Problem Set V

Advanced Statistical Physics – SoSe 2017
Due: Tuesday, May 9, before the lecture

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Exercise 1. Barometer Equation 4 P.

Given the boundary condition $n(0) = n_0$, calculate the chemical potential μ and the space-dependent density $n(\mathbf{r})$ of an inhomogeneous thermostated ideal gas exposed to an external potential, $\mu(\mathbf{r}) = \mu - u(\mathbf{r})$. Use the known expression for the free energy density of an inhomogeneous ideal gas within an external potential $\mu(\mathbf{r}) = \mu - u(\mathbf{r})$. Start from the known free energy functional of an inhomogeneous ideal gas and and minimize the grand canonical potential.

Exercise 2. Denisty Functional Theory 6 P.

Starting from the definition

$$c(\mathbf{r}, \mathbf{r}') \equiv \frac{\delta(\mathbf{r} - \mathbf{r}')}{n(\mathbf{r})} - \beta \frac{\delta \mu(\mathbf{r})}{\delta n(\mathbf{r}')},$$

derive the Ornstein-Zernicke equation.
Hint: Use the relation

$$G(\mathbf{r}, \mathbf{r}') = n(\mathbf{r}) h(\mathbf{r}, \mathbf{r}') n(\mathbf{r}') + n(\mathbf{r}) \delta(\mathbf{r} - \mathbf{r}'),$$

the generalization of $g(r) = h(r) + 1$ to homogeneous fluids.