Problem Set XI

Advanced Statistical Physics – SoSe 2017
Due: Tuesday, June 20, before the lecture Prof. Dr. Klaus Kroy

Exercise 1. Renormalization group transformation II 12 P.
Consider again the Ising model in 2 dimensions on a square lattice as in exercise 1.2 of problem set X.

1.1 Consider an extended model with an additional interaction of next-nearest neighbors with coupling $L_0 \ll 1$. Argue that the RG–equations from exercise 1.2 of problem set X are modified to:

$$\begin{aligned} K_1 &= 2K_0^2 + L_0 \\ L_1 &= K_0^2 \end{aligned}$$

for the first renormalization step? Take these equations as a starting point for the explicit approximative formulation of the RG–equations $K_{n+1}(K_n)$ and $L_{n+1}(L_n)$. Determine the non-trivial fixed point (K^*, L^*). **(2P)**

1.2 To find the critical point, set $L_0 = 0$, vary the initial value K_0 and iterate the RG–equations found in 1.1 until you find an estimate for the critical value of the coupling $K_0 = K_c$ for which $K_n \to K^*$ and $L_n \to L^*$ as $n \to \infty$. Compare your result with Onsager's exact value of $K_c = 0.4407\ldots$. Where do the non-critical starting values $K_0 \neq K_c$ for $L_0 = 0$ converge to under the iteration? **(2P)**

1.3 Sketch the critical trajectory in a (K_0, L_0)-diagram and mark the fixed points and the flow of the coupling (K_n, L_n) on the critical trajectory and in its vicinity. Explain why the introduction of L_0 in problem 1.1 was useful. (Consider how it affects the critical exponents?) **(3P)**

1.4 Calculate the critical exponent ν, which describes the divergence of the correlation length $\xi(K_0, L_0)$ near the critical point, by linearizing the RG–equations about (K_*, L_*) and calculating the eigenvalues of the linearized RGT. Which eigenvalue is the relevant one? **(5P)**

Hint: The RG–transformation leads to a lattice with its lattice constant increased by a factor of $\sqrt{2}$.