Problem Set VI

Advanced Statistical Physics – SoSe 2017

Due: Tuesday, May 16, before the lecture PROF. DR. KLAUS KROY

Exercise 1. Microphase Separation in Random Phase Approximation 12 P.

Within the random phase approximation, the free energy $F^{(0)}$ containing short-ranged interactions ($r \sim \sigma$) is amended by adding a term

$$F_{\text{ex}}[\delta n(\mathbf{r})] = \frac{1}{2}\int \delta n(\mathbf{r})\nu_1(\mathbf{r}-\mathbf{r}')\delta n(\mathbf{r}')\,\mathrm{d}^3r\,\mathrm{d}^3r'$$

accounting in a perturbative manner for the weak long-ranged part ν_1 of the pair potential.

1.1 Using the above expression for the direct correlation function $c(\mathbf{r},\mathbf{r}')$, show that for a homogeneous fluid one has $S_q^{-1} = (S_q^{(0)})^{-1} + n\beta\nu_{1q}$. (**4 P.**)

1.2 Show that the structure factor, for the perturbation potential

$$\nu_1(r) = u_1 \mathrm{e}^{-\varkappa_1 r} - u_2 \mathrm{e}^{-\varkappa_2 r}\,,$$

where $\varkappa_1 < \varkappa_2 \ll \sigma^{-1}$, $0 < \beta u_1 < \beta u_2 \ll 1$, develops a small-angle peak with

$$S^{-1}_{\mathrm{q}\to 0} \sim a + b(q^2/q_0^2 - 1)^2\,.$$

Hint: After rewriting the Fourier transform of the perturbative potential use the approximation

$$\frac{\sin(qr)}{qr} \approx 1 - \frac{q^2r^2}{3!} + \frac{q^4r^4}{5!}.$$

to simplify the integration and minimize $\nu_{1q\to 0}$. (**6 P.**)

1.3 Discuss its physical relevance and the contribution of the long range interaction to the isothermal compressibility, as a function of the potential parameters. (**2 P.**)