Nabla in Kugel- und Zylinderkoordinaten

1 Kugelkoordinaten

$$x = r\,\sin\,\vartheta\,\cos\,\varphi \qquad y = r\,\sin\,\vartheta\,\sin\,\varphi \qquad z = r\,\cos\,\vartheta \tag{1}$$

$$\mathrm{d}V = r^2\,\sin\,\vartheta\,\mathrm{d}r\,\mathrm{d}\vartheta\,\mathrm{d}\varphi. \tag{2}$$

$$\nabla\psi = \frac{\partial\psi}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial\psi}{\partial\vartheta}\vec{e}_\vartheta + \frac{1}{r\sin\vartheta}\frac{\partial\psi}{\partial\varphi}\vec{e}_\varphi \tag{3}$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\vartheta}\frac{\partial(\sin\vartheta\,A_\vartheta)}{\partial\vartheta} + \frac{1}{r\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi} \tag{4}$$

$$\begin{aligned}\nabla\times\vec{A} \;&=\; \frac{1}{r\sin\vartheta}\left[\frac{\partial(\sin\vartheta\,A_\varphi)}{\partial\vartheta} - \frac{\partial A_\vartheta}{\partial\varphi}\right]\vec{e}_r \\ &\quad + \left[\frac{1}{r\sin\vartheta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\vec{e}_\vartheta + \frac{1}{r}\left[\frac{\partial(rA_\vartheta)}{\partial r} - \frac{\partial A_r}{\partial\vartheta}\right]\vec{e}_\varphi\end{aligned} \tag{5}$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\psi}{\partial\vartheta}\right) + \frac{1}{r^2\sin^2\vartheta}\frac{\partial^2\psi}{\partial\varphi^2} \tag{6}$$

$$\text{Man beachte:} \qquad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{r}\frac{\partial^2(r\psi)}{\partial r^2}$$

2 Zylinderkoordinaten

$$x = \rho\,\cos\,\varphi \qquad y = \rho\,\sin\,\varphi \qquad z = z \tag{7}$$

$$\mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\varphi\,\mathrm{d}z. \tag{8}$$

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\vec{e}_\rho + \frac{1}{\rho}\frac{\partial\psi}{\partial\varphi}\vec{e}_\varphi + \frac{\partial\psi}{\partial z}\vec{e}_z \tag{9}$$

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z} \tag{10}$$

$$\nabla\times\vec{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right)\vec{e}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right)\vec{e}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\varphi}\right)\vec{e}_z \tag{11}$$

$$\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\varphi^2} + \frac{\partial^2\psi}{\partial z^2} \tag{12}$$