Problem Set IV

Advanced Statistical Physics – SoSe 2017
Due: Tuesday, May 2, before the lecture

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Exercise 1. Volume of a 4-dimensional spherical shell 4 P.

The volume of high-dimensional bodies or of bodies with complicated shapes can conveniently be calculated with Monte-Carlo methods. However, one needs an algorithm that is able to decide whether a given point belongs to the body or not.

1.1 Find by analytical calculations the volume of the 4-dimensional spherical shell that is defined by $0.8 \leq R \leq 1.0$, where R is the radius of the sphere. (**2 P.**)
Hint: Use the formula for the volume of a 3-dimensional sphere and apply Cavalieri's principle.

1.2 Write a small Monte Carlo simulation code to estimate this volume. Plot the estimated volume for increasing number of points against the analytical result. (**2 P.**)
Hint: Pick random numbers equally distributed within a 4-dimensional box of side length R and check whether they are inside or outside the spherical shell.

Exercise 2. Depletion interactions 10 P.

2.1 Use the low-density approximation for $\Delta w(r_{12})$ given in the lecture to calculate the effective interaction potential of two test spheres with diameter σ_2 dissolved in a dilute fluid of hard spheres with diameter σ_1. Consider in particular the marginal case $\sigma_1 \ll \sigma_2$. For this case, sketch the potential and indicate its characteristic scales. (**4 P.**)

2.2 Give a physical reason for the effective attraction between the test spheres. (**2 P.**)

2.3 Consider three test spheres in the hard-sphere fluid. Which value may the ratio of the radii σ_1/σ_2 not exceed, so that the depletion interactions can be represented by a pair potential $\Delta w(r_{12})$? (**2 P.**)

2.4 Which qualitative changes do you expect for higher densities? Sketch $\Delta w(r)$ and indicate the scales. (**2 P.**)
Hint: Consider first the case $\sigma_1 = \sigma_2$, for which you know the qualitative form of $g(r)$.