On the role of Meta-Properties

Robert Feldmann, Alexander Kiel

May 1, 2003

An important part of an ontology is the taxonomy of the universals. Although in general the building of an appropriate taxonomy is a delicate task, it can be simplified with the help of meta-properties of universals. The considered meta-properties rigidity, identity and dependence may be of fundamental importance in accomplishing this task. We want to introduce these three meta-properties as they are defined by `Guarino/Welty` and present the remarks of `Kaplan` and his suggestions of improvement. Finally we would like to produce some new ideas about identity.

1 Introduction

The paper finishes our talk about `Guarino/Welty`'s paper [NG00a] and the paper of `Kaplan` [Kap01]. We will introduce the meta-properties rigidity, identity and dependence according to `Guarino/Welty` and have a look on `Kaplan`'s remarks on it. Occasionally we will suggest some new ideas. Meta-properties are of fundamental importance in structuring a taxonomy of universals. Hence, we will shortly outline this structuring role in the last sections.

2 Rigidity

`Guarino/Welty` define a rigid property as follows:

Definition 1 (`Guarino/Welty`) *ψ is rigid iff* $\forall x \psi(x) \rightarrow \Box \psi(x)$

`Kaplan` notices that `Guarino/Welty`'s definitions "differ from the standard use of the word 'rigid' in the modal logic literature" . This term should denote the same individual in all possible worlds. Hence `Kaplan` extends the definition of rigidity to

Definition 2 (`Kaplan`) $\forall x \Diamond \psi(x) \rightarrow \Box \psi(x)$

Kaplan then shows, that the following statement made by Guarino/Welty is obviously wrong: A antirigid property can't subsume a rigid one (using Guarino/Welty's definition of subsume). Nevertheless the intuition behind this claim is obvious. In order to account for this idea in a proper way Kaplan changes the definition of subsumption.

Definition 3 (Guarino/Welty) $\forall x \psi(x) \rightarrow \phi(x)$

Definition 4 (Kaplan) $\Box \forall x \psi(x) \rightarrow \phi(x)$

Now he states, while using this new definition of subsumption, that "an anti-rigid property can't subsume a rigid one necessarily; given either definition of rigid". We will see however, that this statement is wrong. Kaplan noticed that Guarino/Welty's definition of rigidity is highly non-standard. In fact, the definition is hardly useful.

Let M be a model with two worlds w_1, w_2 and one individual i (we follow the notation of Kaplan). Let the denotation of ϕ be the empty set in w_1 and i in w_2. The actual world is w_1. We see:

- ϕ is anti-rigid
- ϕ is also rigid according to Guarino/Welty's definition of rigidity

Since ϕ subsumes ϕ (using either definition of subsume) the definition of rigidity does matter. Hence Kaplan's statement above is proven to be wrong. On the other hand the important first part of his statement can be recovered if we consequently apply his definitions of rigidity and subsumption.

Theorem 1 *: An anti-rigid property can't subsume a rigid one; given* Kaplan*'s definition of rigidity and subsumption.*

We will give the proof here and find out an interesting result:

- precondition: ϕ is anti-rigid, i.e. $\forall x \neg \Box \phi(x)$, and ψ is rigid.
- proposition: $\neg \Box \forall x(\psi(x) \rightarrow \phi(x))$
- proof: At first we equivalently transform the proposition: $\neg \Box \forall x(\psi(x) \rightarrow \phi(x)) \longleftrightarrow \Diamond \exists x \neg(\psi(x) \rightarrow \phi(x)) \longleftrightarrow \Diamond \exists x(\psi(x) \wedge \neg \phi(x))$.
  1. We choose a constant a such that $\Box \psi(a)$ is satisfied. This is always possible since ψ is rigid and assumed to be natural, i.e.: $\diamond \exists x \psi(x)$
  2. Now we choose a world w which $\neg \phi(a)$ is not valid in. This is always possible due to the anti-rigidity of ϕ

  Therefore, in w $\Box \psi(a) \wedge \neg \phi(a)$ is satisfied. Thus $\diamond(\psi(a) \wedge \neg \phi(a))$ and finally $\diamond \exists x(\psi(x) \wedge \neg \phi(x))$ are valid.

The interesting point is we did not make use of the rigid property of ψ in 1., rather we used the fact that ψ is semi-rigid (not antirigid), i.e $\exists x \Box \psi(x)$. Therefore we even proved:

Theorem 2 *: An anti-rigid property can neither subsume a rigid nor a semi-rigid property, i.e. it can subsume only anti-rigid properties; given* `Kaplan`*'s definition of rigidity and subsumption.*

3 Identity

The notion of an identity criterion is introduced by `Guarino/Welty` using the following formulation: ρ is an identity criterion for the property ϕ if it is a relation of a suitable type such that

$$\phi(x) \wedge \phi(y) \to (\rho(x,y) \leftrightarrow x = y) \tag{1}$$

They exand this concept further by the introduction of time arguments and the 'actual existence' predicate in order to distinguish between diachronic (with respect to different times) and synchronic (same time) identity. Secondly they split up (1) defining necessary and sufficient identity conditions (IC). In this context an identity criterion is a necessary and sufficient identity condition. The meaning of carrying and supplying is then defined by `Guarino/Welty` as follows

Definition 5 *A rigid property carries an IC iff it carries a sufficient or a necessary IC.*

Definition 6 *It carries a necessary IC $\Gamma(x,y,t,t')$ if Γ contains x,y,t,t' as the only free variables, and :*

- $E(x,t) \wedge \phi(x,t) \wedge E(y,t') \wedge \phi(y,t') \wedge x = y \to \Gamma(x,y,t,t')$
- $\neg \forall xytt' (\Gamma(x,y,t,t') \leftrightarrow x = y)$
- $\neg \forall xy (E(x,t) \wedge \phi(x,t) \wedge E(y,t') \wedge \phi(y,t') \to \Gamma(x,y,t,t'))$

The definition of a sufficient IC is analogous to the case above and can be found in [NG00a].

Definition 7 *A property ϕ supplies an IC Γ iff i) it is rigid ii) it carries Γ iii) Γ is not carried by all the properties subsuming ϕ.*

These conditions are defined for rigid properties, since the authors have in mind a difference between supplying a condition (this should only be possible for rigid properties) and carrying a condition. The given example is that a PERSON (rigid) *supplies* a condition, e.g. "having-the-same-fingerprints", while on the other hand a STUDENT (non-rigid) only *carries* a condition, inheriting it from its subsuming rigid property PERSON. Therefore, a non-rigid property carries an IC Γ iff it is subsumed by a rigid property carrying Γ.

If a property ϕ carries an IC it is denoted ϕ^{+I} and called a "sortal" , otherwise ϕ^{-I} and "non-sortal". Properties ϕ which supply an IC are denoted ϕ^{+O}. Since IC's are inherited along the hierarchy of properties we could use them as a guide for building our taxonomy.

According to `Kaplan` these ideas are not as consistent as intended. The main problem is the definitions of the identity criteria. `Guarino/Welty` use additional axioms to avoid trivial cases. However, `Kaplan` shows that there still exists an infinite class of allowed trivial cases. In this sense almost all properties are sortals and "...therefore the meta-property +/-I cannot play the major role they intend for it in structuring taxonomies" as `Kaplan` noted.

`Kaplan` then shortly considers constraints which make use of the specific identity condition a property carries and not refer to whether or whether not it carries any. This idea comes originally from Guarino and Welty and states "if two properties have 'incompatible' identity conditions, they are disjoint".

Before we go on and decribe our rather different approach we want to add some remarks. The observation that inherited IC's do not change their character may be useful, i.e. necessary conditions are inherited as necessary conditions etc. Thus the +/-I notion should be expanded to:

- $+/\text{-}I^n$ if the property carries (does not carry) a necessary identity condition
- $+/\text{-}I^s$ if the property carries (does not carry) a sufficient identity condition
- $+/\text{-}I^c$ if the property carries (does not carry) an identity criterion (necessary and sufficient)

Please note that if ϕ inherites $+I^n$ from one property and $+I^s$ from a second it usually keeps I^c.

The second remark is about time relations. As `Kaplan` analysis in his paper, the semantics of the modal operators and the relationsship between them, the world and time semantics are not clear. For example, the rigid property is defined using the box operator. Usually this references about all accessible worlds. The example given by `Guarino/Welty`, however, suggest referencing about all times. We decided to keep the troubles involved with this problem out of this paper and therefore will not consider relations with time arguments.

3.1 Our Approach

In our opinion the approach of `Guarino/Welty` cannot work since you can always construct a formal relation Γ which satisfies (1). The question is how to obtain a relation from the inherent structure of the properties considered. An important difference between relations is there position in the taxonomic structure as well as the properties they subsume. Let's think about the property *PERSON*. The individuals of *PERSON* are identical (or at least indistinguishable) if they have the same fingerprint. Since in this sense "having-the-same-fingerprint" is a primary key, we can generalise this statement. Two individuals of a property ϕ are indistinguishable if all considered subsumed properties of ϕ are equal. If the equivalent classes of the indistinguishability relation consist of one or less elements, the property ϕ supplies naturally an identity criterion. By putting it into a more formal way we obtain:

Definition 8 *A property ϕ supplies an identity criterion with respect to X if* $\forall xy(\phi(x) \wedge \phi(y) \rightarrow (\Gamma_{\phi,X}(x,y) \leftrightarrow x = y))$

X is a subset of the complete tree $CT(\phi)$ of ϕ, i.e $CT(\phi) = \{\psi \mid \psi \leq \phi\}$. The subsumption relation is denoted by $\leq$. The identity criterion $\Gamma_{\phi,X}$ is given by

Definition 9 $\Gamma_{\phi,X}(x,y) = \forall\psi \in X : \psi(x) \leftrightarrow \psi(y)$

Γ generates an equivalence relation between individuals. It can be easily seen that the relation defined by $x \sim y \leftrightarrow \Gamma_{\phi,X}(x,y)$ is reflexive, symmetric and transitive. The introduction of relative identity criterions (referencing a subset of the complete tree) is necessary since in the case $X = CT(\phi)$ the equivalence classes of $\Gamma_{\phi,X}$ contain at most one element. As consequence the set X can be enlarged for every property ϕ to X^ϕ such that ϕ supplies an identity criterion with respect to X^ϕ. Therefore, the engineer controls with his choice of X whether a given property ϕ supplies an identity criterion or not. After the definition of supplying a property the weaker meta-property of carrying an identity criterion can be introduced:

Definition 10 *A property ϕ carries an identity criterion if ϕ is subsumed by a property ψ which supplies an identity criterion.*

4 Dependence

`Guarino/Welty` define dependence as follows:

Definition 11 *A property ϕ is* externally dependent *on a property ψ if, for all its instances x, necessarily some instance of ψ must exist, which is neither a part nor a constituent of x:*

$$\forall x \Box \Big(\phi(x) \rightarrow \exists y \big(\psi(y) \wedge \neg P(y,x) \wedge \neg C(y,x)\big)\Big)$$

The part and constituent relations are discussed further in [NG00b]. Generally, a property ϕ is *externally dependent* on a property ψ if the existence of instances of ϕ implies the existence of a instance of the property ψ. For example a *PARENT* is externally dependent on a *CHILD* because it does not make sense to speak of a *PARENT* if no *CHILD* exists. A second example is: *STUDENT* depends on the *ENROLLMENT* at a university. The part-of and constituent relations exclude such examples as a *PERSON* dependsing on its *HEART* or *BODY*, because heart is a part and body is a constituent of a person.

Furthermore `Guarino/Welty` notice that additionally we must exclude qualities (such as colors), things that necessarily exist (such as the universe) and the case where ψ is subsumed by ϕ. This shows that the definition 11 is not precise enough.

The only point where `Guarino/Welty` use the dependence for building up a taxonomy is the dicrimination of roles from other kinds of properties. We will later point out the classification of meta-properties.

5 Constraints and Assumptions

Here we list some constraints and assumptions that follow from the definitions of the meta-properties (according to `Guarino/Welty`), discussed in the corresponding sections.

$$\phi^{\sim R} \text{ can't subsume } \psi^{+R} \tag{2}$$

At this point we remember our own result (see earlier)

$$\phi^{\sim R} \text{ can't subsume } \psi^{+R} \text{ nor } \psi^{\neg R} \tag{3}$$

$$\phi^{+I} \text{ can't subsume } \psi^{-I} \tag{4}$$

$$\text{Properties with incompatible IC's are disjoint.} \tag{5}$$

$$\phi^{+D} \text{ can't subsume } \psi^{-D} \tag{6}$$

6 Property Kinds

`Guarino/Welty` specity the following table: In this table the different metaproperties are

+O	**+I**	**+R**	**+D**	Type	sortal
			−D		
−O	**+I**	**+R**	**+D**	Quasi-type	
			−D		
−O	**+I**	**∼R**	**+D**	Material role	
−O	**+I**	**∼R**	**−D**	Phased sortal	
−O	**+I**	**¬R**	**+D**	Mixin	
			−D		
−O	**−I**	**+R**	**+D**	Category	non-sortal
			−D		
−O	**−I**	**∼R**	**+D**	Formal role	
−O	**−I**	**∼R**	**−D**	Attribution	
		¬R	**+D**		
			−D		

Table 1: Formal ontological property classification

combined into intuitionally imaginable kinds. In [gw00] some additional explanations about the particular kinds can be found. This classification can be used as a starting point for structuring the taxonomy.

7 Building a Taxonomy

Two constraints must be considered in order to build a proper taxonomy. They are the equations (4) and (2) in section *Constraints and Assumptions*.

(4) implies that all non-sortal properties must stay on top of the taxonomy. The non-sortal properties don't carry an identity, thus they cannot refer to particular individials. In other words, non-sortals are abstract, categorizing universals. Examples are attributions like colors (special values) or shapes, formal roles like patient or instrument, and top level ontologie categories.

Constraint (2) accomplishes that rigid properties stands on top or how `Guarino/Welty` note: "in the middle" of the taxonomy. `Guarino/Welty` refer to a *Backbone Taxonomy* build by all rigid properties. So the sortal non-rigid property kinds like roles especially should "hang off" the backbone taxonomy. Thereby the engineer must keep in mind that roles shouldn't be subsumed by rigid properties they depend on. For example *food* (a material role) shouldn't be subsumed by living beings that eat the food or *children* don't subsume *parent*. The two examples are just simple cases. Nobody would say: "food is a living being" or "parent is a children". We can say that *food* is (in the material way) just an amount of matter. All other more special classifications whould be false.

8 Conclusion

From the above mentioned considerations it should be clear, that meta-properties of universals can be of great help when building up a taxonomy. Unfortunately the definitions of these meta-properties are often not as precise as needed. We had a look at the problems involved and tried to emphasise a proper,more formal way of defining the meta-properties rigitiy, identity and dependence.

References

[Kap01] A. N. Kaplan. Towards a consistent logical framework for ontological analysis. 2001.

[NG94] P. Giaretta N. Guarino, M. Carrara. An ontology of meta-level categories. In Jon Doyle, Erik Sandewall, and Pietro Torasso, editors, *KR'94: Principles of Knowledge Representation and Reasoning*, pages 270–280. Morgan Kaufmann, San Francisco, California, 1994.

[NG00a] C. Welthy N. Guarino. A formal ontology of properties. *Proceedings of 12th Int. Conf. on Knowledge Engineering and Knowledge Management, Lecture Notes on Computer Science*, 2000.

[NG00b] C. Welthy N. Guarino. Towards a methodology for ontology-based model engineering. *Proceedings of the ECOOP-2000 Workshop on Model Engineering*, 2000.