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Spins coupled to a Z_2-Regge lattice in 4d

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We study an Ising spin system coupled to a fluctuating four-dimensional Z_2-Regge lattice and compare with the results of the four-dimensional Ising model on a regular lattice. Particular emphasis is placed on the phase transition of the spin system and the associated critical exponents. We present results from finite-size scaling analyses of extensive Monte Carlo simulations which are consistent with mean-field predictions.

1. INTRODUCTION

Spin systems coupled to fluctuating manifolds are studied as a simple example for matter fields coupled to Euclidean quantum gravity. To describe the gravity sector we used the Discrete Regge Model [1] which is both structurally and computationally much simpler than the Standard Regge Calculus with continuous link lengths. Here numerical simulations can be done more efficiently by implementing look-up tables and using the heat-bath algorithm. In the actual computations we took the squared link lengths as $q_{ij} \equiv q_l = b_l(1 + \epsilon\sigma_l)$ with $\sigma_l = \pm 1$ and $\epsilon = 0.0875$. Because a four-dimensional Regge skeleton with equilateral simplices cannot be embedded in flat space, b_l takes different values depending on the type of the edge l. In particular $b_l = 1, 2, 3, 4$ for edges, face diagonals, body diagonals, and the hyperbody diagonal of a hypercube.

2. MODEL AND OBSERVABLES

We investigated the partition function

$$Z = \sum_{\{s\}} \int D[q] \exp[-I(q) - KE(q,s)], \qquad (1)$$

where $I(q)$ is the gravitational action,

$$I(q) = -\beta_g \sum_t A_t \delta_t + \lambda \sum_i V_i. \qquad (2)$$

The first sum runs over all products of triangle areas A_t times corresponding deficit angles δ_t weighted by the gravitational coupling β_g. The second sum extends over the volumes V_i of the 4-simplices of the lattice and allows together with the cosmological constant λ to set an overall scale in the action. The energy of Ising spins $s_i \in Z_2$,

$$E(q,s) = \frac{1}{2} \sum_{\langle ij \rangle} A_{ij} \frac{(s_i - s_j)^2}{q_{ij}}, \qquad (3)$$

is defined as in two dimensions [2], with the barycentric area A_{ij} associated with a link l_{ij}, $A_{ij} = \sum_{t \supset l_{ij}} \frac{1}{3} A_t$. We chose the simple uniform measure as in the pure gravity simulations [1], $D[q] = \prod_l dq_l \mathcal{F}(q_l)$. The function $\mathcal{F}$ ensures that only Euclidean link configurations are taken into account.

For every Monte Carlo simulation run we recorded the time series of the energy density $e = E/N_0$ and the magnetization density $m = \sum_i s_i/N_0$, with the lattice size $N_0 = L^4$. To obtain results for the various observables $\mathcal{O}$ at values of the spin coupling K in an interval around the simulation point K_0, we applied standard reweighting techniques [3].

With the help of the time series we compute the specific heat, $C(K) = K^2 N_0(\langle e^2 \rangle - \langle e \rangle^2)$, the (finite lattice) susceptibility, $\chi(K) = N_0(\langle m^2 \rangle - \langle |m| \rangle^2)$, the Binder parameter, $U_L(K) = 1 - \langle m^4 \rangle / 3 \langle m^2 \rangle^2$, and various derivatives of the magnetization, $d\langle |m| \rangle / dK$, $d \ln \langle |m| \rangle / dK$, and $d \ln \langle m^2 \rangle / dK$. All these quantities exhibit in the infinite-volume limit singularities at K_c which are

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shifted and rounded in finite systems.

3. SIMULATION RESULTS

In four dimensions it is generally accepted that the critical properties of the Ising model on a static lattice are given by mean-field theory, with logarithmic corrections. The finite-size formulas can be written as [4]

$$\xi \propto L(\log L)^{\frac{1}{4}}, \quad (4)$$

$$\chi \propto (L(\log L)^{\frac{1}{4}})^{\gamma/\nu}, \quad (5)$$

$$K_c(\infty) - K_c(L) \propto (L(\log L)^{\frac{1}{12}})^{-1/\nu}, \quad (6)$$

where the critical exponents of mean-field theory are $\alpha = 0$, $\beta = 1/2$, $\gamma = 1$, and $\nu = 1/2$.

The gravitational degrees of freedom of the partition function (1) were updated with the heat-bath algorithm. For the Ising spins we employed the single-cluster algorithm [5]. Between measurements we performed $n = 10$ Monte Carlo steps consisting of one lattice sweep to update the squared link lengths q_{ij} followed by two single-cluster flips to update the spins s_i.

The simulations were done for cosmological constant $\lambda = 0$ and gravitational coupling $\beta_g = -4.665$. This β_g-value corresponds to a phase transition of the pure Discrete Regge Model [1]. The lattice topology is given by triangulated tori of size $N_0 = L^4$ with $L = 3$ up to 10. From short test runs we estimated the location of the phase transition of the spin model and set the spin coupling $K_0 = 0.024 \approx K_c$ in the long runs.

After an initial equilibration time we took about 100 000 measurements for each lattice size. Analyzing the time series we found integrated autocorrelation times for the energy and the magnetization in the range of unity for all lattice sizes. The statistical errors were obtained by the standard Jack-knife method using 50 blocks.

Applying the reweighting technique we first determined the maxima of C, χ, $d\langle|m|\rangle/dK$, $d\ln\langle|m|\rangle/dK$, and $d\ln\langle m^2\rangle/dK$. The locations of the maxima provide us with five sequences of pseudo-transition points $K_{\max}(L)$ for which the scaling variable $x = (K_c - K_{\max}(L))(L(\log L)^{\frac{1}{12}})^{\frac{1}{\nu}}$ should be constant. Using this fact we then have several possibilities to extract the critical exponent ν from (linear) least-square fits of the FSS ansatz with logarithmic corrections (6),

$$dU_L/dK \cong (L(\log L)^{\frac{1}{12}})^{1/\nu} f_0(x), \quad (7)$$

$$d\ln\langle|m|^p\rangle/dK \cong (L(\log L)^{\frac{1}{12}})^{1/\nu} f_p(x), \quad (8)$$

to the data at the various $K_{\max}(L)$ sequences. We also performed fits of a naive power-law FSS ansatz. The exponents $1/\nu$ resulting from fits using the data for $L = 4-10$ are collected in Table 1. Q denotes the standard goodness-of-fit parameter. For our simulations all exponent estimates with the logarithmic corrections and consequently also their weighted average $1/\nu = 2.028(7)$ are in agreement with the mean-field value $1/\nu = 2$. With the naive power-law ansatz one also gets an estimate for $1/\nu$ close to the mean-field value, but clearly separated from it.

Assuming therefore $\nu = 0.5$ we can obtain estimates for K_c from linear least-square fits to the scaling behavior of the various $K_{\max}$ sequences, as shown in Fig. 1. Using the fits with $L \geq 4$, the combined estimate from the five sequences leads to $K_c = 0.02464(4)$.

Knowing the critical coupling we may reconfirm our estimates of $1/\nu$ by evaluating the above quantities at K_c. As can be inspected in Table 1, the statistical errors of the FSS fits at K_c are similar to those using the $K_{\max}$ sequences. However, here we have to take into account the uncertainty in our estimate of K_c. This error is computed by repeating the fits at $K_c \pm \Delta K_c$ and indicated in

Table 1
Fit results for $1/\nu$ in the range $L = 4 - 10$ with a power-law ansatz with logarithmic corrections.

fit type	$1/\nu$	Q
dU/dK at $K^C_{\max}$	1.980(17)	0.70
$d\ln\langle|m|\rangle/dK$ at $K_{\inf}^{\ln\langle|m|\rangle}$	2.032(10)	0.59
$d\ln\langle m^2\rangle/dK$ at $K_{\inf}^{\ln\langle m^2\rangle}$	2.038(10)	0.55
weighted average	2.028(7)	
dU/dK at K_c	1.981(17)[13]	0.70
$d\ln\langle|m|\rangle/dK$ at K_c	2.027(9)[2]	0.95
$d\ln\langle m^2\rangle/dK$ at K_c	2.034(9)[2]	0.85
weighted average	2.025(6)	
overall average	2.026(5)	

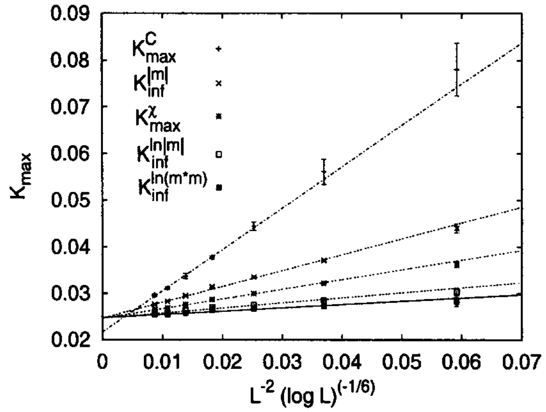

Figure 1. FSS extrapolations of pseudo-transition points $K_{\max}$ vs. $(L(\log L)^{\frac{1}{12}})^{-1/\nu}$, assuming $\nu = 0.5$. The error-weighted average of extrapolations to infinite size yields $K_c = 0.02464(4)$.

Table 1 by the numbers in square brackets. In the computation of the weighted average we assume the two types of errors to be independent. As a result of this combined analysis we obtain strong evidence that the exponent ν agrees with the mean-field value of $\nu = 1/2$.

To extract the critical exponent ratio γ/ν we use the scaling (5) of the susceptibility χ at its maximum as well as at K_c, yielding in the range $L = 4 - 10$ estimates of $\gamma/\nu = 2.039(9)$ $(Q = 0.42)$ and $\gamma/\nu = 2.036(7)[4]$ $(Q = 0.85)$, respectively. These estimates for γ/ν are consistent with the mean-field value of $\gamma/\nu = 2$. In Fig. 2 this is demonstrated graphically by comparing the scaling of $\chi_{\max}$ with a constrained one-parameter fit of the form $\chi_{\max} = c(L(\log L)^{\frac{1}{4}})^2$ with $c = 4.006(10)$ $(Q = 0.17,\ L \geq 6)$.

4. CONCLUSIONS

We have performed a study of the Ising model coupled to fluctuating manifolds via Regge Calculus. Analyzing the Discrete Regge Model with two permissible edge lengths it turns out that the Ising transition shows the expected logarithmic corrections to the mean-field theory. We have also studied the pure Ising model on a rigid lattice without presenting the results in this short note. The critical exponents of the phase transition of the Ising spins on a static lattice as well as on a discrete Regge skeleton are both consistent with the exponents of mean-field theory, $\alpha = 0$, $\beta = 1/2$, $\gamma = 1$, and $\nu = 1/2$. In summary, from our comparative analysis with uniform computer codes we conclude that the phase transition of the Ising spin model coupled to a discrete Regge skeleton exhibits the same critical exponents and the same logarithmic corrections as on a static lattice.

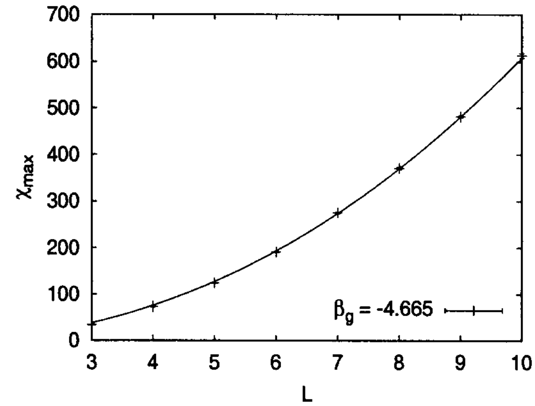

Figure 2. FSS of the susceptibility maxima $\chi_{\max}$. The exponent entering the curve is set to the mean-field value $\gamma/\nu = 2$ for regular static lattices.

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