Problem Set XII

Advanced Statistical Physics – SoSe 2017
Due: Tuesday, June 27, before the lecture

Prof. Dr. Klaus Kroy

Exercise 1. Langevin equation 9 (+3) P.

Consider the 1-dimensional Langevin equation

$$m\dot{v}(t) + \zeta v(t) = \xi(t)$$

as a model for a Brownian particle, where $\xi(t)$ is a Gaussian random force, for which $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = 2\zeta k_{\mathrm{B}} T \delta(t - t')$ holds.

1.1 Determine $v(t)$ for the initial condition $v(t = 0) = v_0$. Calculate the conditional average values $\langle v(t) \rangle_{v_0}$ and $\langle v(t)v(t') \rangle_{v_0}$, and sketch them as a function of t. (**4 P.**)

1.2 Show that the steady state is reached for long times $t, t' \gg m/\zeta$, that is, the initial condition is "forgotten", the velocity correlation function becomes time-translation invariant, and the equipartition theorem is recovered. (**2 P.**)

1.3 Calculate the mean position $\langle x(t) \rangle_{v_0 x_0}$ of a Brownian particle, that starts with the velocity $v(t = 0) = v_0$ at $x(t = 0) = x_0$, and the mean-square displacement $\Delta x(t, t') \equiv \langle [x(t) - x(t')]^2 \rangle_{v_0}$ for $t - t' > 0$. Determine the variance $\Delta_x(t) \equiv \langle [x(t) - \langle x(t) \rangle]^2 \rangle$ by reducing it to known quantities. (**3 P.**)

1.4* Analyze the long-time behavior $(t, t' \gg m/\zeta)$ of $\Delta x(t, t')$ and distinguish the cases $|t - t'| \gg m/\zeta$ and $|t - t'| \ll m/\zeta$. Show that the diffusion coefficient defined via a reasonable limit fulfills the Einstein relation. (**+3 P.**)