Diffusion of the transverse magnetization profil in quantum spin chains

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- ◊ The quantum Ising and XX-model
- ◊ The initial state
- ◊ Expectation values
- ◊ The kink like initial state
- ◊ The droplet relaxation

The XX and Ising model

$$\mathcal{H}^{XX} = -\frac{1}{4}\sum_{n=1}^{L-1}\left[\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right] - \frac{h}{2}\sum_{n=1}^{L}\sigma_n^z$$

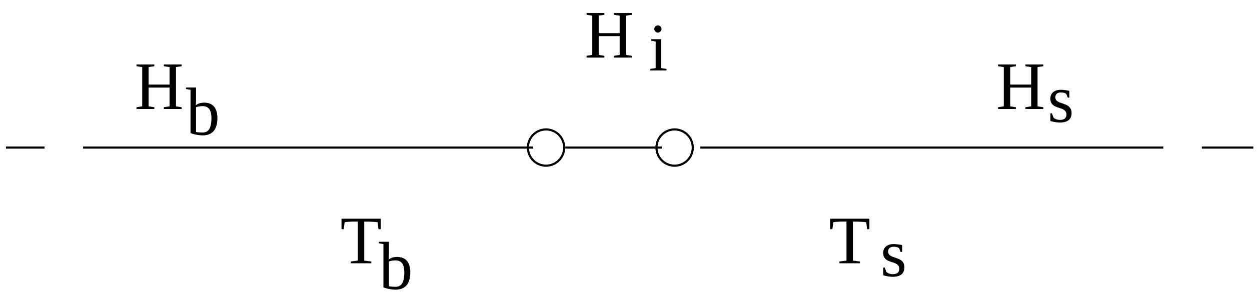
$M^z = \sum_n \sigma_n^z \qquad [\mathcal{H}^{XX}, M^z] = 0$ conserved quantity

$$\mathcal{H}^{Ising} = -\frac{1}{2}\sum_{n=1}^{L-1}\sigma_n^x\sigma_{n+1}^x - \frac{h}{2}\sum_{n=1}^{L}\sigma_n^z$$

$[\mathcal{H}^{Ising}, M^z] \neq 0$ non conserved quantity

Factorized initial state

$$\rho(0) = \rho_b(0)\rho_s(0), \qquad \rho_j(0) = \frac{1}{Z_j} e^{-\beta_j \mathcal{H}_j}$$



Expectation values

The expectation value of an observable $\mathcal{O}$ at time t

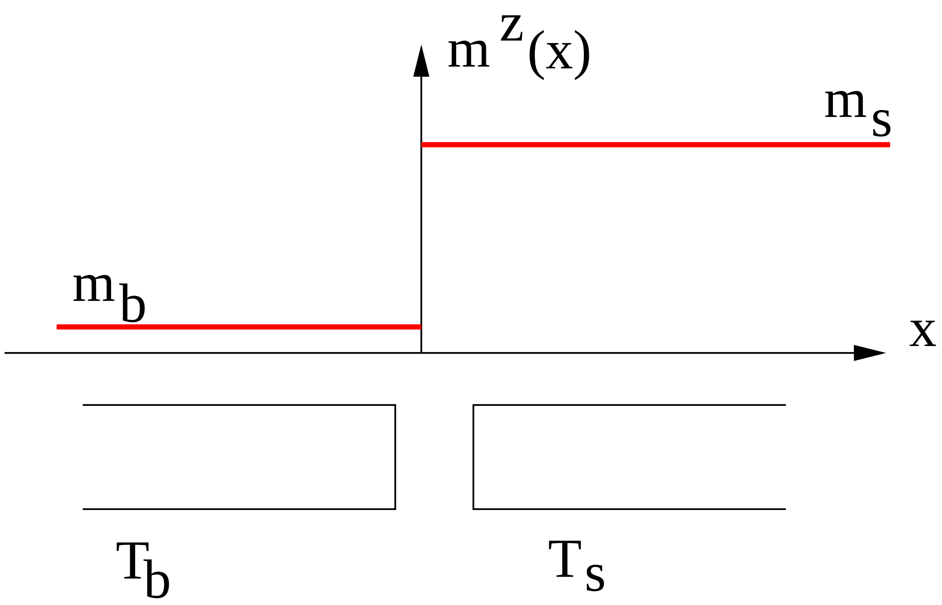
$$\langle \mathcal{O}(t) \rangle = Tr\{\rho(0)\mathcal{O}(t)\} = Tr\{\rho(t)\mathcal{O}(0)\}$$

The transverse magnetization $m^z(n,t) = \langle \sigma_n^z(t) \rangle$

$$m^z(x,t) = (G_t * m^z(0))\,(x)$$

where $G_t(x)$ is the Green function associocieted to m^z.

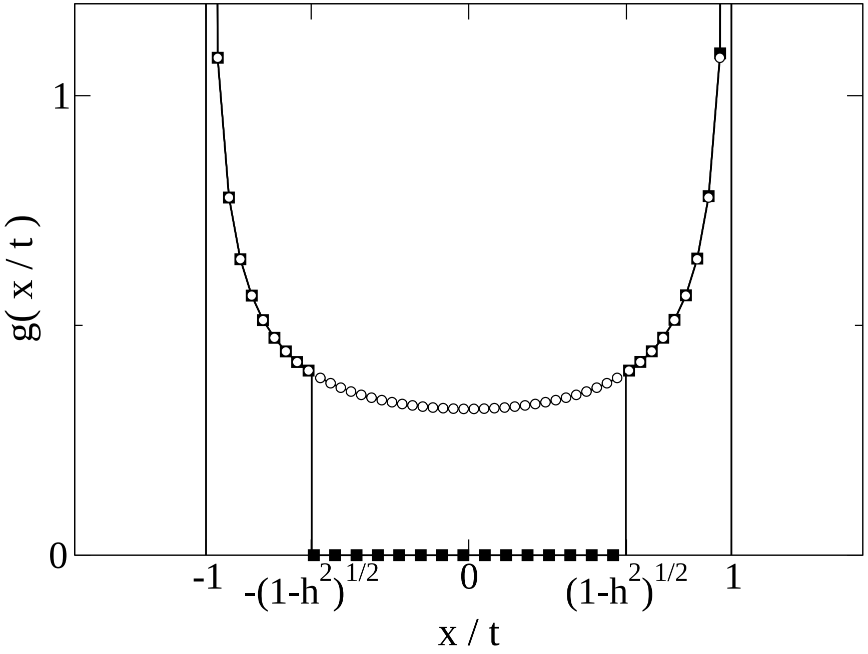
The System-Bath situation



$$
\begin{aligned}
m^z(x,t) &= (G_t * m^z(0)H)(n) \\
\partial_x m^z(x,t) &= (G_t * m^z(0)H')(x) = m^z(0)G_t(x)
\end{aligned}
$$

The zero temperature case $T_s = 0$

$$G_t(x) = \frac{1}{t} g\left(\frac{x}{t}\right)$$



In the region $h < 1$.

$$g\left(u\right) = \frac{1}{2 \arcsin(h) \sqrt{1 - u^2}},$$

$$\sqrt{1 - h^2} < |u| < 1$$

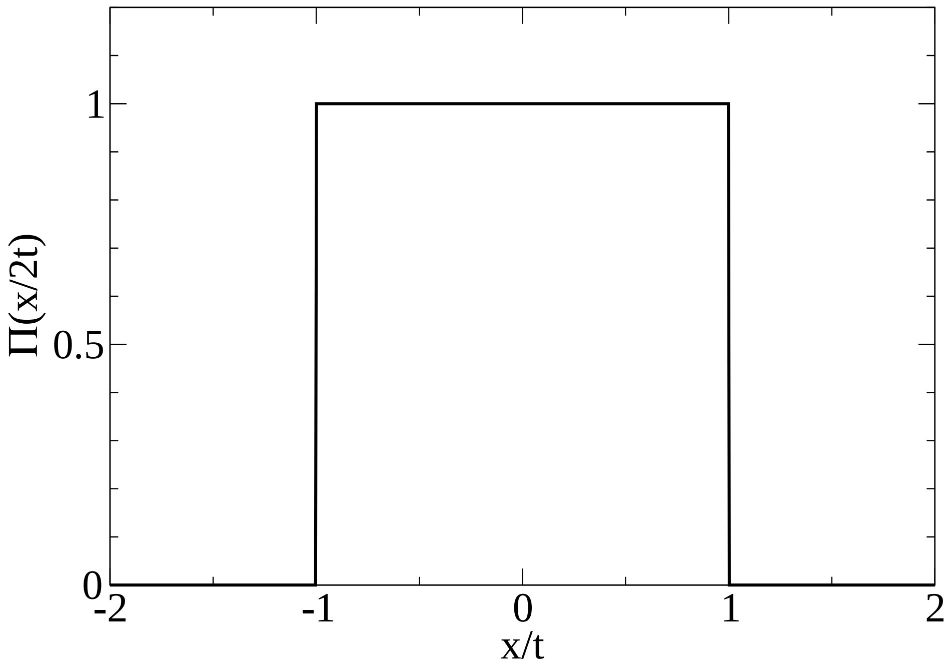
In the region $h \geq 1$.

$$g(u) = \frac{1}{\pi \sqrt{1 - u^2}}, \qquad |u| < 1$$

The zero temperature case $T_s = 0$

In the Ising model, at the critical poit $h = 1$.

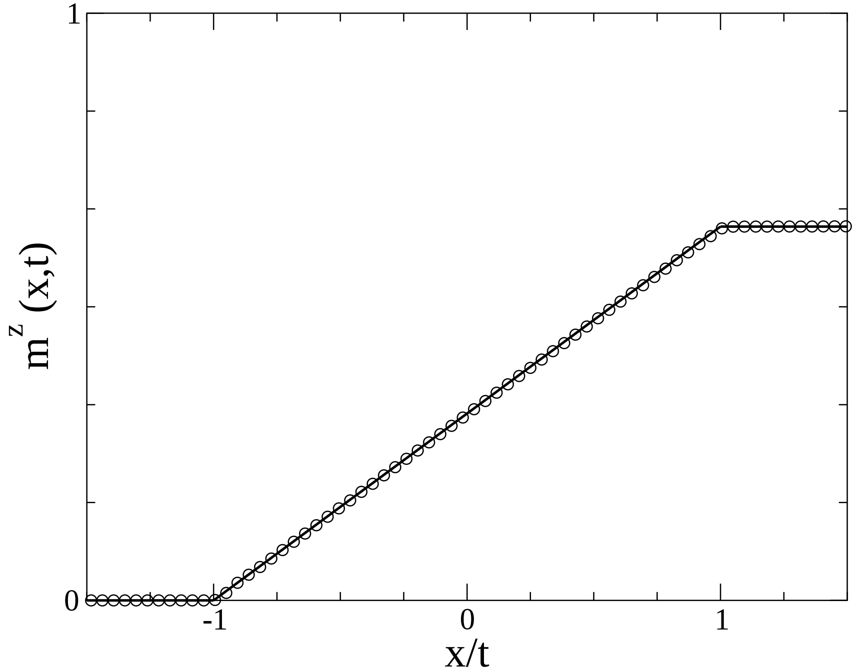
$$G_t(x) = \frac{1}{2t}\Pi\left(\frac{x}{2t}\right)$$

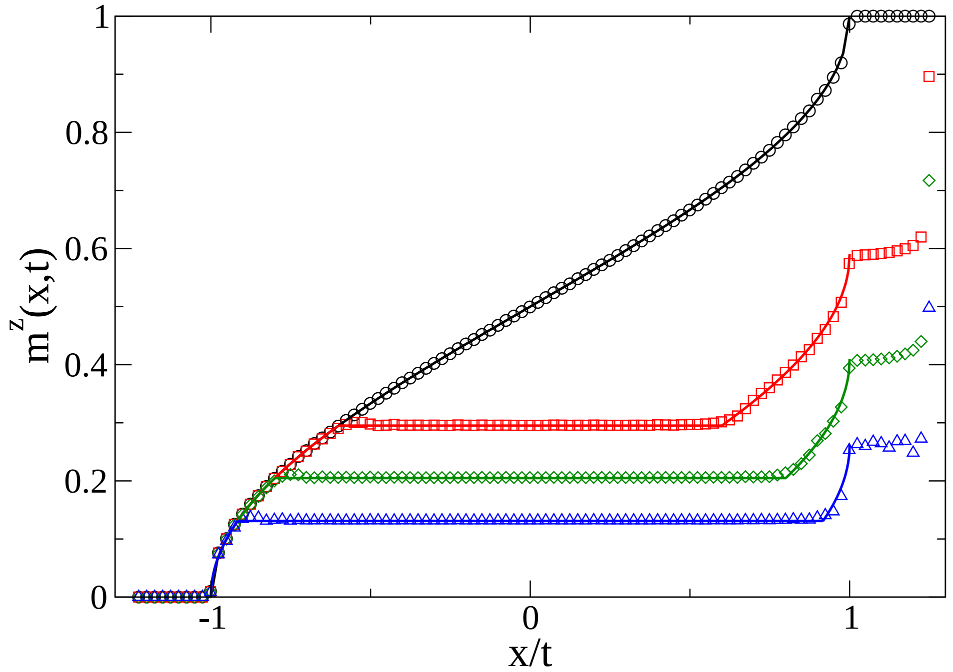


T. Platini D. Karevski, *Eur. Phys. J.* **B27**, 147 (2005).

The kink situation

$$m^z(x,t) = (G_t * m^z(0)H)(x) = \Phi(x/t) + m^z(0)/2$$





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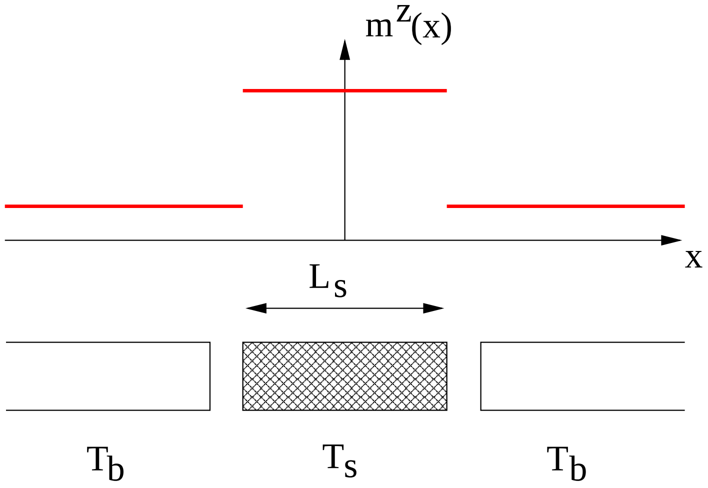
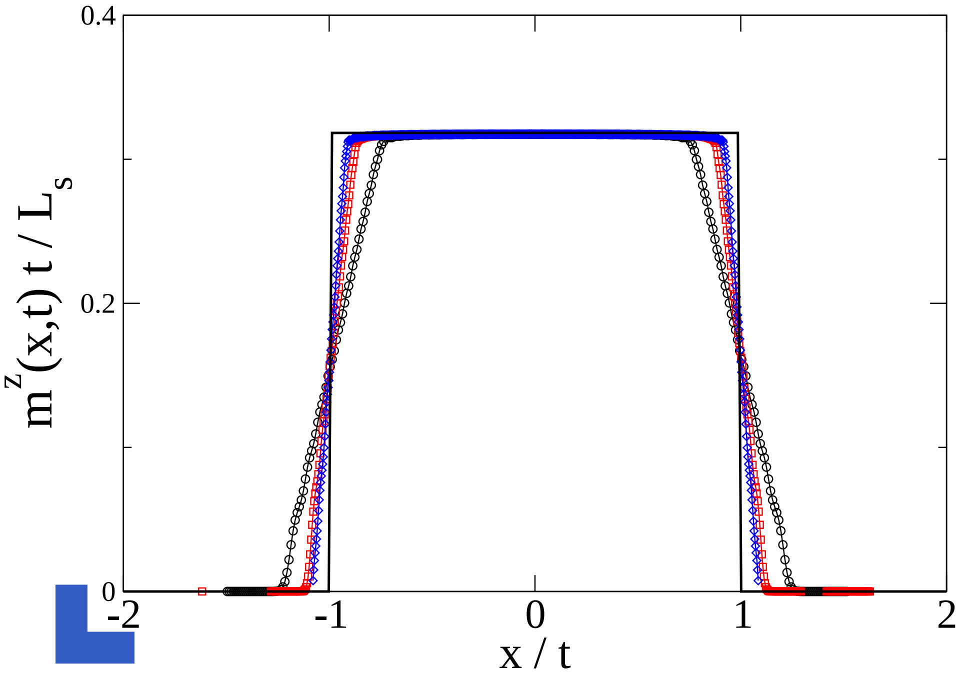
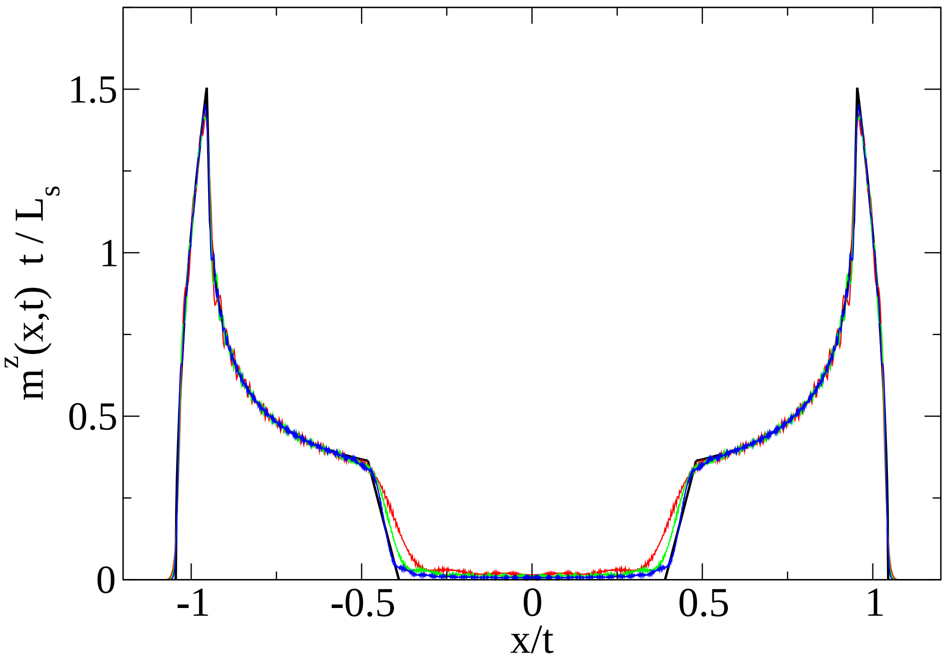
Droplet relaxation



$$m^z(x,t) \sim \frac{m^z(0) L_s}{t} g\left(\frac{x}{t}\right)$$





Droplet relaxation

$$m^z(t) = \frac{1}{L_s} \sum_{k=-L_s/2}^{L_s/2} m^z(k,t)$$

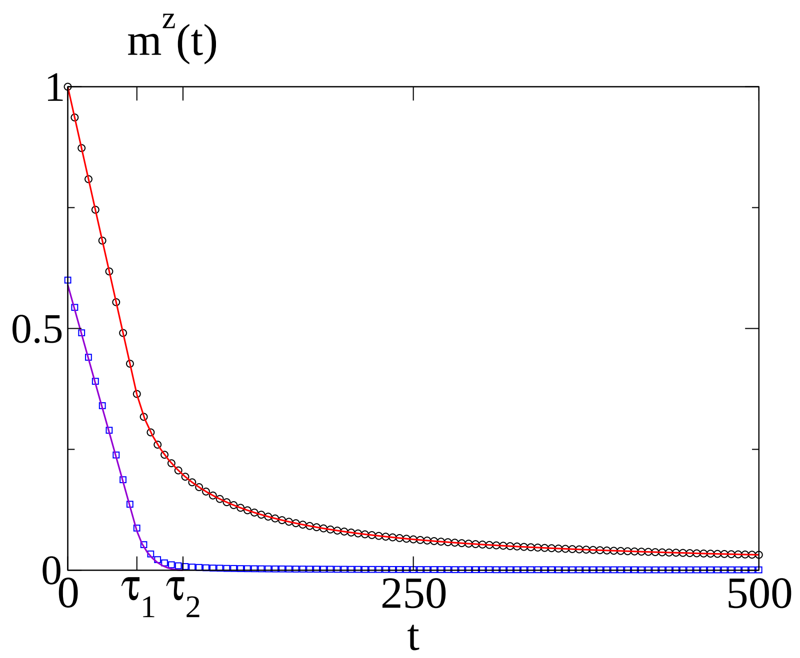
For the XX chain.

$$m^z(t) \simeq \frac{L_s}{\pi t} \qquad h \geq 1$$

$$m^z(t) \simeq \mathcal{O}\left(\frac{1}{t}\right) \qquad h < 1$$

For the Ising chain.

$$m^z(t) \simeq \frac{L_s}{\pi t} \qquad h = 1$$



Summary

An exact expression of the Green function for the XX and Ising model.

$$G_t(x) = \frac{1}{t} g\left(\frac{x}{t}\right)$$

$\Diamond$ Anisotropic XY-model ? $G_t(x)$?

$\Diamond$ Influence of disorder on the relaxation properties ?

$\Diamond$ Two-point functions $\langle \sigma^z(t)\sigma^z(s)\rangle$? Ageing ?