Problem Set II

Advanced Statistical Physics – SoSe 2017
Due: Tuesday, April 18, before the lecture PROF. DR. KLAUS KROY

Exercise 1. Canonical partition function for fermions and bosons 4 P.
Consider a system of two identical particles with the same spin (*e.g.* $+1/2$) from which each can occupy states with the energy 0, $\mathcal{E}$ or $2\mathcal{E}$. Let the lowest energy level be two-fold degenerate.

1.1 Count the possible configurations carefully and calculate the canonical partition function as well as the average energy for

i) fermions (**1 P.**)

ii) bosons (**1 P.**)

iii) classically distinguishable particles. (**1 P.**)

1.2 Under which physical conditions can fermions and bosons be regarded classically? (**1 P.**)

Exercise 2. 1-point density 4 P.
Show by explicit calculation that the canonical average value $\langle \hat{n}(\mathbf{r}) \rangle$ of the microscopic particle concentration

$$\hat{n}(\mathbf{r}) \equiv \sum_{i=1}^{N} \delta(\mathbf{r} - \mathbf{r}_i)$$

for a *homogeneous* system of pairwise interacting particles just becomes the constant overall density $n = N/V$.

Exercise 3. Structure factor 4 P.
As sketched in the lecture, the structure factor of a *homogeneous* system is given by

$$S_{\mathbf{q}} \equiv \frac{1}{\langle N \rangle} \left\langle \sum_{i,j=1}^{N} \mathrm{e}^{-i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \right\rangle = 1 + (2\pi)^d n \delta(\mathbf{q}) + n \int_{\mathbb{R}^d} \mathrm{d}\mathbf{r}\, h(\mathbf{r}) \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{r}}.$$

Present a detailed step-by-step derivation of this result starting from the definition of $S_{\mathbf{q}}$.