Collaboration

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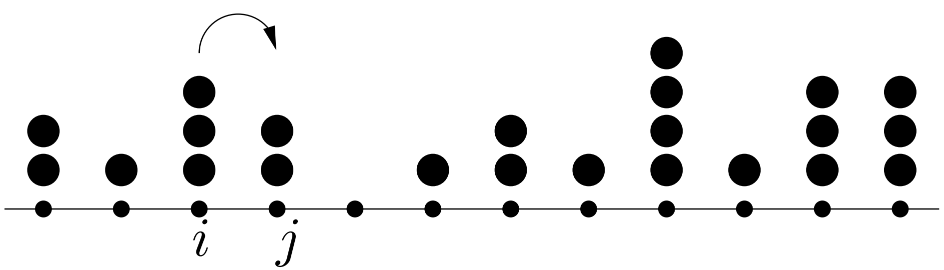
Motivation:

Solvable and non-trivial model of non-equilibrium dynamics

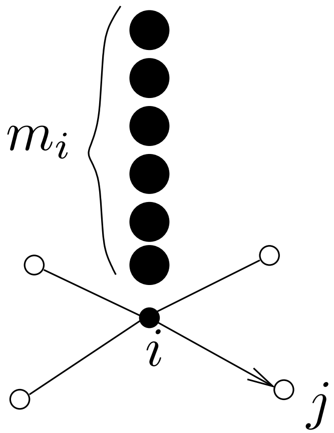
Outline:

- zero-range process (ZRP);
- steady state and condensation (balls-in-boxes model);
- free ZRP + random geometry;
- summary;

$$W_{i\to j} = W_i$$



Zero-range processes



$m_i = 6$

$q_i = 4$

Identical balls $\Longrightarrow$ State of the system: $C = \{m_1, m_2, \ldots, m_N\}$

Hopping rate: $$W_{i \to j} = W_i = \frac{1}{q_i} u(m_i)$$

Dynamics

State probability: $P_t(C) = P_t(m_1, m_2, \ldots, m_N)$

Dynamics:

$$\frac{dP_t(C)}{dt} = \sum_A \left(P_t(A)W(A \to C) - P_t(C)W(C \to A)\right)$$

Steady state: $\frac{dP_t(C)}{dt} = 0$?

Off-equilibrium dynamics: $P_t(C) \to P(C)$

Steady state exists! M.R. Evans

$$P(C) = p_1(m_1)p_2(m_2)\dots p_N(m_N)/\mathcal{N}$$

$$p_i(m) = q_i^m p(m)$$

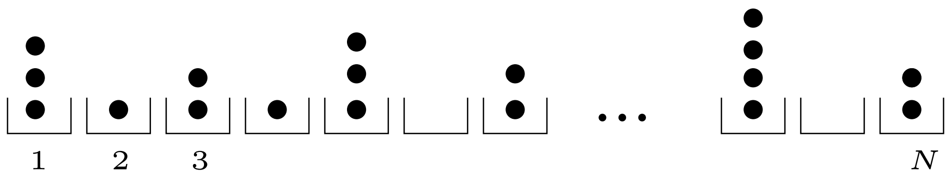
$$p(0) = 1, \quad p(m) = \frac{1}{u(1)u(2)\dots u(m)}$$

Balls-in-boxes model:

$$Z_{M,N} = \sum_{\{m_i\}} p_1(m_1)p_2(m_2)\dots p_N(m_N)\delta_{m_1+m_2+\dots+m_N,M}$$

Condensation P.Bialas, Z.Burda, D.Johnston

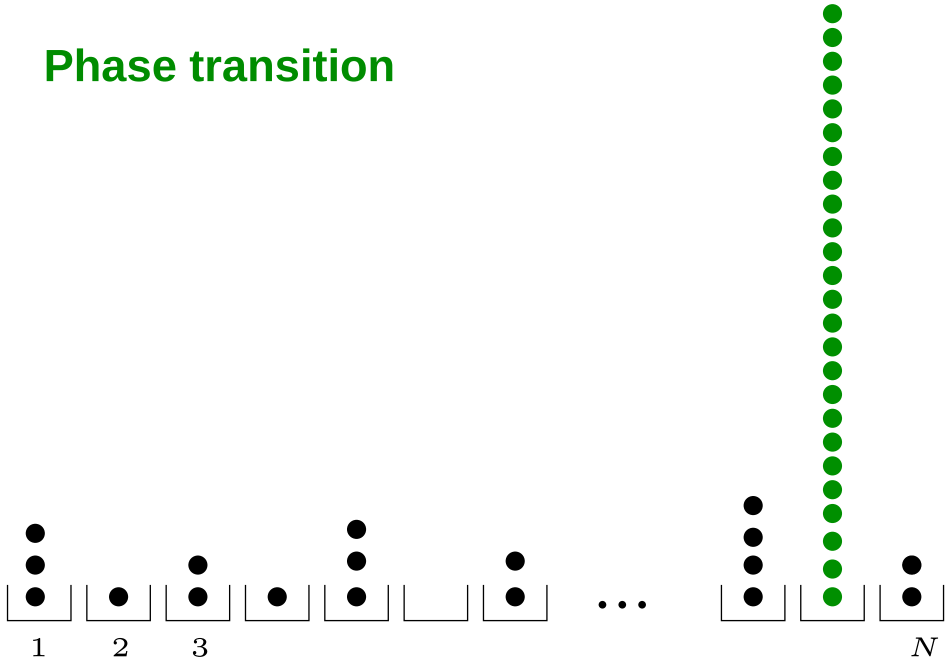
$$Z_{M,N} = \sum_{\{m_i\}} p(m_1)p(m_2)\dots p(m_N)\delta_{m_1+m_2+\dots+m_N,M}$$



Condensation P.Bialas, Z.Burda, D.Johnston

$$Z_{M,N} = \sum_{\{m_i\}} p(m_1)p(m_2)\dots p(m_N)\delta_{m_1+m_2+\dots+m_N,M}$$

Phase transition



Condensation mechanism

$$Z_{M,N} = \int_{-\pi}^{\pi} \frac{\mathrm{d}\phi}{2\pi} e^{-i\phi M} \, P(\phi)^N = \oint \frac{\mathrm{d}z}{2\pi i z^{M+1}} P(z)^N$$

where $P(\phi) = \sum_m p(m)\, e^{i\phi m} \equiv P(z) = \sum_m p(m)\, z^m$

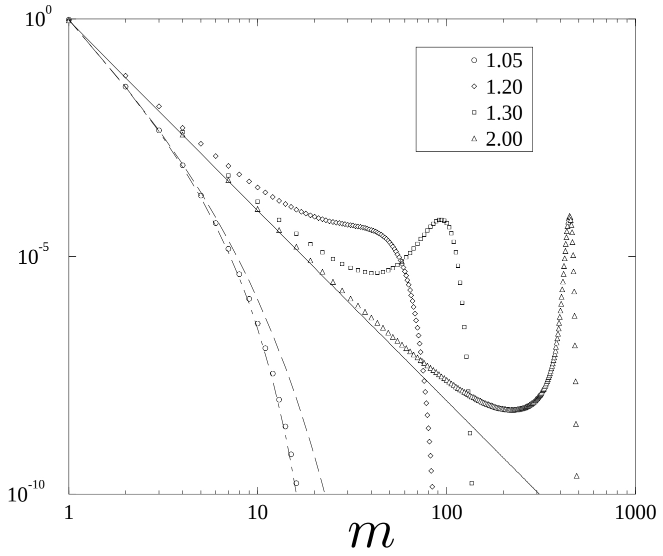
Phase transition in density $\rho = \frac{M}{N}$

Saddle point solution z_{sp} breaks down for $\rho \longrightarrow \rho_{cr}$ because the saddle point hits the circle of convergence of $P(z)$.

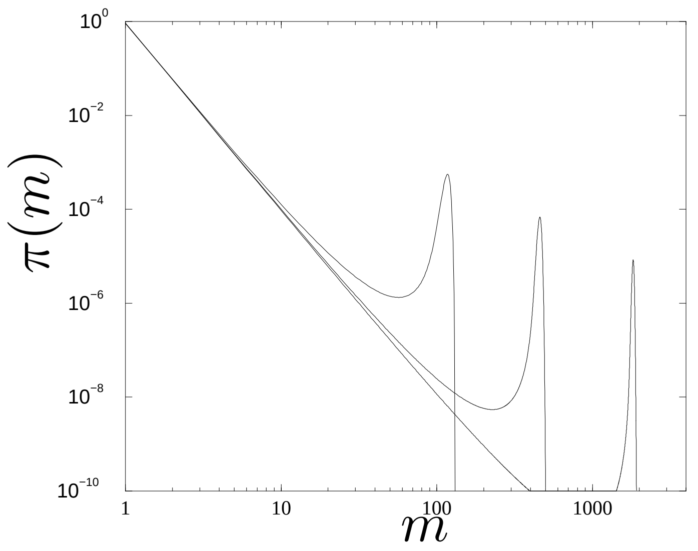
For $\rho > \rho_{cr}$ one box contains $(\rho - \rho_{cr})N$ balls!

Occupation probability

Probability $\pi(m)$ that a randomly choosen box has m balls:



N-fixed, ρ-varying



ρ-fixed, N-varying ($\times 4$)

Free ZRP

$W_{i \to j} = \frac{1}{q_i}$

Steady state:

$$Z_{M,N} = \sum_{\{m_i\}} p_1(m_1) p_2(m_2) \ldots p_N(m_N) \delta_{m_1+m_2+\cdots+m_N,M}$$

with $p_i(m) = q_i^m$

Relevant parameter: $\alpha = \frac{\langle q \rangle}{q_{max}}$

Critical density: $\rho_{cr} \sim \alpha/(1-\alpha)$

Single inhomogeneity graphs / Scale free networks

Averaging over graphs

"Uncorrelated networks":

$\Pi(q_1, q_2, \ldots, q_N) \sim \Pi(q_1)\Pi(q_2)\ldots\Pi(q_N) \quad N \to \infty$

$$Z(M,N) = \sum_{\{m_i\}} \mu(m_1)\mu(m_2)\ldots\mu(m_N)\delta_{m_1+m_2+\cdots+m_N,M}$$

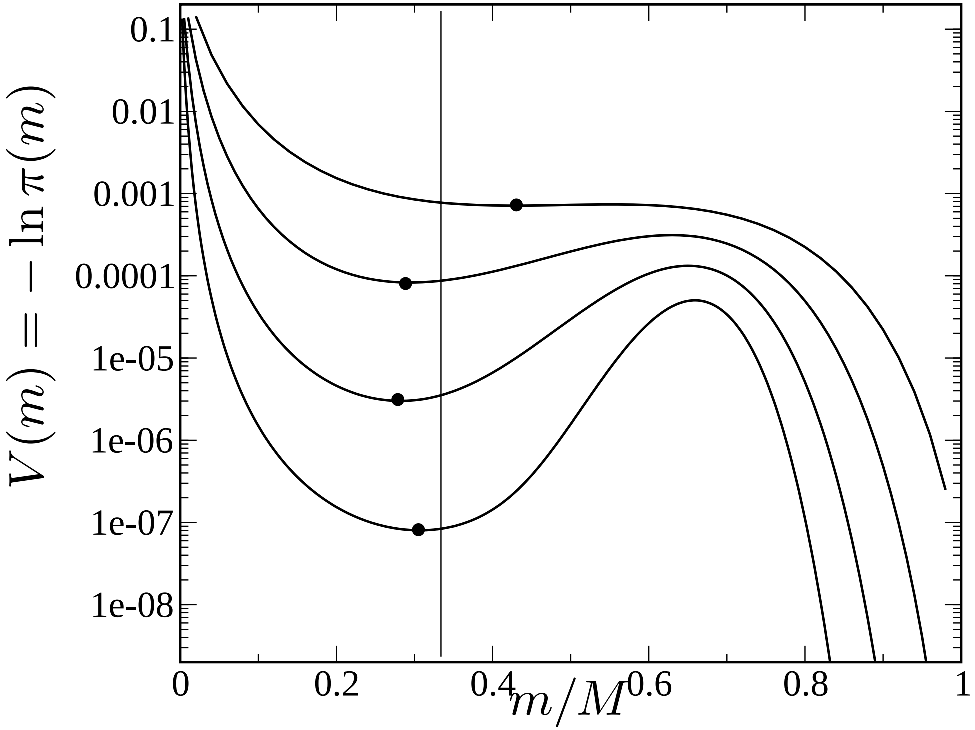
where $\mu(m) = \sum_q \Pi(q) q^m$

In particular: $\Pi(q) \sim (k_0 - k)^{b-1} \;\longrightarrow\; \pi(m) \sim m^{-b}$

Dynamics C. Godreche, J.M. Luck

Condensate formation/evaporation

Typical time scale (Arrhenius law) $\tau \sim e^{V_{min}} \sim 1/\pi(m_{min})$



$\tau \sim N^{\beta}$ or $\tau \sim e^{\alpha \Delta \rho N}$

Mean-field dynamics

Summary

- simple solvable model with a non-trivial behaviour
- steady state (balls-in-boxes model)
- dynamics (condensate formation and evaporation)
- interactions with network geometry

Balls-in-boxes condensation / related models

- quantum gravity
- random-trees (graphs)
- wealth condensation
- urn models
- Hagedorn transition
- Kac-Berlin solution of spherical model
- Bose-Einstein condensation