On the phase structure of the 3D Abelian Two-Higgs model on the lattice

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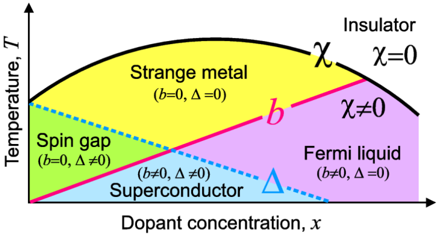
Motivation

- Higgs model(s):
- in Standard Model of particle physics: mass
- recent years: strongly correlated electron systems → gauge theories
- Lee, Nagaosa, Wen; Rev. Mod. Phys.; 2006:
  3d-A2HM as effective model for ground state of
  2d quantum-spin Hamiltonian of overdoped high T_c superconductors

- starting from tJ-model,
  four phases predicted:
  - SG: spin gap phase
  - SC: superconducting phase
  - FL: Fermi liquid phase
  - SM: strange metal phase
  - b: holon condensate
  - Δ: spinon condensate (cooper pair)
  - χ: (neglected) resonating valence bond coupling



Can one identify those four phases in the A2HM?

Compact Abelian Two-Higgs Model

with frozen Higgs-field radii

Action:

$$S = -\beta \sum_P \cos\left(\theta_P\right) - \kappa_h \sum_{\mathbf{n}} \sum_{\mu} \cos\left(\varphi^{(h)}_{\mathbf{n}+\hat{\mu}} + Q_h \theta_{\mathbf{n},\mu} - \varphi^{(h)}_{\mathbf{n}}\right)$$
$$-\kappa_s \sum_{\mathbf{n}} \sum_{\mu} \cos\left(\varphi^{(s)}_{\mathbf{n}+\hat{\mu}} + Q_s \theta_{\mathbf{n},\mu} - \varphi^{(s)}_{\mathbf{n}}\right)$$

- $\varphi^{(h)}_{\mathbf{n}}, \varphi^{(s)}_{\mathbf{n}}$ holon, spinon fields, $\theta_{\mathbf{n},\mu}$ gauge field
- $Q_h = 1, Q_s = 2$ holon, spinon field charge
- $\theta_P = \theta_{\mathbf{n},\mu} + \theta_{\mathbf{n}+\hat{\mu},\nu} - \theta_{\mathbf{n}+\hat{\nu},\mu} - \theta_{\mathbf{n},\nu}$ plaquette angle
- $\beta, \kappa_h, \kappa_s$ coupling parameters
- $\varphi^{(h)}_{\mathbf{n}}, \varphi^{(s)}_{\mathbf{n}}, \theta_{\mathbf{n},\mu} \in [-\pi; \pi)$
- $Z = \int \prod_{\mathbf{n}} \frac{\mathrm{d}\varphi^{(h)}_{\mathbf{n}}}{2\pi} \prod_{\mathbf{n}} \frac{\mathrm{d}\varphi^{(s)}_{\mathbf{n}}}{2\pi} \prod_{\mathbf{n},\mu} \frac{\mathrm{d}\theta_{\mathbf{n},\mu}}{2\pi} e^{-S}$

Topological excitations

$$m_P \in \mathbb{Z}: \ \theta_P + 2\pi m_P \in [-\pi;\pi)$$

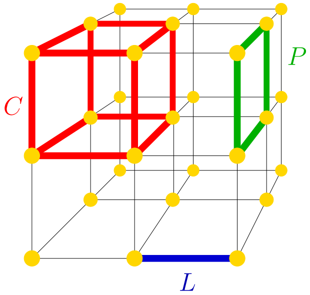
$$l_L \in \mathbb{Z}: \varphi_{\mathbf{n}+\hat{\mu}} + Q\theta_{\mathbf{n},\mu} - \varphi_{\mathbf{n}} + 2\pi l_L \in [-\pi;\pi)$$

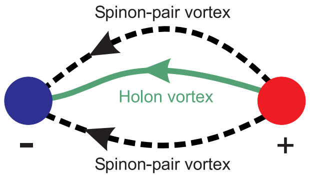
Monopole charge:

$$j = \sum_{\partial C} m_P$$

Holon/Spinon vortex current:

$$\sigma_{h/s} = -Q_{h/s} m_P + \sum_{\partial P} l_L$$





Observables

Action parts:

$$\langle E_{h/s} \rangle = \left\langle \frac{S_{h/s}}{3N} \right\rangle$$

$$\langle E_P \rangle = \left\langle \frac{S_P}{3N} \right\rangle$$

Topological Excitations:

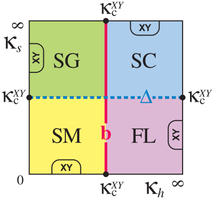
$$\langle \rho_{h/s} \rangle = \left\langle \frac{1}{3N} \sum |\sigma_{h/s}| \right\rangle$$

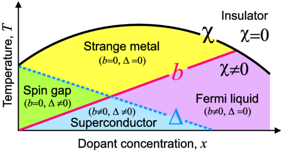
$$\langle \rho_m \rangle = \left\langle \frac{1}{N} \sum |j| \right\rangle$$

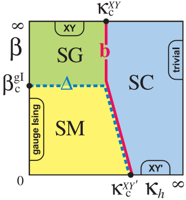
$$S = -\beta S_P - \kappa_h S_h - \kappa_s S_s$$

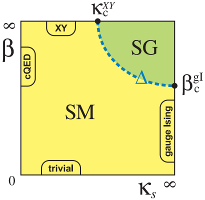
Suscepibilities:

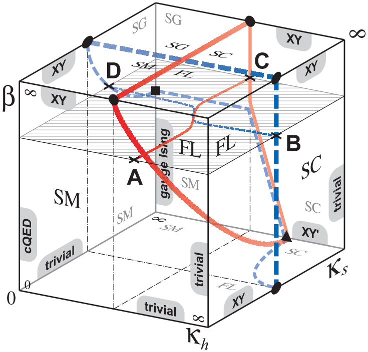
$$\chi_O = (3)\, N \left(\langle O^2 \rangle - \langle O \rangle^2 \right)$$

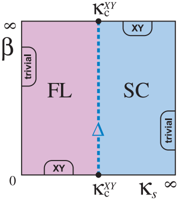


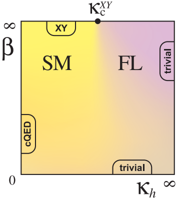


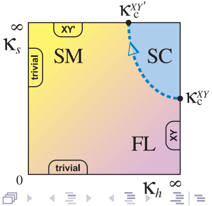












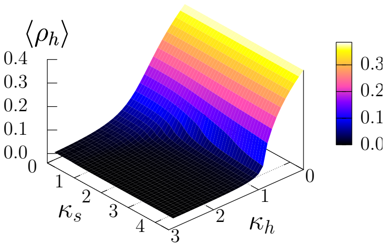
All transitions

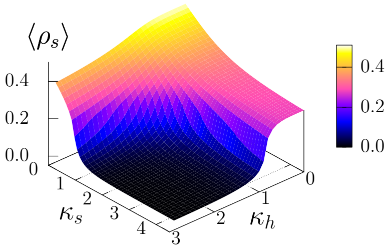
2nd order ,

or pure percolation

transitions!

Phase characterization $\beta = 0.5$



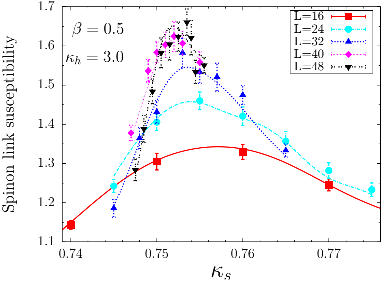


- two phases:
- SC phase off κ_h, κ_s axes: low $\langle\rho_s\rangle$
- SM/FL phase along κ_h, κ_s axes: high $\langle\rho_s\rangle$

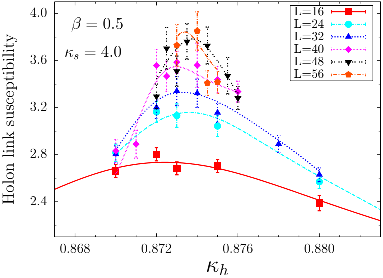
- analytic connection of SM/FL phases
- SM-like at $\kappa_h \approx 0, \kappa_s \approx \infty$
- FL-like at $\kappa_h \approx \infty, \kappa_s \approx 0$

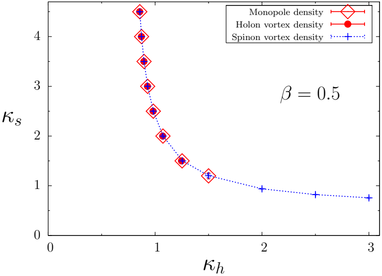
boundary cases

$\kappa_h - \kappa_s$ phase diagram at $\beta = 0.5$

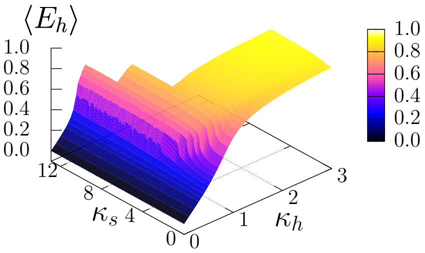


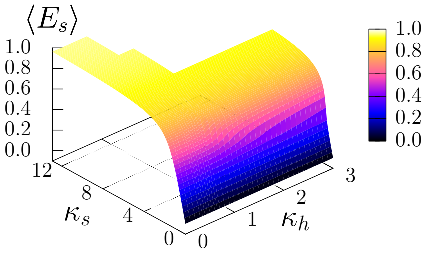
- fine-grided sampling across transition line, fixing either κ_h or κ_s
- build susceptibility peak by Ferrenberg-Swendsen reweighting
- identify maximum $\rightarrow$ position of transition
- all phase transitions appear to be 2$^{\text{nd}}$ order; clear from boundary cases





Phase characterization $\beta = 0.8$

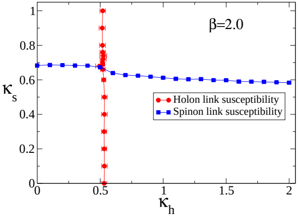


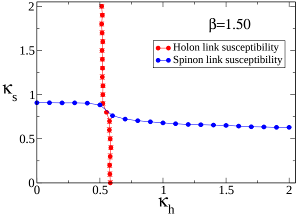


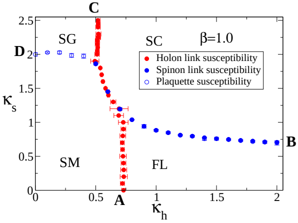
- four phases:
- SM phase $(\kappa_h, \kappa_s) \approx (0, 0)$: low $\langle E_h \rangle$, low $\langle E_s \rangle$
- SG phase $\kappa_h \approx 0, \kappa_s \approx \infty$: low $\langle E_h \rangle$ and high $\langle E_s \rangle$
- FL phase $\kappa_h \approx \infty, \kappa_s \approx 0$: high $\langle E_h \rangle$ and low $\langle E_s \rangle$
- SC phase $(\kappa_h, \kappa_s) \approx (\infty, \infty)$: high $\langle E_h \rangle$ and high $\langle E_s \rangle$

boundary cases

Trend in β:

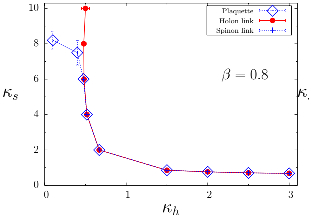


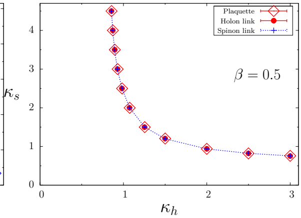




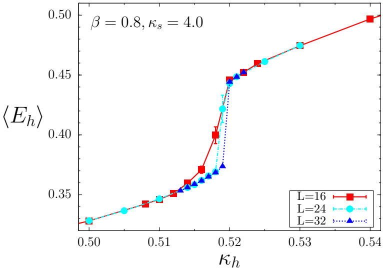
- $\beta \approx \infty$: rectangular section of red and blue phase transition lines
- decreasing β: emergence of χ-like structure
- phase transition lines merge
- below $\beta = \beta_c^{\mathrm{gI}} \approx 0.7613$: transition line at low κ_h vanishes

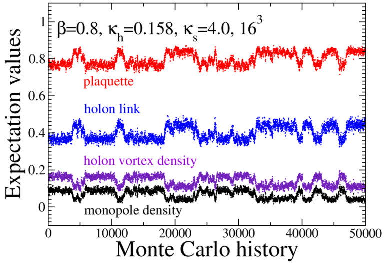
▸ boundary cases





Signature of 1^{st} order transition





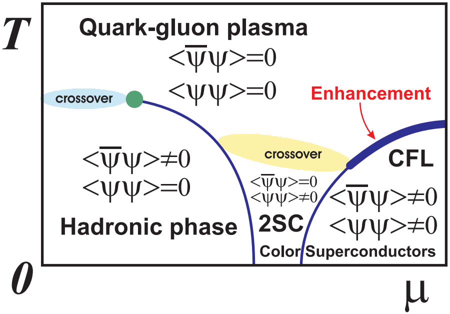
- in the region $\infty < \beta < \beta_c^{\mathrm{gI}}$ two transition lines merge (χ structure)
- lines joint $\rightarrow$ region of 1^{st} order transition
- results from activity of gauge field coupling holon and spinon field
- published in Physical Review B: Bock, Chernodub, Ilgenfritz, Schiller (November 2007)

boundary cases

Conclusions

- considered cA2HM as effective model for some aspects of high-T_c superconductors
- identified SG, SC, FL, SM phases as proposed by Lee, Nagaosa, Wen
- found new effect: merging of second order phase transition lines may lead first order transition
- this effect may have impact on other fields of physics (e.g. QCD)

Phase transition enhancement in QCD



- quark $\langle \psi\bar{\psi} \rangle$ and diquark $\langle \psi\psi \rangle$ condensates
- both $= 0$ in quark-gluon plasma phase
- both $\neq 0$ in color-flavor locked (CFL) phase
- 2SC two color superconducting phase
- e.g. Nononha et al., Physical Review D, 2006