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Quantum Field Theory — Problem Sheet 9

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Problem 9.1
Compute the wave front set of the following distributions on $\mathbb{R}$:

$$\delta(x)\,, \qquad \Theta(x)\,, \qquad \lim_{\epsilon\to 0^+}\frac{1}{x\pm i\epsilon}\,, \qquad \lim_{\epsilon\to 0^+}\log(x\pm i\epsilon)\,.$$

Decide which of them can be multiplied.

Problem 9.2
The 2-point Wightman function of the β-KMS state ω_β on the Borchers-Uhlmann algebra of the real Klein-Gordon field on Minkowski spacetime is

$$\begin{aligned}\mathcal{W}_{2,\beta}(x,y) &\doteq \omega_\beta(\phi(x)\phi(y))\\ &= \lim_{\epsilon\to 0^+}\frac{1}{(2\pi)^3}\int_{\mathbb{R}^3}\frac{d^3p}{2\omega_{\vec{p}}}\left(\frac{\exp(-i\omega_{\vec{p}}z_0)}{1-\exp(-\beta\omega_{\vec{p}})}+\frac{\exp(i\omega_{\vec{p}}z_0)}{\exp(\beta\omega_{\vec{p}})-1}\right)\exp(i\vec{p}\cdot\vec{z}-\omega_{\vec{p}}\epsilon)\end{aligned}$$

$$z \doteq x-y$$

Show that $\mathrm{WF}(\mathcal{W}_{2,\beta})=\mathrm{WF}(\mathcal{W}_{2,\infty})$, i.e. that the wave front set of the β-KMS 2-point-function and of the vacuum 2-point-function coincide for all β.

Problem 9.3
The normal-ordered square of the Klein-Gordon field on Minkowski spacetime is defined by

$$:\phi(x)^2: \doteq \lim_{x\to y}\left(\phi(x)\phi(y)-\mathcal{W}_{2,\infty}(x,y)\mathbb{1}\right)$$

where the limit is meant in the sense of expectation values. Compute the expectation value

$$\omega_\beta(:\phi(x)^2:)$$

in the massless case.