Operator product expansion on the lattice: analytic Wilson coefficients

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Outline

Introduction

Lattice OPE and structure functions

Calculation

Results

0^{th} order

1^{st} order

Conclusions

Introduction
Lattice OPE and structure functions
Calculation
Results
0^{th} order
1^{st} order
Conclusions

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Results

0^{th} order

1^{st} order

Conclusions

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Calculation

Results

0^{th} order

1^{st} order

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1^{st} order

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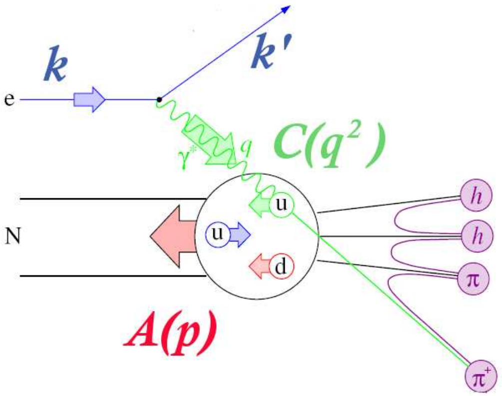
DIS, structure functions, OPE

Scheme of deep inelastic scattering



Lattice OPE

Moments of structure functions ($\mathcal{M}_n(q^2)$) are related to operator product expansion (OPE) of the product of two conserved vector currents between states of particles

$$\mathcal{M}_n(q^2) = C_n^{(2)}(a,q)\, A_n^{(2)}(a) + \frac{C_n^{(4)}(a,q)}{q^2}\, A_n^{(4)}(a) + \ldots,$$

$A_n^{(2)}$, $A_n^{(4)}$: reduced nucleon matrix elements of local operators of twist-two and four
$C_n^{(2)}$, $C_n^{(4)}$: Wilson coefficients

$A_n^{(i)}$: computed on the lattice
$C_n^{(i)}$: usually computed perturbatively in $\overline{MS}$-scheme

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Problems:

- Operator mixing (higher and leading twist) on the lattice
- Renormalon contributions: coefficients of leading twist operators and expectation values of higher twist operators $\rightarrow$ cancel in the complete OPE sum if calculated in the same framework: lattice

$\longrightarrow$ it is recommended to compute the Wilson coefficients on the lattice as well
$\rightarrow$ need to control possible lattice artefacts ($\mathcal{O}(a^2)$ -effects)

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Wilson coefficients and $\mathcal{O}(a^2)$ - effects

Reduced Wilson coefficients $c(q^2)$:

$$C(a,q) = c(q^2)\, C_{BORN}(a,q)\,,$$

On the lattice both the non-perturbative $C(a,q)$ and $C_{BORN}(a,q)$ have corrections of $O(a^2)$:

$$\begin{aligned} C_{BORN}(a,q) &= C^{(0)}_{BORN} + (aq)^2\, C^{(2)}_{BORN} + \dots \\ C(a,q) &= c^{(0)}(q^2)\, C^{(0)}_{BORN} + (aq)^2\, c^{(2)}(q^2)\, C^{(2)}_{BORN} + \dots\,, \end{aligned}$$

Taking the the ratio

$$\frac{C(a,q)}{C_{BORN}(a,q)} = c^{(0)}(q^2) + \left(c^{(2)}(q^2) - c^{(0)}(q^2)\right) \frac{C^{(2)}_{BORN}}{C^{(0)}_{BORN}}\,(aq)^2 + \dots\,,$$

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Symbolic calculation of $C^{(2)}_{BORN}$

Starting point:
tree level Compton scattering amplitude $\mathcal{W}_{\mu\nu}(a,p,q)$ with off-shell quark states of momentum p in Wilson's formulation:

$$\begin{aligned}\mathcal{W}_{\mu\nu}(a,p,q) &= \langle p|\hat{J}_\mu(q)\hat{J}^\dagger_\nu(q)|p\rangle \\ &= \langle p|\mathcal{T}_{\mu\nu}|p\rangle \\ &= \sum_{m,n} C^m_{\mu\nu,\mu_1,\ldots,\mu_n}(aq)\,\langle p\,|\mathcal{O}(a)^m_{\mu_1,\ldots,\mu_n}|p\rangle\,.\end{aligned}$$

- Expansion into powers of $p_\nu\,(\leftrightarrow \overleftrightarrow{D}_\nu)$ up to third order (restriction due to numerical simulations)
- Identification of the possible local operators $\mathcal{O}(a)^m_{\mu_1,\ldots,\mu_n}$ contributing to $\mathcal{W}_{\mu\nu}(a,p,q)$
- Unpolarised case: $\bar\psi 1\psi\,,\ \bar\psi\gamma_\mu \overleftrightarrow{D}_\nu\psi\,,\ \bar\psi \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu\psi\,,\ \bar\psi\gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\omega \overleftrightarrow{D}_\rho\psi$
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- Expansion of the operators into the bases of irreducible representations of hypercubic group H(4)
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Scattering amplitude in 0^{th} order

The Compton scattering amplitude can be given in general form:

$$
\begin{aligned}
\mathcal{T}^{(0)}_{\mu\nu}(a,q)/e_\gamma^2 \quad = \quad & -a\,r\,\delta_{\mu\nu}\,\bar\psi 1\psi - \frac{8\,a\,r\,\cos(aq_\mu/2)^2}{Q^2}\,\delta_{\mu\nu}\,\bar\psi 1\psi + \\
& \sum_\tau \frac{2\,a\,r\ \cos(aq_\mu/2)^2\cos(aq_\tau)}{Q^2}\,\delta_{\mu\nu}\,\bar\psi 1\psi + \\
& \frac{8\,a\,r^3\ \sin(aq_\mu/2)\sin(aq_\nu/2)}{Q^2}\,\bar\psi 1\psi - \\
& \sum_\tau \frac{2\,a\,r^3\ \cos(aq_\tau)\sin(aq_\mu/2)\sin(aq_\nu/2)}{Q^2}\,\bar\psi 1\psi + \\
& \frac{2\,a\,r\ \cos(aq_\mu/2)\sin(aq_\nu/2)\sin(aq_\mu)}{Q^2}\,\bar\psi 1\psi + \\
& \frac{2\,a\,r\ \cos(aq_\nu/2)\sin(aq_\mu/2)\sin(aq_\nu)}{Q^2}\,\bar\psi 1\psi + \\
& \sum_{\tau,\sigma} \frac{2\,i\,a\ \cos(aq_\mu/2)\cos(aq_\nu/2)\sin(aq_\sigma)}{Q^2}\,\bar\psi\gamma_\tau\gamma_5\epsilon_{\mu\sigma\nu\tau}\psi\,,
\end{aligned}
$$

with

$$
Q^2 = Q^2(a,q) = \sum_\tau \sin(aq_\tau)^2 + r^2\left(\sum_\tau(1-\cos(aq_\tau))\right)^2 .
$$

Wilson coefficients: 0^{th} order

In order to save space we give the following results for the special momentum transfer
$q = (f, f, f, f), \quad s = \sin f, \quad c = \cos f, \quad Q_f^2 = 4s^2 + 16(1-c)^2$.
(If not shown explicitly we set $a = 1$ and $r = 1$.)

Diagonal part ($\mathcal{T}_{11}^{(0)}$):

operator	representation	Wilson coefficient	a expansion
$\bar{\psi} 1 \psi$	$\tau_1^{(1)}$	$-\frac{6(3-c)(1-c)}{Q_f^2}$	$-\frac{3a}{2}(1 - \frac{5}{12}(af)^2)$

Off-diagonal part ($\mathcal{T}_{12}^{(0)}$):

operator	representation	Wilson coefficient	a expansion
$\bar{\psi} 1 \psi$	$\tau_1^{(1)}$	$\frac{2(3-c)(1-c)}{Q_f^2}$	$\frac{a}{2}(1 - \frac{5}{12}(af)^2)$
$\bar{\psi}\gamma_3\gamma_5\psi, -\bar{\psi}\gamma_4\gamma_5\psi$	$\tau_4^{(4)}$	$\frac{i(1+c)s}{Q_f^2}$	$\frac{i}{2f}(1 - \frac{13}{12}(af)^2)$

Wilson coefficients: 1^{st} order, diagonal part

The expansion up to one covariant derivative cannot be given in readable form. Here, I present the Wilson coefficients only.

$\mathcal{T}_{11}^{(1)}$:

Defining the combinations

$$\begin{aligned} b_1 &= 4i(1-c)(74-126c+63c^2-9c^3)/Q_f^4 \\ b_2 &= -4i(6-8c+3c^2)s^2/Q_f^4 \\ b_3 &= 4i(4-3c)s^2/Q_f^4 \\ b_4 &= -4i(1-c)(4-9c+3c^2)/Q_f^4 \end{aligned}$$

operator	repr.	Wilson coeff.	a expansion
$\frac{1}{2}(\mathcal{O}_{11}+\mathcal{O}_{22}+\mathcal{O}_{33}+\mathcal{O}_{44})$	$\tau_1^{(1)}$	$\frac{1}{2}(b_1+3b_3)$	$\frac{i}{2f^2}(1+\frac{29}{24}(af)^2)$
$\frac{1}{2}(\mathcal{O}_{11}+\mathcal{O}_{22}-\mathcal{O}_{33}-\mathcal{O}_{44})$	$\tau_1^{(3)}$	$\frac{1}{2}(b_1-b_4)$	$\frac{15i\,a^2}{16}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{11}-\mathcal{O}_{22})$	$\tau_1^{(3)}$	$\frac{1}{2}(b_1-b_4)$	$\frac{15i\,a^2}{16}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{12}+\mathcal{O}_{21})$, $\frac{1}{\sqrt{2}}(\mathcal{O}_{13}+\mathcal{O}_{31})$	$\tau_3^{(6)}$	$\frac{1}{\sqrt{2}}(b_2+b_3)$	$\frac{i\,a^2}{8\sqrt{2}}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{14}+\mathcal{O}_{41})$	$\tau_3^{(6)}$	$\frac{1}{\sqrt{2}}(b_2+b_3)$	$\frac{i\,a^2}{8\sqrt{2}}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{23}+\mathcal{O}_{32})$, $\frac{1}{\sqrt{2}}(\mathcal{O}_{24}+\mathcal{O}_{42})$	$\tau_3^{(6)}$	$\sqrt{2}b_3$	$\frac{i}{2\sqrt{2}f^2}(1-\frac{1}{6}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{34}+\mathcal{O}_{43})$	$\tau_3^{(6)}$	$\sqrt{2}b_3$	$\frac{i}{2\sqrt{2}f^2}(1-\frac{1}{6}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{12}-\mathcal{O}_{21})$, $\frac{1}{\sqrt{2}}(\mathcal{O}_{13}-\mathcal{O}_{31})$	$\tau_1^{(6)}$	$\frac{1}{2}(b_2-b_3)$	$-\frac{i}{4f^2}(1-\frac{5}{12}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{14}-\mathcal{O}_{41})$	$\tau_1^{(6)}$	$\frac{1}{2}(b_2-b_3)$	$-\frac{i}{4f^2}(1-\frac{5}{12}(af)^2)$

$(\mathcal{O}_{\mu\nu} = \bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi)$

Wilson coefficients: 1^{st} order, off-diagonal part

$\mathcal{T}_{12}^{(1)}$:

We find the combinations

$$
\begin{aligned}
b_1 &= -4i(1-c)(33-52c+24c^2-3c^3)/Q_f^4 \\
b_2 &= 4i(7-9c+3c^2)s^2/Q_f^4 \\
b_3 &= -4i(3-2c)s^2/Q_f^4 \\
b_4 &= 6i(1-c)^2(1+c)(2-c)/Q_f^4 \\
b_5 &= 2i(1-c)^2(4-9c+3c^2)/Q_f^4 \\
b_6 &= 2i(1-c)^2(1+c)(4-3c)/Q_f^4 \\
b_7 &= -4(1-c)(19-18c+3c^2)s/Q_f^4 \\
b_8 &= 2(1-c)(14-15c+3c^2)s/Q_f^4 \\
b_9 &= 12(2-c)s^3/Q_f^4 \\
b_{10} &= -4s^3/Q_f^4 \\
b_{11} &= -2(1-c)(4-9c+3c^2)s/Q_f^4 \\
b_{12} &= -2(4-3c)s^3/Q_f^4
\end{aligned}
$$

Wilson coefficients: 1^{st} order, off-diagonal part

We give some selected examples for the Wilson coefficients

operator	repr.	Wilson coeff.	a expansion
$\frac{1}{2}(\mathcal{O}_{11}+\mathcal{O}_{22}+\mathcal{O}_{33}+\mathcal{O}_{44})$	$\tau_1^{(1)}$	b_1+b_5	$-\frac{i}{4f^2}(1+\frac{25}{12}(af)^2)$
$\frac{1}{2}(\mathcal{O}_{11}+\mathcal{O}_{22}-\mathcal{O}_{33}-\mathcal{O}_{44})$	$\tau_1^{(3)}$	b_1-b_5	$-\frac{i}{4f^2}(1+\frac{19}{12}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{12}+\mathcal{O}_{21})$	$\tau_3^{(6)}$	$\sqrt{2}b_2$	$\frac{i}{2\sqrt{2}f^2}(1-\frac{1}{6}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{13}+\mathcal{O}_{31}),\ \frac{1}{\sqrt{2}}(\mathcal{O}_{14}+\mathcal{O}_{41})$	$\tau_3^{(6)}$	$\frac{1}{\sqrt{2}}(b_3+b_4)$	$\frac{i}{4\sqrt{2}f^2}(1+\frac{1}{12}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{23}+\mathcal{O}_{32}),\ \frac{1}{\sqrt{2}}(\mathcal{O}_{24}+\mathcal{O}_{42})$	$\tau_3^{(6)}$	$\frac{1}{\sqrt{2}}(b_3+b_4)$	$\frac{i}{4\sqrt{2}f^2}(1+\frac{1}{12}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{34}+\mathcal{O}_{43})$	$\tau_3^{(6)}$	$\sqrt{2}b_6$	$\frac{ia^2}{8\sqrt{2}}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{13}-\mathcal{O}_{31}),\ \frac{1}{\sqrt{2}}(\mathcal{O}_{14}-\mathcal{O}_{41})$	$\tau_1^{(6)}$	$\frac{1}{\sqrt{2}}(b_3-b_4)$	$\frac{i}{4\sqrt{2}f^2}(1-\frac{17}{12}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}_{23}-\mathcal{O}_{32}),\ \frac{1}{\sqrt{2}}(\mathcal{O}_{24}-\mathcal{O}_{42})$	$\tau_1^{(6)}$	$\frac{1}{\sqrt{2}}(b_3-b_4)$	$\frac{i}{4\sqrt{2}f^2}(1-\frac{17}{12}(af)^2)$
$\sqrt{\frac{2}{3}}(\mathcal{O}^T_{1\{23\}}+\mathcal{O}^T_{2\{13\}})$	$\tau_2^{(8)}$	$\sqrt{\frac{2}{3}}(2b_{10}+b_9)$	$\frac{a}{2\sqrt{6}f}(1-\frac{4}{3}(af)^2)$
$\sqrt{\frac{2}{3}}(\mathcal{O}^T_{1\{34\}}+\mathcal{O}^T_{3\{14\}})$	$\tau_2^{(8)}$	$\sqrt{6}b_{12}$	$-\frac{\sqrt{3}a}{4\sqrt{2}f}(1-\frac{4}{3}(af)^2)$
$\sqrt{2}\mathcal{O}^T_{2\{13\}},\ \sqrt{2}\mathcal{O}^T_{2\{14\}}$	$\tau_2^{(8)}$	$-\sqrt{6}b_9$	$-\frac{3\sqrt{3}a}{2\sqrt{2}f}(1-\frac{4}{3}(af)^2)$
$\sqrt{2}\mathcal{O}^T_{3\{14\}},\ -\sqrt{2}\mathcal{O}^T_{3\{24\}}$	$\tau_2^{(8)}$	$-\sqrt{2}b_{12}$	$\frac{a}{4\sqrt{2}f}(1-\frac{4}{3}(af)^2)$
$-\frac{1}{\sqrt{6}}(\mathcal{O}^T_{122}+\mathcal{O}^T_{133}-2\mathcal{O}^T_{144})$	$\tau_1^{(8)}$	$\sqrt{2}(b_{11}+b_7)$	$-\frac{3a}{4\sqrt{2}f}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{2}}(\mathcal{O}^T_{211}+\mathcal{O}^T_{233}-2\mathcal{O}^T_{244})$	$\tau_1^{(8)}$	$\sqrt{2}b_{11}$	$\frac{a}{4\sqrt{2}f}(1-\frac{4}{3}(af)^2)$
$\frac{1}{6}\sum_{p\in(1,2,3)}\mathrm{sgn}(p)\mathcal{O}^T_{123}$	$\tau_4^{(4)}$	$\sqrt{\frac{2}{3}}(b_9-b_{10})$	$\sqrt{\frac{2}{3}}\frac{a}{f}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{3}}(\mathcal{O}^T_{122}+\mathcal{O}^T_{133}+\mathcal{O}^T_{144})$	$\tau_1^{(4)}$	$\frac{4}{\sqrt{3}}(4b_{11}-2b_7)$	$\frac{2\sqrt{3}a}{f}(1-\frac{4}{3}(af)^2)$
$\frac{1}{\sqrt{3}}(\mathcal{O}^T_{211}+\mathcal{O}^T_{233}+\mathcal{O}^T_{244})$	$\tau_1^{(4)}$	$-\frac{4}{\sqrt{3}}b_{11}$	$-\frac{a}{2\sqrt{3}f}(1-\frac{4}{3}(af)^2)$

$(\mathcal{O}_{\mu\nu}=\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu\psi,\ \mathcal{O}^T_{\mu\nu\omega}=\bar{\psi}\sigma_{\mu\nu}\overleftrightarrow{D}_\omega\psi)$

Conclusions

- We have developed a program for expanding tree-level scattering amplitudes symbolically
- We have found analytic expressions for Wilson coefficients corresponding to local operators up to third order in covariant derivatives
- We have classified them according to irreducible representations
- Next steps: O(a)-improved fermions, overlap fermions

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