Frustration from Fat Graphs

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In collaboration with: Des Johnston (Heriot-Watt, Edinburgh)

Random graphs and surfaces: a genealogy

A whole zoo of random graphs and surfaces has recently attracted the attention of physicists.

Generic random graphs

Erdös-Rényi: N nodes with n randomly set edges, Poissonian degrees

Barabási-Albert: network growing with *preferrential attachment*, power-law degrees

Watts-Strogatz: interpolation between regular lattice and Erdös-Rényi through re-wiring

In general: *uncorrelated* co-ordination numbers, completely determined by degree distributions, no topology.

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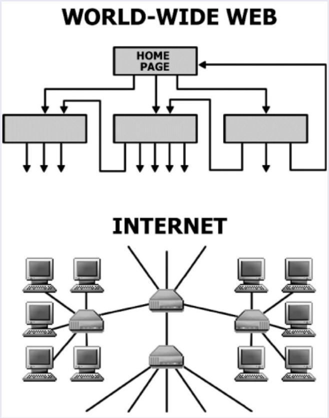

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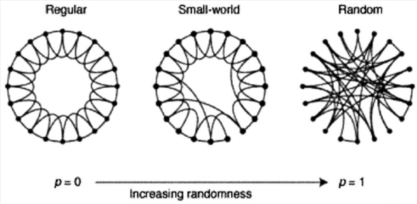

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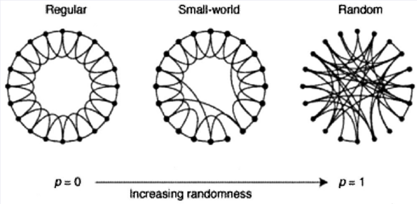

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$$\frac{1}{2\pi i}\oint \frac{dg}{g^{2N+1}} \int_{-\infty}^{\infty} \frac{d\phi}{2\pi} \exp[-\frac{1}{2}\phi^2 + \frac{1}{6}g\phi^3]$$

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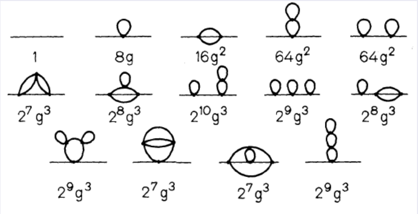

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Fat Graphs

Generalize to matrix model:

$$\int \mathrm{d}\phi\, e^{-\mathcal{N}\mathrm{Tr}V(\phi,g)},$$

with, e.g., $V(\phi,g) = \frac{1}{2}\phi^2 - \frac{g}{3}\phi^3$ or $V(\phi,g) = \frac{1}{2}\phi^2 - \frac{g}{4}\phi^4$ and for $\mathcal{N} \to \infty$.

- well-defined topology (i.e., planar)
- fixed co-ordination number
- *not* power-law
- model of Euclidean quantum gravity
- fractal dimension $d_h = 4$

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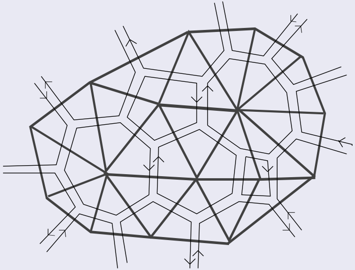

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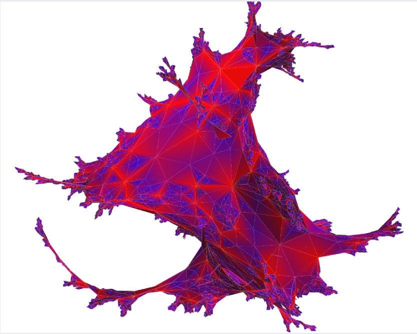

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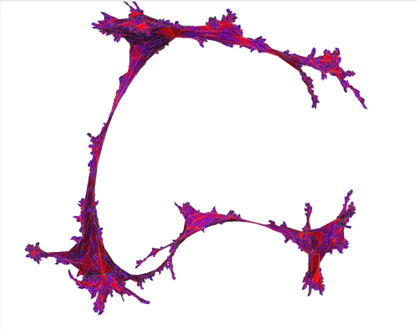

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Voronoï-Delaunay

Poissonian Voronoï-Delaunay triangulation:

- neighborhood construction from point set
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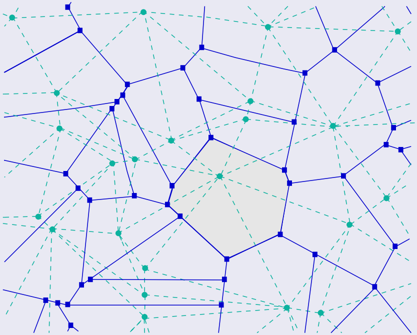

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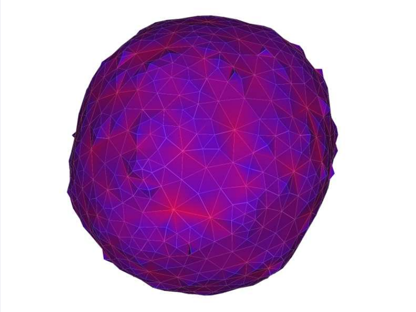

Spin models on random graphs

Measure of the "strength" of this geometric randomness: effect on Ising or Potts ferromagnet.

Generic random graphs

In general, one finds mean-field results. Well tractable with generating function techniques.

- deviations from mean-field, when moments of $P(k)$ diverge (Dorogovtsev et al., 2003)
- interesting dynamical phenomena for the non-static ensembles
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- mean-field like
- useful since tractable with field-theoretical methods
- sometimes more convenient than Bethe lattice, since no boundaries

(Johnston, Janke et al., 1994–2002)

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Weakest form of disorder considered here.

- apparently unchanged exponents for 2D, 3D Ising models (Janke et al., 1994–2002)
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Fat graphs

Can solve a series of matrix models.

annealed:

- the quantum gravity case
- exponents change according to KPZ/DDK formula

$$\tilde{\Delta} = \frac{\sqrt{1-c+24\Delta} - \sqrt{1-c}}{\sqrt{25-c} - \sqrt{1-c}}$$

- exponents change, but not to mean-field values
- many exact results: Ising, Potts, $O(n)$ models

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quenched:

- exponents should always change according to relevance criterion
- consistent with exact result for percolation
- consistent with numerical results for $q \leq 4$ Potts model (Janke, Johnston, Wernecke)
- first-order transitions of $q > 4$ Potts model softened to second order (Janke, Johnston)

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Spin models on random graphs

Effect on ferromagnets is rather weak. What about *antiferromagnets*, where frustration comes to play?

Regular lattices

- **square lattice**: Néel order with phase transition equivalent to the ferromagnet as seen from the Mattis transformation of one sub-lattice
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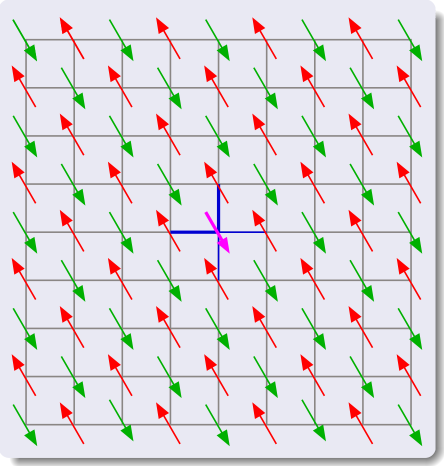

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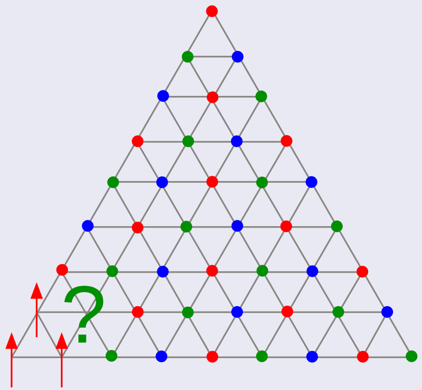

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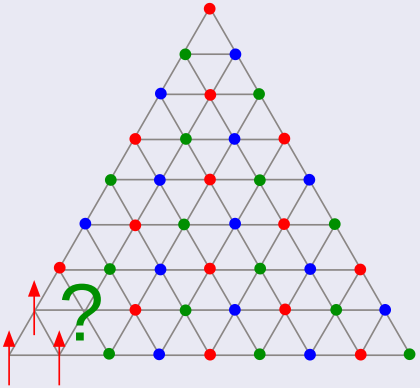
order depends on *bipartiteness*



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- **Voronoï-Delaunay**: not studied
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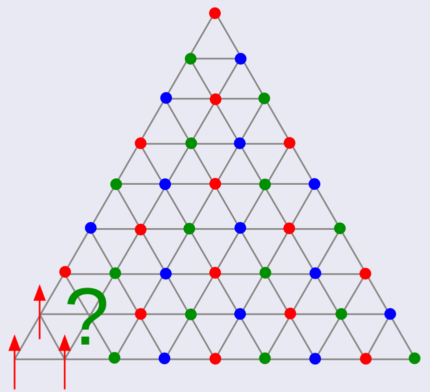

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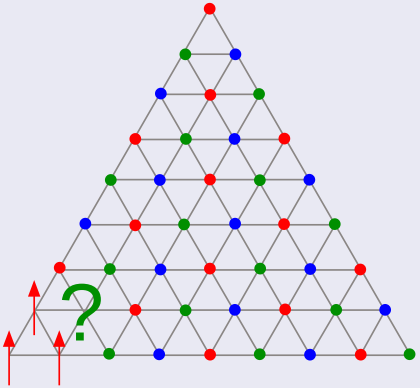

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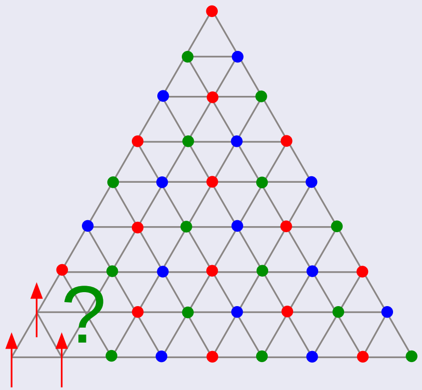

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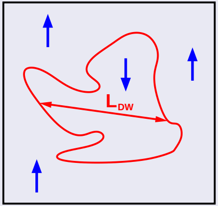
Fat graphs

Type	Bipartite	Annealed	Quenched
quadrangulations	✓	equivalent to FM	equivalent to FM
triangulations	–	all triangles are frustrated, even at $T = 0$ $\Rightarrow$ PM everywhere	PM everywhere
ϕ^4 graphs	–	ground state is square lattice with Néel order $\Rightarrow$ finite-T phase transition?	spin-glass order at $T = 0$?
ϕ^3 graphs	–	GS is honeycomb lat. with Néel order	spin-glass order at $T = 0$?

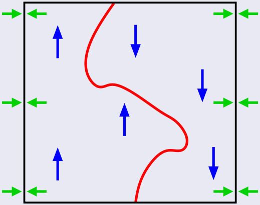
Spin stiffness and zero-temperature scaling

Ferromagnet

(Peierls)



$\Delta E \sim L^{d-1}$ resp. L^{d-2}



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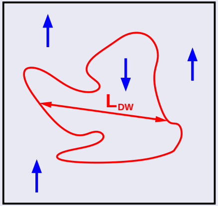

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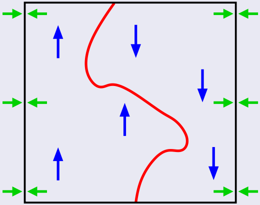

Spin glass

(Bray/Moore, 1987)

Distribution of couplings evolving under RG transformations, asymptotic width scales as

$$J(L) \sim JL^{\theta(d)}.$$

Spin-stiffness exponent θ determines lower critical dimension. For $\theta < 0$,

$$\xi \sim T^{-\nu}, \quad \nu = -1/\theta.$$

Numerically, θ can be determined from inducing droplets or domain walls with a change of *boundary conditions*,

$$\Delta E = |E_{\mathrm{AP}} - E_{\mathrm{P}}| \sim L^{\theta}.$$

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2D Ising

- ground-state problem is polynomial $\rightarrow$ large systems tractable

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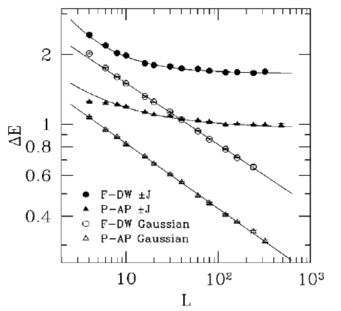

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- ground-state problem is polynomial $\rightarrow$ large systems tractable, $\theta \approx -0.28$ resp. $\theta = 0$

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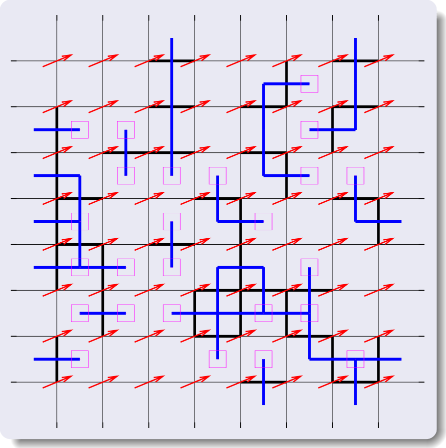
Ising ground states as perfect matchings

System energy equals total weight of energy strings pairing frustrated plaquettes (Toulouse, 1977),

$$E = -\sum_{\text{strings}} |J_{ij}| + \text{const.}$$



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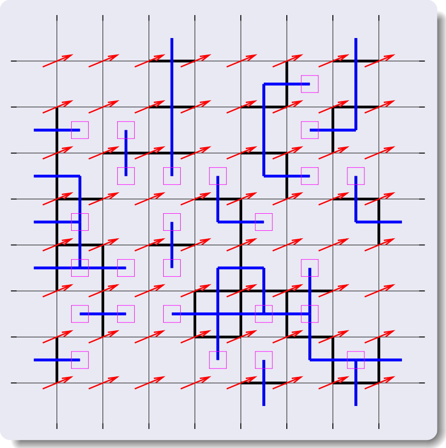

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- matching solution always corresponds to spin configuration for planar graphs
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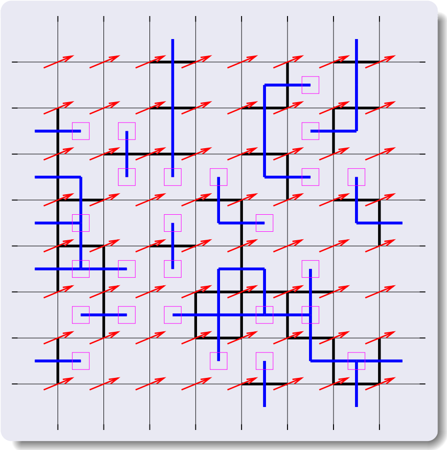

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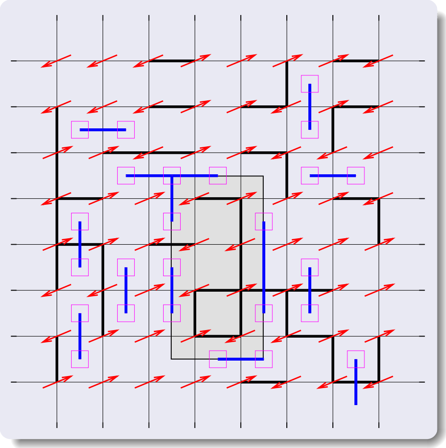

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Problem: how to generate fat graphs with boundaries?

Homotopy

Possible solutions:

- keep torus boundaries fixed (we start out with a hexagonal lattice) $\Rightarrow$ strong boundary effects
- keep track of the boundaries as we update $\Rightarrow$ becomes very ugly
- find the boundaries from intrinsic properties: compute an *optimal homotopy basis* (Erickson, Whittlesley, 2005), possible in $O(n^2 \log n)$ time

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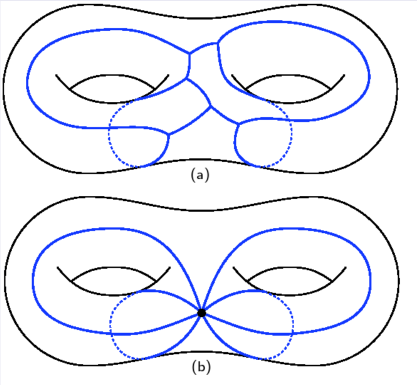

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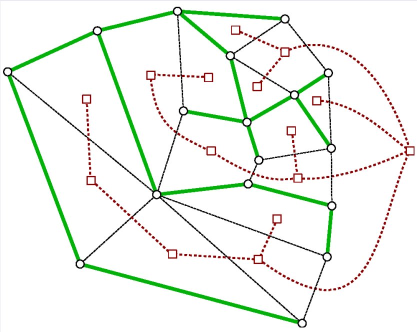
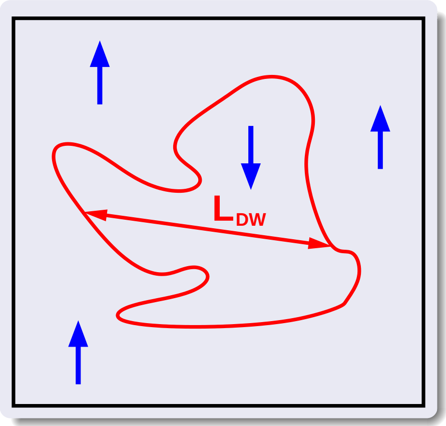


Length scales

What is the length scale relevant for domain-wall defects?



Domain-wall scaling

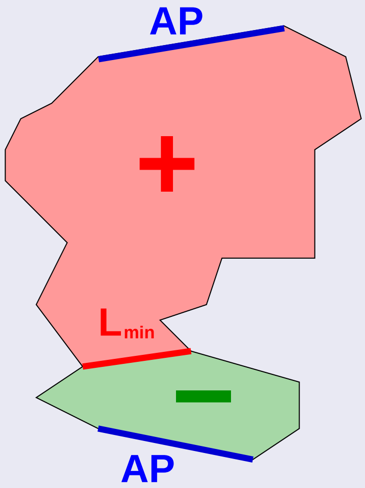
Problem of non-trivial Hausdorff dimension. Domain-wall energies should scale as

$$|\Delta E_{\mathrm{DW}}| \sim L_{\mathrm{DW}}^{\theta_s} \overset{?}{\sim} L_{\mathrm{eff}}^{\theta_s} \sim N^{\theta_s/d_h}$$

But scaling of L_{DW} can be determined from the ferromagnet: $L_{\min} \leq L_{\mathrm{DW}}$.

Length scales

What is the length scale relevant for domain-wall defects?



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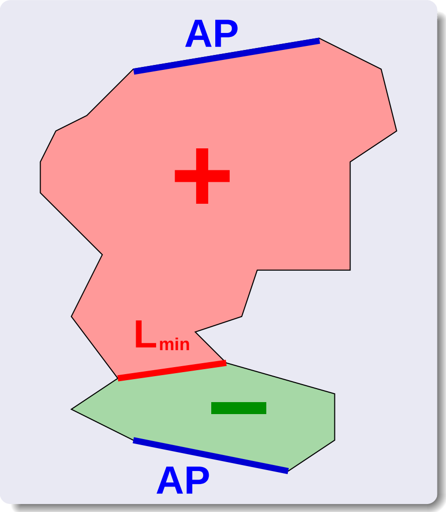

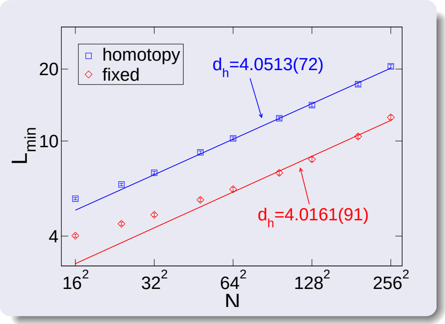

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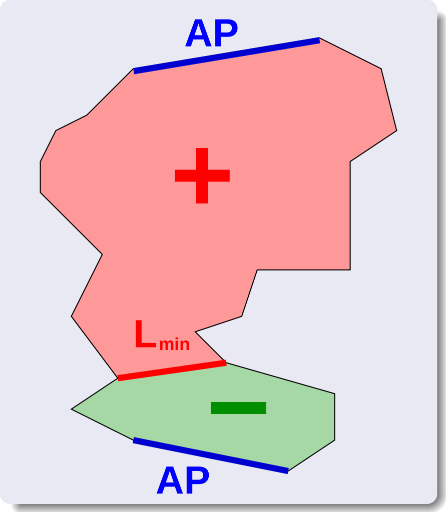
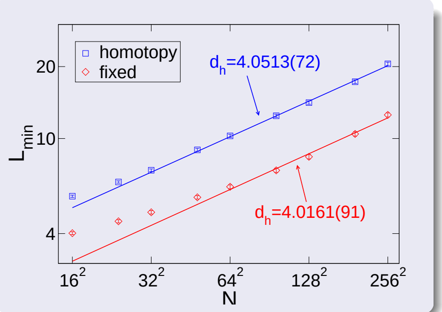


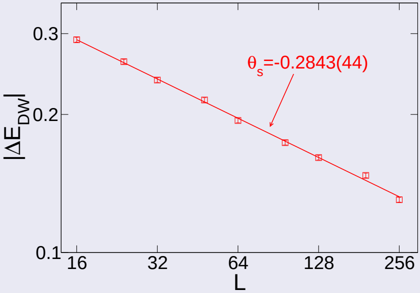


$L_{\mathrm{min}} \sim N^{1/d_h}$ with far less corrections than conventional approach!

Spin-stiffness exponent

A benchmark and universality test: the honeycomb lattice

Gaussian bond distribution



$$|\Delta E_{\mathrm{DW}}(L)| = E_0 + S_E L^{\theta_s}$$

$E_0 = 0$: $\theta_s = -0.2843(44)$, $Q = 0.29$

$E_0 \neq 0$: $\theta_s = -0.288(40)$
$E_0 = 0.0026(261)$, $Q = 0.20$

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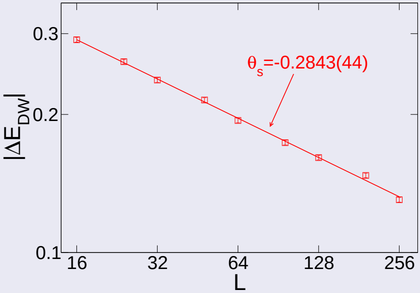
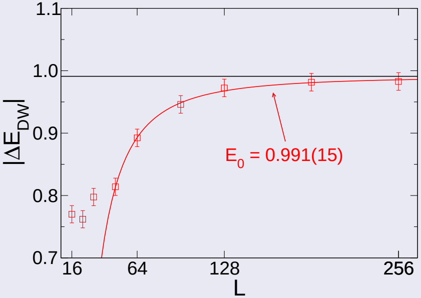


Bimodal bond distribution



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$E_0 = 0.0026(261)$, $Q = 0.20$

$E_0 = 0.991(15)$, $\theta_s' = -2.06(59)$, $Q = 0.98$

$\Rightarrow \theta_s^{\mathrm{eff}} = 0$

KPZ exercise

Spin stiffness for the random-lattice case.

KPZ/DDK

If the spin glass on a regular lattice corresponds to a $c = 0$ CFT, then

$$\tilde{\Delta} = \frac{\sqrt{1 + 24\Delta} - 1}{4}.$$

Conjecture for θ_s/d_h:

Bonds	Regular	KPZ $c = 0$
$\pm J$	0	0
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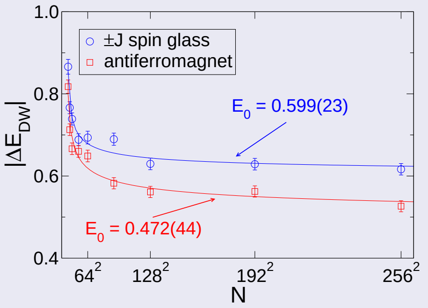

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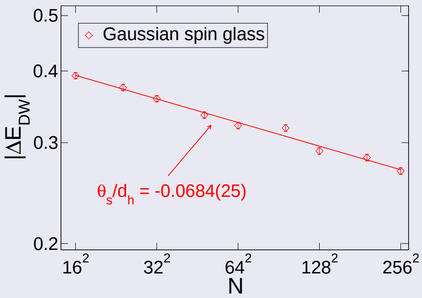

Domain-wall fractal dimension

Domain walls are fractal curves (SLE curves, actually).

Link overlap

$$q_l = \frac{1}{N_1} \sum_{\langle i,j \rangle} (\boldsymbol{S}_i^{(1)} \cdot \boldsymbol{S}_j^{(1)})(\boldsymbol{S}_i^{(2)} \cdot \boldsymbol{S}_j^{(2)}),$$

should scale as

$$[1 - q_l]_J \sim N^{-(1-d_s/d_h)},$$

Conjecture for d_s/d_h:

Bonds	Regular	KPZ $c = 0$
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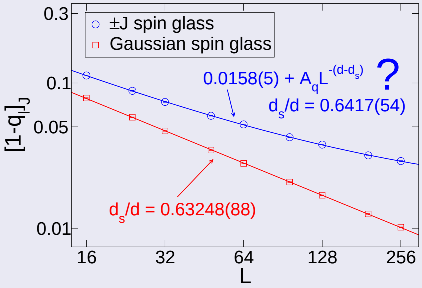

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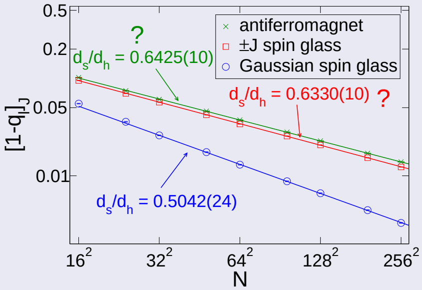
ϕ^3 graphs



Conclusions

- connectivity disorder from random graphs has much more dramatic effects on antiferromagnets than on ferromagnets
- the Ising antiferromagnet on the fat ϕ^3 graphs of the dynamical triangulations model shows $T = 0$ spin-glass order in the universality class of the $\pm J$ spin glass
- the quenched approximation to the KPZ/DDK formula is certainly not exact, but indicates the right tendency
- the ferromagnet with antiperiodic boundaries opens up a new method to precisely determine the Hausdorff dimension d_h

Outlook:

- determine the domain-wall fractal dimension for the discrete cases more properly (see O. Melchert's talk)
- annealed cases:
  - do they show Néel order?
  - finite-temperature transitions?
  - which universality class? relation to KPZ/DDK?
- all the other graphs ...