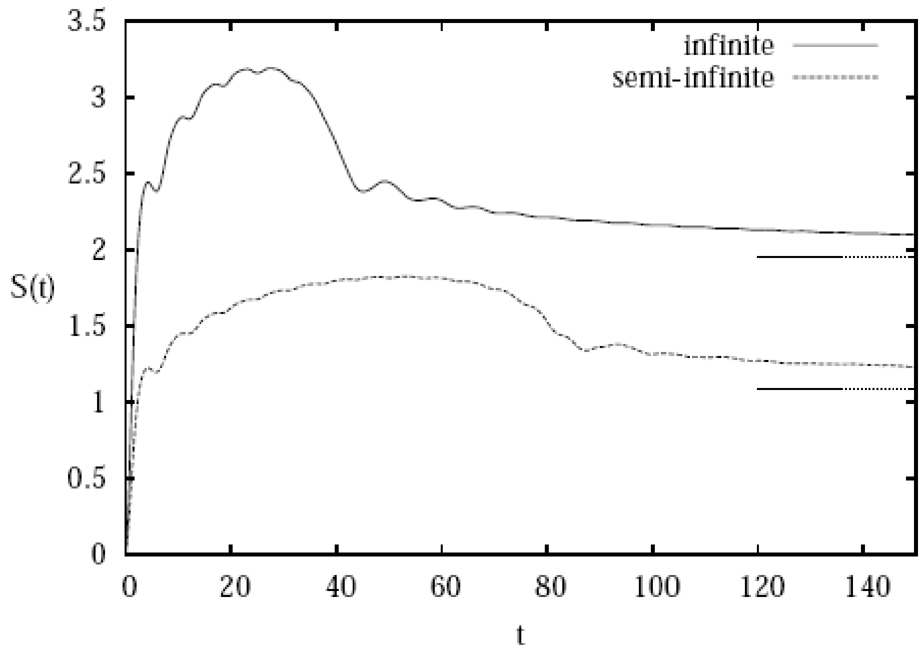
Reduced density matrix has the simple form

$$\rho = \frac{1}{Z}\, e^{-\mathcal{H}} \,, \quad \mathcal{H} = \sum_{k=1}^{L} \varepsilon_k(t) f_k^\dagger f_k$$

Entropy is given by $S = -\mathrm{Tr}\,(\rho \ln \rho)$

- eigenvalues are obtained from time dependent fermionic correlation matrices
- calculate time evolution of correlations
- diagonalize L x L (XX case) or 2L x 2L (TI case) matrices

General results – Critical case



- rises quickly to a plateau initially (overshooting)
- slow relaxation towards equilibrium
- time scales: L for infinite and 2L for semi-infinite case
- connection between XX and critical TRI: $S_{XX}(L,t) = 2\ S_{TI}(L/2, t/2)$
(extending results of Juhász & Iglói)

Comparison with CFT

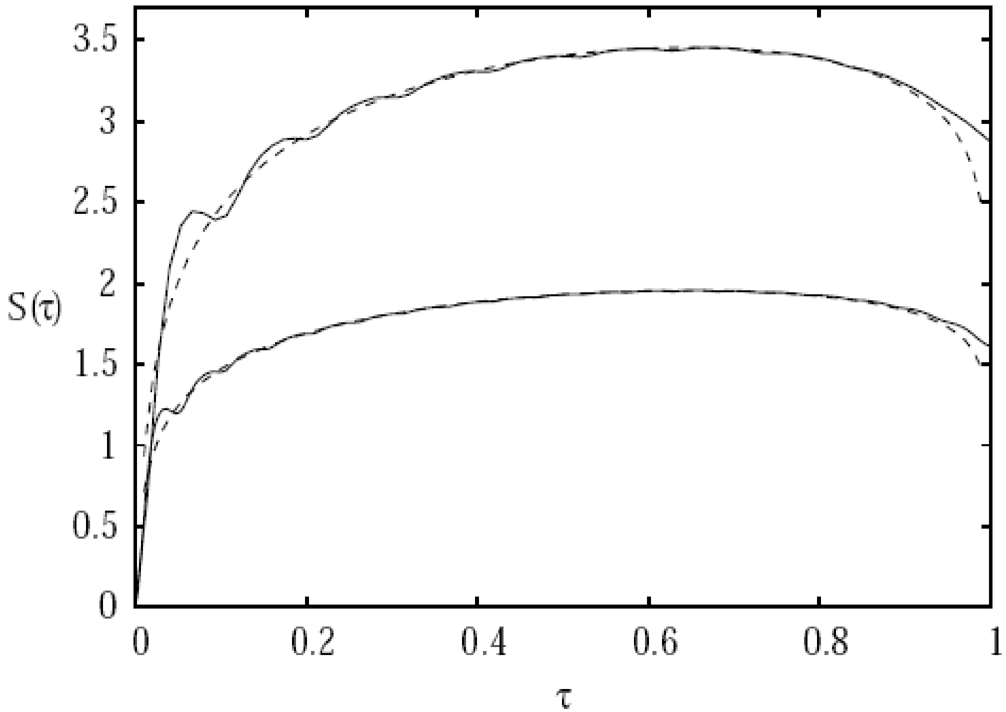
- plateau region described by **fast** excitations
- CFT treatment possible in critical case (Calabrese & Cardy)

Semi-infinite case (C&C)

$$S(t) = \frac{c}{6} \ln \left[\frac{4L}{\pi} t \sin\left(\frac{\pi t}{2L} \right) \right] + k'$$

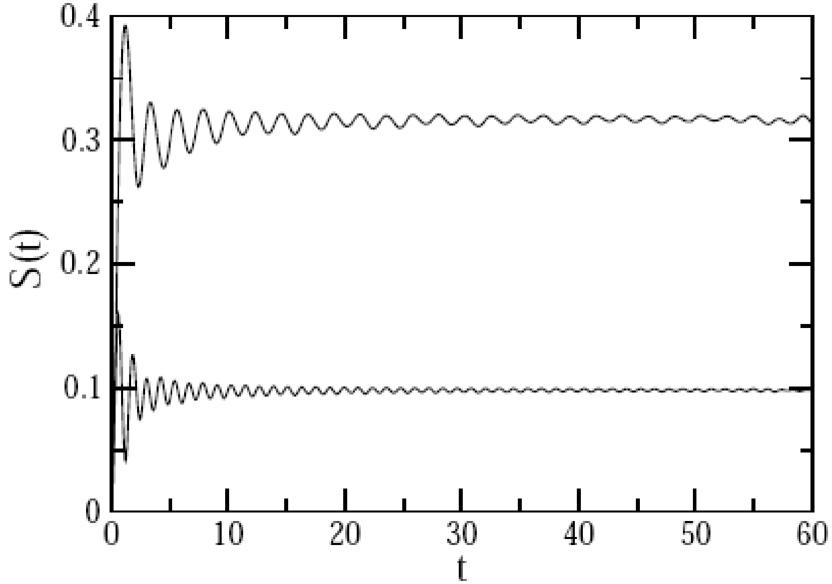
Generalized to infinite case:

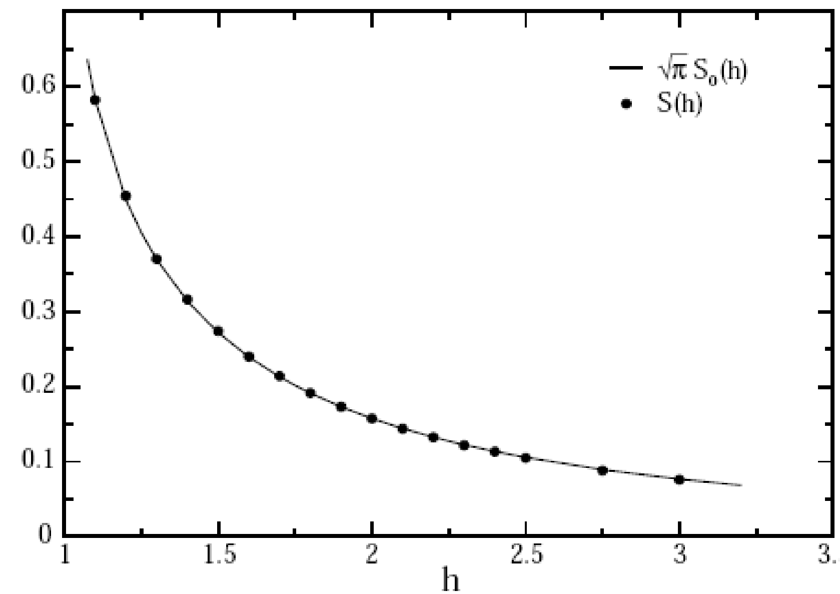
$$S(t) = \frac{c}{3} \ln \left[\frac{2L}{\pi} t \sin\left(\frac{\pi t}{L} \right) \right] + k$$



Excellent agreement with lattice results! (except oscillations)

Plateaus in the noncritical case



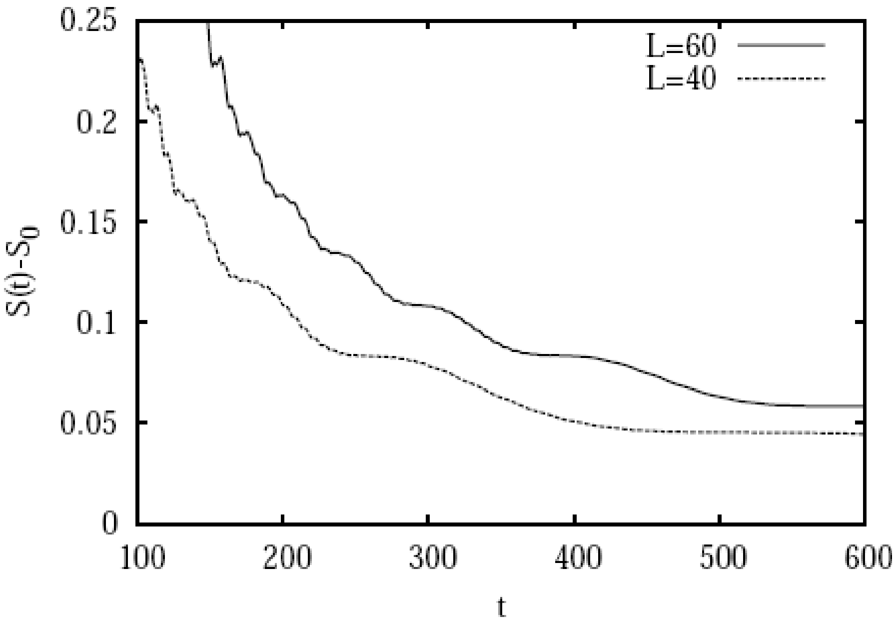


- flat plateau region, independent of L
- heigth of the plateu can be connected with equilibrium value
- oscillations are more prominent due to the small height

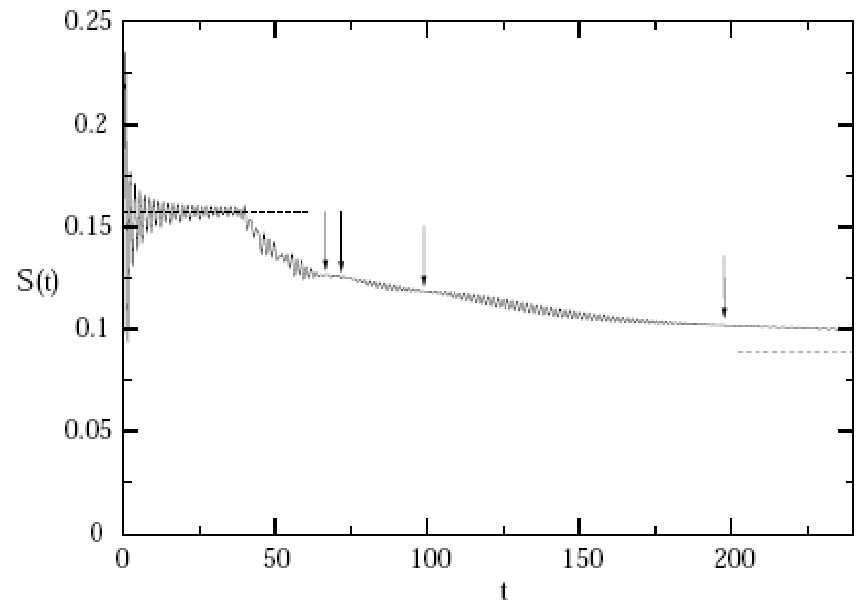
$$S(t) \simeq \sqrt{\pi} S_0(h) + \frac{1}{t}\left[A + B\cos(2ht)\right]$$

Long time behaviour

Critical



Non-critical



- clear step-structure in critical case
- smaller but visible effects in non-critical-case
- characteristic times are connected with **slow** excitations: $T_n = \dfrac{L}{v_n}$

Conclusions

- typical behaviour for all geometries: entanglement “pulses”
- critical behaviour well described by CFT for the *initial stage* of evolution
- non-critical plateaus can be well described, though not yet understood
- long time step-structure carries memory of the initial *discrete* spectrum
- interpretation in terms of simple particle picture??