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Standard and Z_2-Regge theory in two dimensions*

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We qualitatively compare two versions of quantum Regge calculus by means of Monte Carlo simulations. In Standard Regge Calculus the quadratic link lengths of the triangulation vary continuously, whereas in the Z_2-Regge Model they are restricted to two possible values. The goal is to determine whether the computationally more easily accessible Z_2 model retains the characteristics of standard Regge theory.

Standard Regge Calculus (SRC) [1] provides an interesting method to explore quantum gravity in a non-perturbative fashion [2]. Although its code can be efficiently vectorized for large scale computing, it is still a very time demanding enterprise. One therefore seeks for suitable approximations which will simplify the SRC and yet retain most of its universal features. The Z_2-Regge Model (Z_2RM) [3] could be such a desired simplification. Here the quadratic link lengths of the simplicial complexes are restricted to take on only the two values

$$q_l = 1 + \epsilon\sigma_l \ , \quad 0 < \epsilon < \epsilon_{max} \ , \quad \sigma_l = \pm 1 \ , \qquad (1)$$

in close analogy to the ancestor of all lattice models, the Ising-Lenz model. To test whether this simpler model is in a reasonable sense still similar to SRC, we study both models in two dimensions and compare a number of observables for one particular lattice size.

Starting point for both SRC and Z_2RM is Regge's discrete description of General Relativity [1] in which a given continuum manifold is replaced by a piecewise flat simplicial space. In two dimensions this procedure is easily illustrated by choosing a triangulation of the surface under consideration. Every triangle then represents a part of a piecewise linear manifold.

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Quantization of SRC proceeds by evaluating the Euclidean path integral

$$Z = \left[\prod_l \int_0^\infty dq_l q_l^{-m} \mathcal{F}(\{q_l\})\right] e^{-\lambda \sum_t A_t} \ . \qquad (2)$$

In principle the functional integration should extend over all metrics on all possible topologies, but, as is usually done, we restrict ourselves to one specific topology, the torus, whose Euler characteristic is zero. Consequently the action in the exponent of (2) consists only of a cosmological constant λ times the sum over all triangle areas A_t. Moreover the path-integral approach suffers from a non-uniqueness of the integration measure and it is even claimed that the true measure is of non-local nature [4]. We used as a trial functional integration measure the expression within the square brackets of (2) with $m \in \mathbb{R}$ permitting to investigate a 1-parameter family of measures. The function $\mathcal{F}$ constrains the integration to those Euclidean configurations of link lengths which do not violate the triangle inequalities.

In the Z_2RM [3] the squared link lengths as well as functions of them are rewritten with respect to (1). Thus the area of a triangle with edges q_1, q_2, q_l is expressed as

$$\begin{aligned} A_t &= c_0 + c_1(\sigma_1 + \sigma_2 + \sigma_l) + c_2(\sigma_1\sigma_2 + \\ &\quad +\sigma_1\sigma_l + \sigma_2\sigma_l) + c_3\sigma_1\sigma_2\sigma_l \ . \qquad (3) \end{aligned}$$

The coefficients c_i depend on ϵ only and impose the condition $\epsilon < \frac{3}{5} = \epsilon_{max}$ in order

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to have real and positive triangle areas [3], i.e. $\mathcal{F} = 1$ for all possible configurations. The measure $\prod_l \int dq_l q_l^{-m}$ is replaced by

$$\sum_{\sigma_l=\pm 1} \exp[-m \sum_l \ln(1+\epsilon\sigma_l)] =$$
$$\sum_{\sigma_l=\pm 1} \exp[-N_1 m_0(\epsilon) - \sum_l m_1(\epsilon)\sigma_l] \ , \quad (4)$$

with $m_1 = mM$ and $M = \sum_{i=1}^{\infty} \frac{\epsilon^{2i-1}}{2i-1}$. N_1 is the total number of links and $m_0 = -\frac{1}{2}m\epsilon^2 + O(\epsilon^4)$. Hence the partition function of the Z_2RM results in

$$Z = \sum_{\sigma_l=\pm 1} J \exp\{-\sum_l (2\lambda c_1 + m_1)\sigma_l -$$
$$\lambda \sum_t [c_2(\sigma_1\sigma_2 + \sigma_1\sigma_l + \sigma_2\sigma_l) + c_3\sigma_1\sigma_2\sigma_l]\} \ , \quad (5)$$

with a constant J. A particular simple form of (5) is obtained if $m_1 = -2\lambda c_1$ and therefore

$$m = \frac{-2\lambda c_1}{M} \quad (6)$$

is henceforth used for the measure in the Z_2RM as well as in SRC. We set the parameter $\epsilon = 0.5$ in the following.

To compare both models we examined the quadratic link lengths and the area fluctuations on the simplicial lattice. Furthermore the Liouville mode is of special interest because it represents the only degree of freedom of pure 2d-gravity. The discrete analogue of the continuum Liouville field $\varphi(x) = \ln\sqrt{g(x)}$ is defined by $\phi = \frac{1}{A}\sum_i \ln A_i$, where $A_i = \frac{1}{3}\sum_{t\supset i} A_t$ is the area element of site i and A the total area [5]. Figure 1 displays the corresponding expectation values as a function of the cosmological constant λ measured from 100k Monte Carlo sweeps after thermalization on lattices with 16×16 vertices.

Within the SRC the area increases with decreasing λ in perfect agreement with the scaling relation

$$\langle A \rangle = N_1 \frac{1-m}{\lambda} \ . \quad (7)$$

One also expects that $\langle q \rangle$ will increase as λ tends to zero. The Liouville field $\langle \phi \rangle$ behaves accordingly. Actually we observe that the system thermalizes extremely slowly for very small λ and therefore display only statistically reliable data points for $\lambda \geq 1$ in the upper plots of Fig. 1.

Whereas the SRC becomes ill-defined for negative couplings λ, the Z_2RM is well-defined for all values of the cosmological constant. The phase transition the Z_2RM undergoes at $\lambda_c \approx -11$ can be viewed as the relic of the transition from a well- to an ill-defined regime of SRC. So $\lambda_c \to 0$ if we allow for more than two link lengths. Altogether there is already in our simple case correspondence of observables for $\lambda > \lambda_c$.

Another interesting quantity to consider would be the Liouville susceptibility

$$\chi_\phi = \langle A \rangle [\langle \phi^2 \rangle - \langle \phi \rangle^2] \ . \quad (8)$$

From continuum field theory it is known that for fixed total area A the susceptibility scales according to

$$\ln \chi_\phi(L) \overset{L\to\infty}{\sim} c + (2 - \eta_\phi) \ln L \ , \quad (9)$$

with $L = \sqrt{A}$ and the Liouville field critical exponent $\eta_\phi = 0$. This has indeed been observed for SRC with the dq/q scale invariant measure and fixed area constraint [5]. It is, however, a priori not clear if this feature will persist in the present model due to the fluctuating area and the non-scale invariant measure. This point is presently under investigation.

To conclude, physical observables like the Liouville field and the squared curvature behave similar for the bare coupling $\lambda > \lambda_c$. The phase transition of the Z_2RM in the negative coupling regime is interpreted as the remnant of the $\lambda = 0$ singularity of SRC. There remains the interesting question if by allowing for more than two link lengths the phase transition of such extended Z_2RM approaches that of SRC. Then the situation might resemble the more involved four-dimensional case where one has to deal with 10 edges per simplex and the nontrivial Einstein-Hilbert action $\sum_{t\supset i} \delta_t A_t$ with 50 triangles t per vertex i. Thus the action takes on a large variety of values already for Z_2RM and therefore SRC can be approximated more accurately [6].

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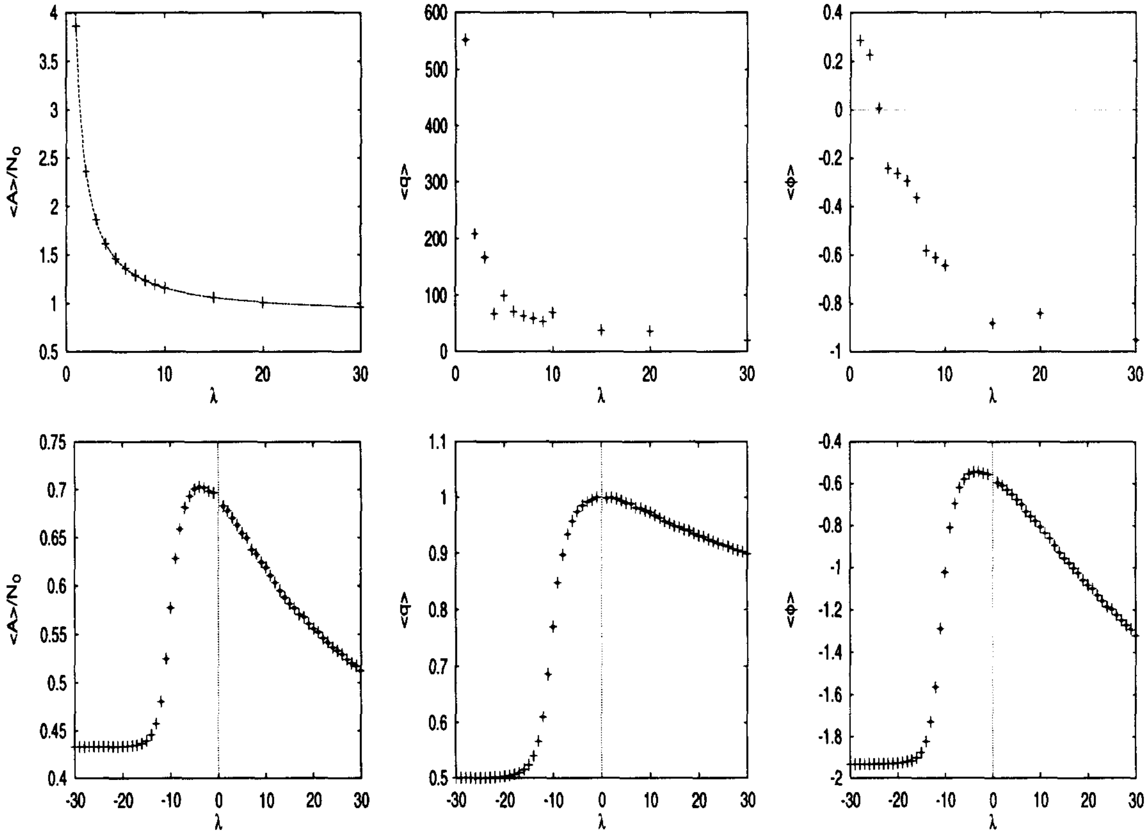

Figure 1. Expectation values of the area A normalized to the total number of vertices, the average squared link length q, and the Liouville field ϕ as a function of the cosmological constant λ for Standard Regge Calculus (upper plots) and the Z_2-Regge Model (lower plots).

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