SELF - AVOIDING WALKS ON FRACTALS: SCALING LAWS

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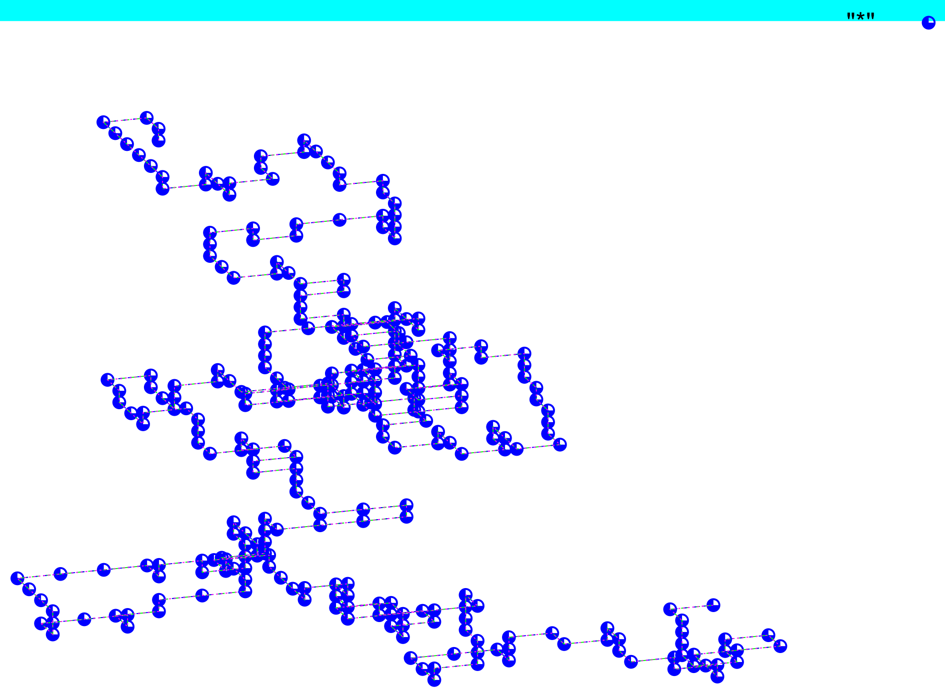
CONFIGURATIONAL PROPERTIES OF SAWS



End-to-end distance:

$$\langle R^2 \rangle \sim N^{2\nu}$$

$d = 3$: $\nu = 0.588$

Number of configurations:

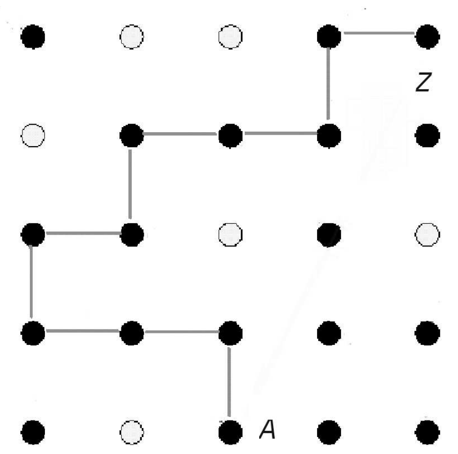
$$Z_N \sim z^N N^{\gamma - 1}$$

$\gamma = 1.159$ (Guida '98)

Fractal dimension of SAWs: $d_{SAW} = 1/\nu$

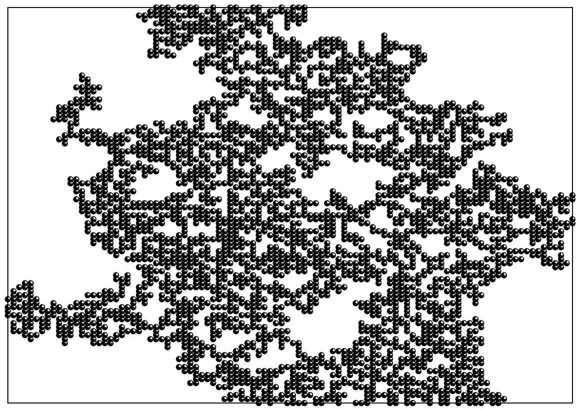
THE INFLUENCE OF DISORDER

Weak disorder, point-like defects



$\nu_{dil} = \nu_{pure}$(Harris '83, Kim'83).

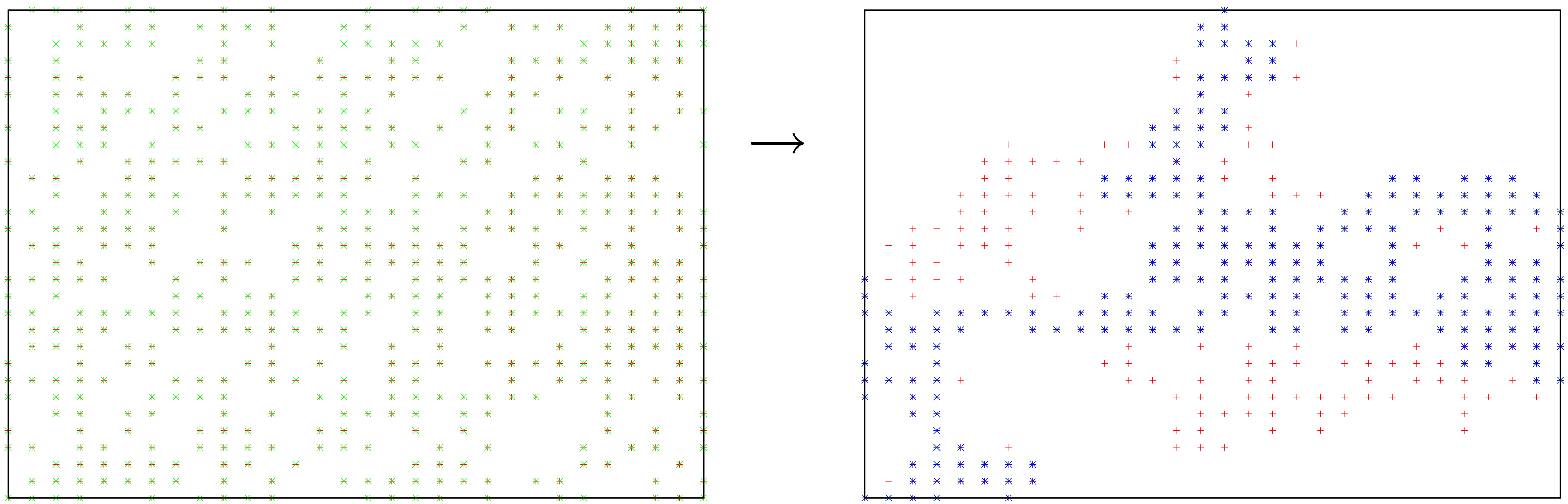
Percolation threshold, $p = p_c$



$\nu_{p_c} \neq \nu_{pure}, d_{p_c} \neq d$

RG: (V.B., von Ferber, Holovatch '04)

BACKBONE OF THE PERCOLATION CLUSTER

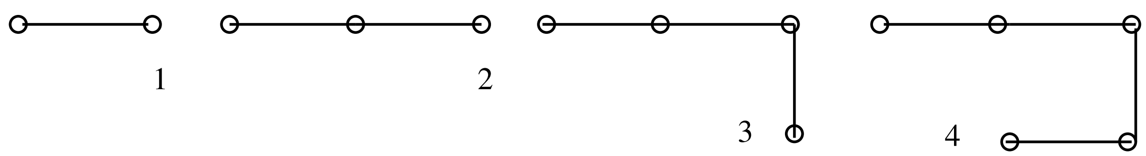


$d = 2 : p_c = 0.592$ (Ziff '94), $\quad d_B^f = 1.64$ (Porto'97)

$d = 3 : p_c = 0.312$ (Grassberger'92), $\quad d_B^f = 1.87$ (Porto '97)

$d = 4 : p_c = 0.197$ (Paul '01), $\quad d_B^f = 2.23$ (Paul '01)

PRUNED-ENRICHED ROSENBLUTH METHOD (PERM)



$W_1 = 4$; $W_2 = W_1 \cdot 3$; $W_3 = W_2 \cdot 3$; $W_4 = W_3 \cdot 3$; (Rosenbluth' 59)

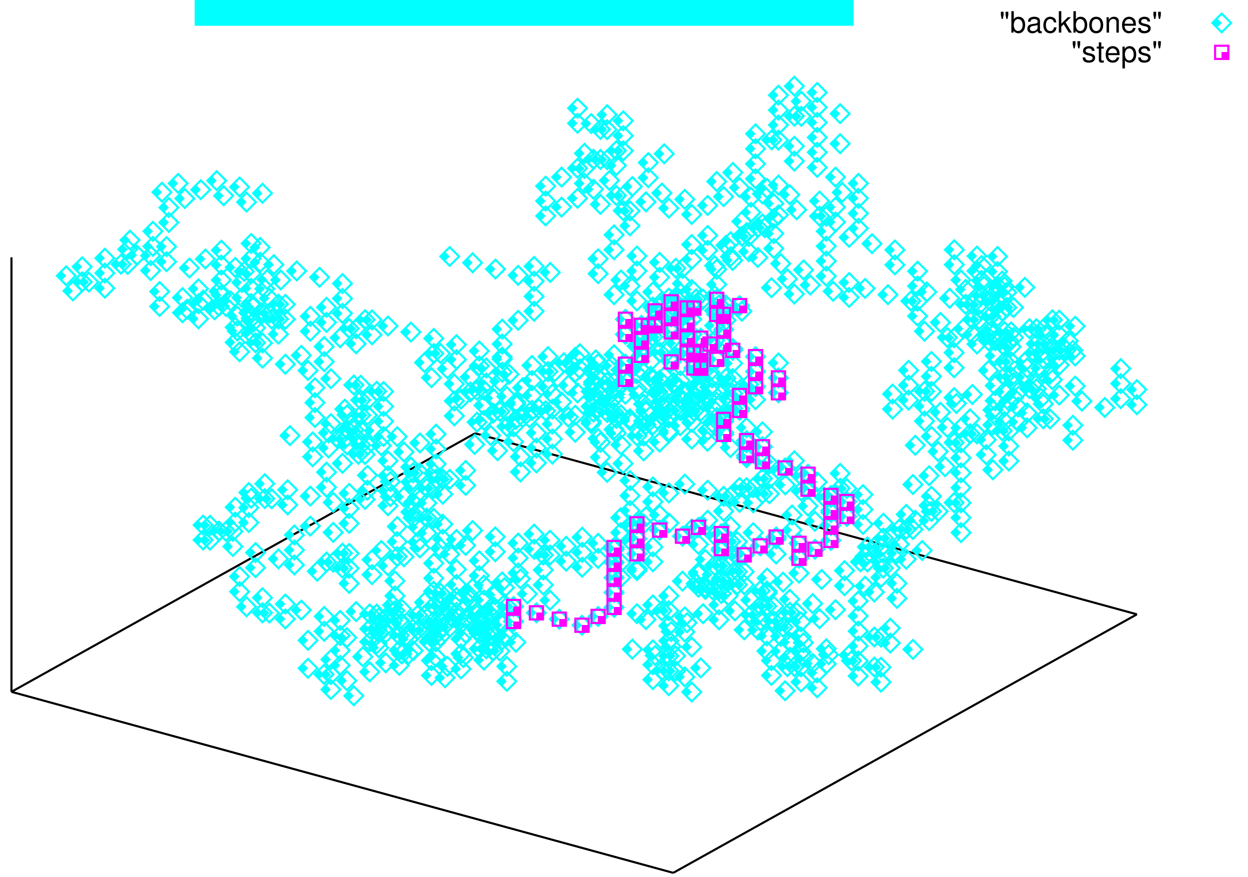
$$W_N^{conf} = \prod_{l=1}^{N} m_l, \quad \langle (...) \rangle = \frac{\sum_{conf} (...) W_N^{conf}}{\sum_{conf} W_N^{conf}}$$

$W_N^{max} = a \cdot Z_N / Z_1$, $W_N^{min} = b \cdot Z_N / Z_1$ (Grassberger '97)

$W_N^{conf} < W_N^{min}$ - prunning with probability 1/2, $W_N^{conf} = 2W_N^{conf}$

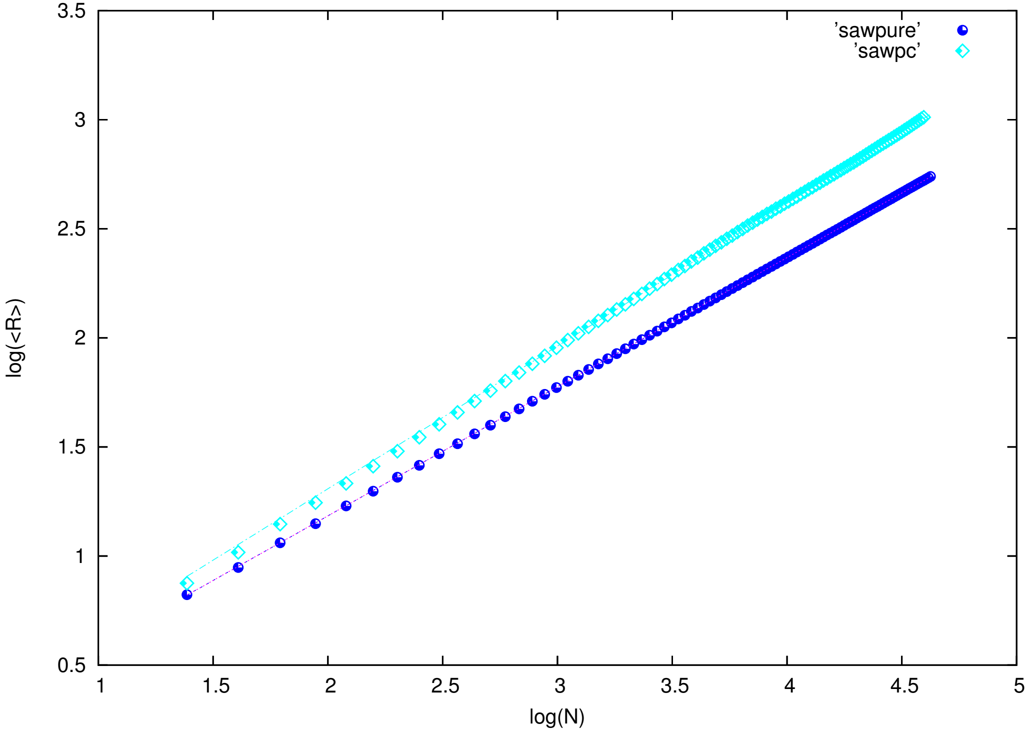
$W_N^{conf} > W_N^{max}$ - enrichment, $W_N^{conf} = W_N^{conf}/2$

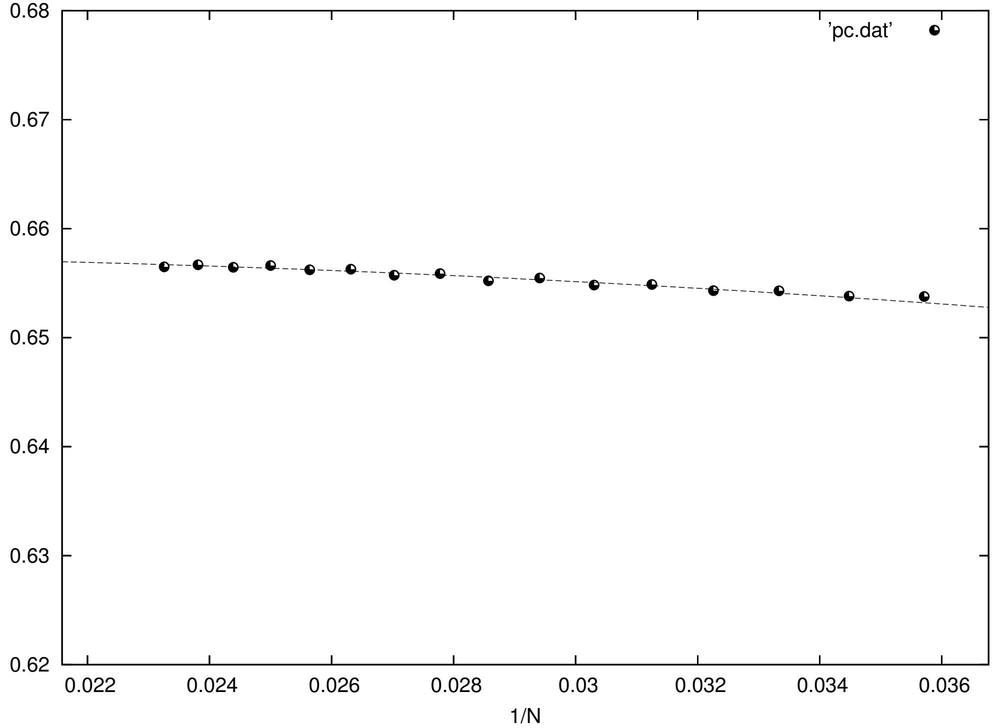
REALIZATION



SAWs on backbone of percolation cluster in 3 dimensions $L = 150$, $N = 100$.

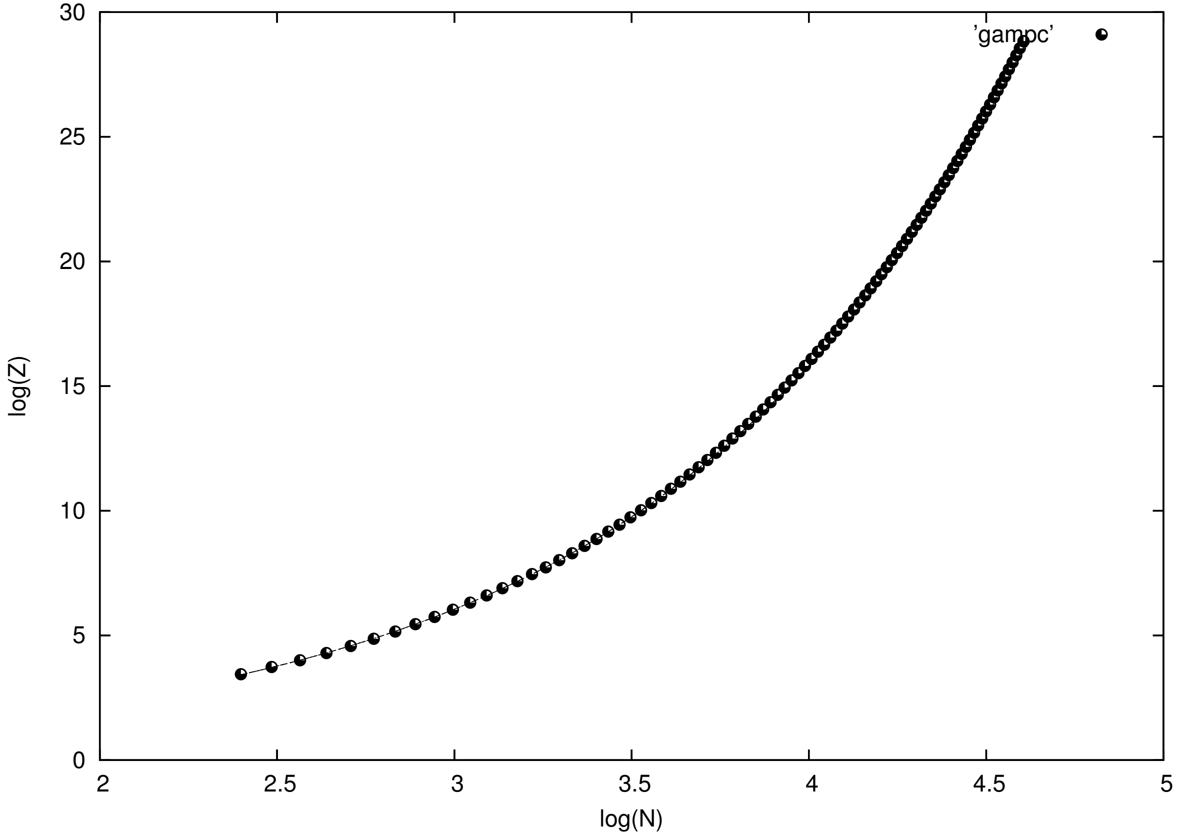
REZULTS





1: End-to-end distance $\sqrt{\langle R^2 \rangle}$ as function of N in double logarithmic scale.
2: Extrapolation of exponent ν_{p_c} as function of $1/N$. An estimate: $\nu_{p_c} = 0.658 \pm 0.001$.

$$\langle (R^2)^2 \rangle - (\langle R^2 \rangle)^2 \sim N^{\nu_1} \qquad \nu_1 = 0.821 \pm 0.002.$$



Number of SAWs configurations as function of N in double logarithmic scale.

$$Z_N \sim {z_{p_c}}^N N^{\gamma_{p_c}-1} \qquad z_{p_c} = 1.319 \pm 0.002, \gamma_{p_c} = 0.160 \pm 0.002$$

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d	2	3	4	5
ν_{pure}	$\frac{3}{4}$	0.588	$\frac{1}{2}$	$\frac{1}{2}$
ν_{p_c}, Flory-like formulas				
Roy'90	0.778	0.662	0.593	0.543
Lam'90	0.77 ± 0.01	0.65 ± 0.01	0.57 ± 0.01	0.52 ± 0.01
Kim'90	0.76 ± 0.01	0.63 ± 0.01	0.58 ± 0.01	0.55 ± 0.03
ν_{p_c}, Exact enumeration				
Lyklema'84	0.76 ± 0.08	0.67 ± 0.04	0.63 ± 0.02	0.54 ± 0.02
Rintoul'94	0.81 ± 0.03			
LeDoussal'91	0.745 ± 0.01	0.635 ± 0.01		
ν_{p_c}, RG				
ν_{p_c}, our result	0.78	0.67	0.59	
ν_{p_c}, MC				
Grassberger'93	0.78(3)			
ν_{p_c}, our result	0.775 ± 0.002	0.658 ± 0.001	0.598 ± 0.001	

Exponent ν_p, governing the mean-square end-to-end distance of SAWs on a percolation cluster.