

The Ising transition in 2D simplicial quantum gravity - can Regge calculus be right?*

Christian Holm^a and Wolfhard Janke^{ab†}

^aInstitut für Theoretische Physik, Freie Universität Berlin, 14195 Berlin, Germany

^bInstitut für Physik, Johannes Gutenberg-Universität Mainz, 55099 Mainz, Germany

We report a high statistics simulation of Ising spins coupled to 2D quantum gravity in the Regge calculus approach using triangulated tori with up to 512^2 vertices. For the constant area ensemble and the dl/l functional measure we definitively can exclude the critical exponents of the Ising phase transition as predicted for dynamically triangulated surfaces. We rather find clear evidence that the critical exponents agree with the Onsager values for static regular lattices, independent of the coupling strength of an R^2 interaction term. For exploratory simulations using the lattice version of the Misner measure the situation is less clear.

1. INTRODUCTION

Ising spins coupled to two-dimensional (2D) Euclidean quantum gravity in the dynamically triangulated surface (DTS) formulation exhibit a third-order phase transition with critical exponents which are very different from the Onsager values for static 2D lattices. This follows from exact results for matrix models [1] and numerical simulations [2], and is in perfect agreement with predictions from conformal field theory for matter of central charge $c = 1/2$ in the continuum limit [3].

An alternative formulation is the Regge calculus [4] where the connectivity of the discretized surface is fixed but the link lengths vary. Recent Monte Carlo (MC) simulations [5] of pure 2D gravity were interpreted in accordance with the DTS results. When coupled to Ising spins, however, evidence for Onsager critical exponents was claimed [5]. Since this would be a severe failure of the Regge approach we decided to investigate this question again with much higher precision [6]. Finally it should be remarked that already the consistency of pure gravity with the dl/l measure was recently questioned in Ref.[7].

2. MODEL AND SIMULATION

Employing the usual transcription [8] of continuum quantities like the metric g and the scalar curvature R , the Regge partition function reads

$$Z = \sum_{\{s\}} \int \mathcal{D}\mu(l) \exp(-I(l) - KE(l, s)), \quad (1)$$

where

$$I(l) = \sum_i \left(\lambda A_i + a \frac{\delta_i^2}{A_i} \right) \quad (2)$$

is the discretized gravitational action and

$$E(l, s) = \frac{1}{2} \sum_{\langle ij \rangle} A_{ij} \left(\frac{s_i - s_j}{l_{ij}} \right)^2 \quad (3)$$

is the energy of Ising spins $s_i = \pm 1$, which are located at the $N = L^2$ vertices i of a triangulated torus with fixed coordination number $q = 6$. Here $\delta_i = 2\pi - \sum_{t \supset i} \theta_i(t)$ is the deficit angle and $A_i = \sum_{t \supset i} A_t/3$ is the barycentric area, where $\theta_i(t)$ denotes the dihedral angle at vertex i and A_t the area of a triangle t . Finally, $A_{ij} = \sum_{t \supset \langle ij \rangle} A_t/3$ is the area associated with a link $\langle ij \rangle$. In the first set of simulations we used the commonly employed simplest scale invariant functional measure $\mathcal{D}\mu(l) = \prod_{\langle ij \rangle} dl_{ij}/l_{ij}$. To investigate the dependence of our results on this choice we also performed exploratory simulations using the physically more motivated lattice version of the Misner

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Table 1
Critical parameters of the Ising transition.

a	K_c	U^*	$1/\nu$	γ/ν	β/ν	α/ν
0.000	1.0234(2)	0.609(33)	0.95(2)	1.717(6)	0.123(4)	-0.06(13)
0.001	1.0265(1)	0.612(5)	1.00(1)	1.735(5)	0.127(3)	-0.06(5)
0.100	1.0295(1)	0.615(6)	0.98(1)	1.728(3)	0.123(2)	-0.07(9)

measure $\mathcal{D}\mu(l) = \prod_{\langle ij \rangle} dl_{ij} / \sqrt{A_{ij}}$ [5]. Since both measures are scale invariant the total area is easily kept fixed at its initial value $A = \sum_i A_i = N$ by rescaling all link lengths when proposing a link update. In our simulations one MC step consisted of a single-hit Metropolis lattice sweep to update the link lengths l_{ij} followed by a single-cluster flip to update (a fraction of) the spins s_i . In each run we recorded the energy density $e = E/A$ and the magnetization density $m = \sum_i A_i s_i / A$ in a time-series file. The statistical errors are computed by the jack-knife method.

3. RESULTS

3.1. dl/l measure

Here we concentrated on finite-size scaling (FSS) analyses for the R^2 couplings $a = 0$, 0.001, and 0.1, and performed long simulations with about 50 000 measurements near the critical coupling K_c for various lattice sizes up to $L = 512$ [6]. We did not encounter any equilibration problems and the autocorrelation times of e and m turned out to be about 1–4 measurements for all lattice sizes. By applying standard reweighting techniques we first determined the maxima of the susceptibility, $\chi = A(\langle m^2 \rangle - \langle |m| \rangle^2)$, the specific heat, $C = K^2 A(\langle e^2 \rangle - \langle e \rangle^2)$, as well as of $d\langle |m| \rangle / dK$, $d \ln \langle |m| \rangle / dK$, and $d \ln \langle m^2 \rangle / dK$. This defines five sequences of pseudo-transition points $K_{\max}(L)$ for which the scaling variable $x = (K_{\max}(L) - K_c)L^{1/\nu}$ should approach a constant for large L .

The critical exponent ν can then be estimated from (linear) least square fits to the FSS Ansatz $dU_L/dK \cong L^{1/\nu} f_0(x)$ or $d \ln \langle |m|^p \rangle / dK \cong L^{1/\nu} f_p(x)$ to the data at the various $K_{\max}(L)$, where $U_L = 1 - \langle m^4 \rangle / 3 \langle m^2 \rangle^2$. By averaging these estimates we obtain the values given in Table 1 which are all compatible with the Onsager value of $\nu = 1$. Assuming thus $\nu = 1$ we next de-

termined K_c from fits to $K_{\max}(L) = K_c + c/L$. Another, more precise method is to analyze the crossing points K^\times of the curves $U_L(K)$ with L and $L' = bL$. Combining this information we obtained the values quoted in Table 1 which were used in further analyses such as the asymptotic limit U^* of $U_L(K_c)$. Within error bars our values in Table 1 agree with the very precise estimate for the regular square lattice, $U^* = 0.611(1)$ [9].

The exponent ratios γ/ν and β/ν follow from the FSS $\chi \cong L^{\gamma/\nu} f_3(x)$ and $\langle |m| \rangle \cong L^{-\beta/\nu} f_4(x)$ or $d\langle |m| \rangle / dK \cong L^{(1-\beta)/\nu} f_5(x)$, respectively. The final averages are again collected in Table 1. Also here we see little influence of the R^2 term. While our estimates for γ/ν are slightly below the Onsager value of 1.75, we obtain almost perfect agreement with the Onsager result $\beta/\nu = 0.125$. Finally we have checked that 3-parameter fits of the form $C_{\max} = A + BL^{\alpha/\nu}$ yield values of α/ν consistent with zero; compare Table 1. Similar fits of C at the other $K_{\max}(L)$ sequences as well as at K_c gave compatible results.

Since the R^2 interaction term does not affect the critical exponents we give in Table 2 as final estimates the weighted average of the three simulations with different γ coupling a . For the dl/l measure we definitely can exclude the DTS exponents and find strong evidence for the Onsager universality class. We conclude by mentioning that in the meantime we have also confirmed the failure of the Regge approach with dl/l measure already for pure gravity [10].

Table 2
Critical exponents of the Ising transition.

	α	β	γ	ν
DTS	-1	0.5	2	1.5*
Onsager	0	0.125	1.75	1
Regge	≈ 0	0.126(2)	1.75(2)	1.01(1)

3.2. Misner measure

In order to investigate the importance of the functional measure, we ran for the R^2 couplings $a = 0$ and 0.001 also simulations with the Misner measure. Using the same analysis techniques as before we observed indeed a qualitatively *completely* different behaviour and thus a strong dependence on the measure. In particular the critical couplings K_c are shifted now to much smaller values around $K_c \approx 0.4$, which might be taken as an indication that with the Misner measure we are much further away from the flat triangular case (where $K_c = \sqrt{3} \ln 3/2 \approx 0.9514$).

The autocorrelation times are now extremely long and in some cases we cannot even be sure that the system has equilibrated at all. We observe long-lived metastabilities similar to what one expects, e.g., in spin glasses. It is not yet clear to us whether this is a generic feature of the Regge partition function with Misner measure or an artifact caused by our update procedure. As an example of this curious behaviour we show in Fig. 1 the distribution $P(\delta_i)$ of local deficit angles δ_i for $a = 0.001$ in the vicinity of K_c . While for the dl/l measure $P(\delta_i)$ is a smooth curve, for the Misner measure it is strongly peaked at zero and displays an extra peak near the maximal possible value of $\delta_i = 2\pi$. This means that in the Misner case the discretized surface is composed of large, almost flat and small, highly curved regions. We have the feeling that once a region of high curvature has formed it is very improbable to relax it again, which could explain the extremely slow dynamics observed in the simulations.

With this caveat in mind we have nevertheless analyzed the data for the Ising observables. As a result we obtain critical exponents which seem not to agree with the Onsager values but could possibly match the DTS values.

4. CONCLUDING REMARKS

Summarizing, we obtain strong numerical evidence that the phase transition in an Ising model coupled to the Regge calculus approach with dl/l measure belongs to the Onsager universality class. The Regge approach is, however, extremely sensitive to the choice of measure and, since first

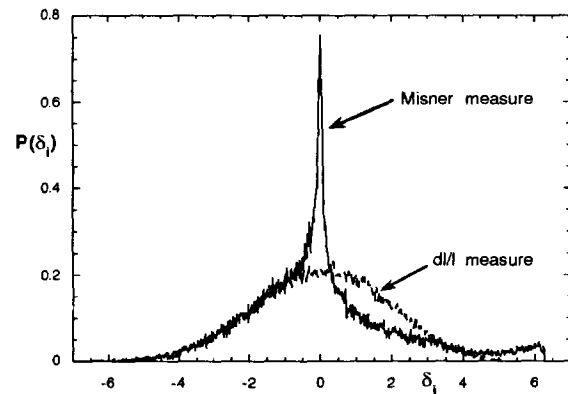


Figure 1. Distribution of local deficit angles.

analyses of simulations with the Misner measure do not exclude the DTS exponents, we still have the hope that the Regge approach can survive as an alternative tool to describe quantum gravity.

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