Problem Set I

Advanced Statistical Physics – SoSe 2017
Due: Friday, April 7, before the seminar

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Exercise 1. Fluctuations and response (+3 P.)

Show that for the energy fluctuations $\sigma_E^2 \equiv \langle (E - \langle E \rangle)^2 \rangle$ of a canonical ensemble one finds $\sigma_E^2 = k_\mathrm{B} T^2 C_V$ as well as $\sigma_E^2 / \langle E \rangle^2 \propto 1/N$, where C_V denotes the heat capacity at constant volume.

Exercise 2. The molecular zipper (+7 P.)

Consider a molecule consisting of N links in succession. As with a zipper, links can only be opened consecutively from one end, *i.e.*, the i-th link can only be open if all links $1, 2, \ldots, i-1$ before are also open. Opening the i-th link would then require the energy $\mathcal{E}$. The final link shall never be open.

2.1 Determine the Hamiltonian of the system and calculate the corresponding canonical partition function from it. (**3 P.**)

2.2 What is the average number $\langle n \rangle$ of open links? Discuss the limit of high and low temperatures. (**4 P.**)

Exercise 3. Isotropic oscillator (+8 P.)

The Hamiltonian of a three-dimensional harmonic oscillator is given by

$$H = \frac{|\mathbf{p}|^2}{2m} + \frac{m\omega^2 |\mathbf{x}|^2}{2},$$

where $\mathbf{x}, \mathbf{p} \in \mathbb{R}^3$ denote position and momentum of the oscillator respectively.

3.1 Calculate the *classical* canonical partition function, the internal energy and the specific heat of the oscillator at constant volume. (**4 P.**)

3.2 Calculate the *quantum-statistical* canonical partition function and show that it reduces to the classical expression for very high temperatures. (**4 P.**)