Critical Exponents of the Classical Heisenberg Ferromagnet

In a recent Letter, Brown and Ciftan (BC) [1] reported high precision Monte Carlo (MC) estimates of the static critical exponents of the classical 3D Heisenberg model. While their finite-size scaling (FSS) analysis yields values for the critical temperature $T_c = 1/K_c$ and the critical exponent ratios $\beta/\nu, \gamma/\nu$, which are compatible with all recent findings, BC claim that the specific heat C of this model is divergent at T_c, which is in strong disagreement with other recent high statistics MC simulations, high-temperature (HT) series analyses, field theoretic methods [2–6], and experimental studies [7], which all find a finite cusplike behavior.

In their ansatz (2) BC use a nonlinear six parameter fit to 14 data points for C on lattices of linear size $2 \leq L \leq 32$. The fit resulting in $\alpha/\nu = 0.117(4)$ has still a total $\chi^2 \approx 60$, and therefore is by standard reasoning not acceptable. This indicates that either the statistical errors of their data are underestimated or the use of such small lattices as $L = 2$ requires the inclusion of even more correction terms. Alternatively, one may ask how the fit parameters would change if the smallest lattices are successively discarded. By hyperscaling BC deduce from this value a nonstandard exponent $\nu = 0.642(2)$, leading in turn to nonstandard estimates of β and γ.

We find it very dangerous to base such an incisive conclusion solely on the very delicate FSS behavior of the specific heat. In particular, we strongly disagree with the statement of BC that ν is extremely difficult to measure directly. The derivative of the Binder parameter dU/dK and the logarithmic derivatives $d\ \ln\langle m\rangle/dK$ and $d\ \ln\langle m^2\rangle/dK$ all scale like $L^{1/\nu}$ and, using fluctuation formulas, can be as easily measured as C, and the statistical errors are straightforward to control. Already our data for dU/dK in Ref. [2] gave an estimate of $\nu = 0.704(6)$ which agrees with our estimate of α/ν by hyperscaling. Moreover, it is compatible with the value $\nu = 0.698(2)$ derived from fits to the critical behavior of independent correlation length data in the high-temperature phase [2], as well as with the value $\nu = 0.73(4)$ obtained in the broken phase [6]. In Ref. [4] we studied this model with emphasis on topological excitations on much larger lattices with $8 \leq L \leq 80$. By analyzing the new data with the above three quantities at $K = 0.6930 \approx K_c$ we obtain from FSS fits a prediction of $\nu = 0.699(3)$ (cp. Fig. 1).

For our large lattices of Ref. [4] a fit of the cusp form $C(L) = C^{\text{reg}} + C_0 L^{\alpha/\nu}$ to 12 data points yielded $\alpha/\nu = -0.225(80)$ ($\chi^2 = 7.84$). A more precise estimate can be obtained by analyzing the energy directly according to $E(L) = E^{\text{reg}} + E_0 L^{(\alpha-1)/\nu}$, which definitely has a regular term. This yielded again a negative exponent ratio $\alpha/\nu = -0.166(31)$ ($\chi^2 = 11.3$), which by hyperscaling is completely consistent with our value of ν.

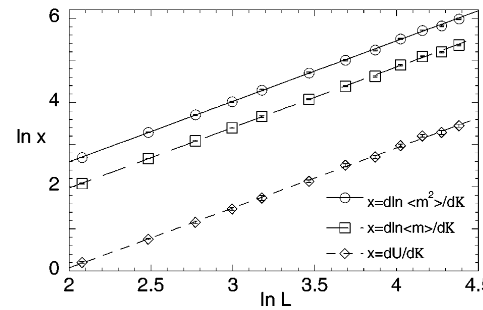

FIG. 1. Double logarithmic plots of data and fits at $K = 0.6930$ for dU/dK [yielding $\nu = 0.703(6)$], $d\ \ln\langle m\rangle/dK$ [$\nu = 0.698(3)$], and $d\ \ln\langle m^2\rangle/dK$ [$\nu = 0.697(3)$] versus L.

If we allow for a confluent correction $E_1 L^{(\alpha_1-1)/\nu}$, then the fit only slightly improves ($\chi^2 = 10.9$) with an almost vanishing amplitude $E_1 = -0.8 \times 10^{-13}$, showing that there really is no need for an additional term. If we add a term $C_1 L^{\alpha_1/\nu}$ to the C fit, we may get a marginally improved fit ($\chi^2 = 7.79$) with a positive exponent, $\alpha/\nu = 0.09$, and $\alpha_1/\nu = -0.6$, but there are also different, slightly better solutions ($\chi^2 = 7.68$) with $\alpha/\nu = -0.17$ and $\alpha_1/\nu = -4.7$, showing the danger of being misled by a too flexible C-fit ansatz.

The "universality scaling" of Fig. 3 in [1] indeed looks best for $\nu = 0.642$, but this plot cannot serve as an independent determination of ν. The predicted data collapse is an *asymptotic* statement for large L near T_c, and neither should it be expected far beyond T_c nor for very small lattice sizes, even if the correct exponents are used.

To summarize, we feel that the numerical data analysis presented in [1] does not provide sufficient evidence to question the current critical exponent estimates of the 3D Heisenberg model, as consistently obtained by field theory, HT series analyses, and MC methods.

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